

Advance Mathematical Models for Cosmological Problems: A Comprehensive Study

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Abstract

The work offers a very inclusive theoretical examination of the sophisticated mathematical models functional in the study of cosmological problems. The paper distillates on anisotropic cosmologies, reformed gravity, undercurrents of the scalar field, particle formation theories, and formulations of the dark energy, and establishes a wide category of cosmological models, utilizing the field equations of Einstein, Brans -Dickes theory, Lyra geometry, Chaplygin gas models, and different varieties of the dark energy. The article produces detailed cosmological solutions to various situations and examines the physical soundness of these clarifications and dissimilarities model forecasts with persisting astronomical data.

1. Introduction

The paper starts by presenting a summary of the cosmological principles, which is the shift of classical cosmology to relativistic cosmology through the current advancements that include the Cosmic Microwave Background Radiation (CMBR) and the large-scale arrangement conclusions. The predictable Big Bang Model (SBBM) is a efficacious model but has weaknesses in amplification complications of horizons, flatness and distinctiveness. To incredulous these shortcomings, the research work uses anisotropic models, inflationary cosmology and modified gravity backgrounds.

2. Mathematical Framework and Bianchi Cosmologies

The research work builds cosmological elucidations in Bianchi-type cosmoses, a spatially homogeneous nonetheless anisotropic universe. These are representations that are allowances of the Friedmann-Robertson-Walker (FRW) model, certification both rotational and shear crescendos in early cosmology, and simplify drawing insinuations of structure-formation.

$$ds^2 = -dt^2 + G_{a,b}\omega^{a,b}$$

These metrics are spatially dependent on time factors.

$$d\omega^a = -[C_{bc}^a\omega^b \wedge \omega^c]/2$$

3. Einstein Field Equations with Variable Λ and G

The momentous route of the research work is the study of cosmological facsimiles in the context of adjustable gravitational continual (G) and cosmological persistent (Λ). The author demonstrates that any replicas where $\Lambda(t)$ is declining in time and $G(t)$ is cumulative give natural explanations of cosmic acceleration, age fortitudes and structure creation.

Major assumptions:

$$\mathfrak{G}_{ij} = 8\left(\prod \varphi^{-1}\right)\mathfrak{t}_{ij} + \frac{1}{\theta}(\theta_{ij} - \mathfrak{g}_{ij}) - \frac{v}{2\theta}\mathfrak{g}_{ij}$$

4. Brans–Dicke Gravity and Scalar-Field Cosmology

It gives substantial attention to the analysis of the BransDicke theory, a scalar-tensor irregular of General Relativity, in the research work. Accurate clarifications of Bianchi models in BD gravity are attained representing that BD models can maintenance different gravitational strengths and accelerated expansion periods.

The scalar tensor gravitation theory evaluated by space metric influence as:

$$s = \int \left\{ \mathfrak{g} r - \omega * \left(\frac{\Delta}{\phi}\right) \phi K_{ij} - v(\phi) \right\} d^4x$$

5. Particle Creation and Bulk Viscosity

There is a study of cosmological models involving matter creation and bulk viscous pressures. Bulk viscosity iron the initial anisotropies, and the form of particles simulates dark-energy-like behavior, fabricating physically realistic early-universe histories.

In projected cosmology, the universe is exhibited as a perfect fluid. When element creation or bulk viscosity is extant, the cosmic fluid becomes non-equilibrium, fabricating a improved effective compression.

The basic Einstein field compilation for an FLRW universe are:

$$3h'^2 = 8(\pi)g * \rho,$$

$$2\dot{h}' + 3h^2 = -8 \pi g * p_{EFF},$$

For \dot{h}' is the Hubble parameter and p_{EFF} as the effective pressure.

Bulk goeyness model with pressure in negative presentation:

$$p_{vsc} + \mathfrak{p} = p_{EFF}$$

For $p_{vsc} = -3\kappa h$

κ – Constant of bulk viscosity.

Preservation equation for energy:

$$\kappa = \kappa_0 + \kappa_1 * h + \kappa_2 * h'$$

6. Dark Energy Models and Chaplygin Gas Cosmology

The work of the research contains the analysis of the models of the dark energy using Chaplygin Gas, the Modified Chaplygin Gas, the Holographic Dark Energy (HDE) and the time-varying Equation of State (EoS) parameters.

$$\rho = \mathcal{B}p - \frac{\dot{A}}{\rho^a}$$

These models are able to explain accelerating universe at late times and matter dominated universe at early times.

7. Main Findings and Contributions

- Development of a large class of anisotropic cosmological models Zero potential: $\nabla\phi = 0$

- Construction of exact cosmological solutions across multiple gravity theories Zero potential: $\nabla\phi = 0$
Large convolution: $\varpi \rightarrow \infty$
- Advancement of variable G and Λ cosmology
- Integration of particle creation and viscous effects
- Analysis of dark energy with time-variable EoS
- Comparative analysis with SNIa observations Potent inflation drive: $\nabla(\varphi) = \nabla\phi_o(\varphi)^k$

Conclusion

The study gives a detailed and mathematically sound study of non-standard cosmological models that are not in the framework of FRW. The models made are consistent, observable and can solve long-standing cosmological issues.

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