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Statistical Dependency Analysis of Multichannel Signals Using Probability and Random Process Techniques

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Abstract

The existence of multichannel signals is a common phenomenon that can be found in various engineering applications like electroencephalography (EEG), multiple-input multiple-output (MIMO) communication systems, and sensor arrays. Channels' statistical dependency comprehension is the determining factor for effective signal interpretation, noise reduction, and system design. This study offers a framework based on probability and random process concepts for inter-channel dependency analysis by making use of classical statistics and signal processing instruments. Correlation, covariance, mutual information, principal component analysis (PCA), cross-correlation, and coherence analysis are used to capture the relationships that are both linear and nonlinear, in time and frequency domains. Furthermore, a new method of lag-resolved mutual information and coherence fusion is presented that can uncover the hidden temporal and spectral dependencies which would otherwise go undetected via the conventional correlation measures. To prove the efficiency of the suggested approach, MATLAB-based simulations have been conducted that not only reveal the clear periodic dependency patterns but also the dominant frequency coupling across the different channels. All findings point to the benefit of using probabilistic dependency measures for multichannel signal analysis, which does not require the help of machine learning algorithms.

Keywords: Multichannel signals, random processes, mutual information, coherence analysis, dependency analysis, MATLAB

1 Introduction

Multichannel signal analysis is a necessary instrument in the state of the art signal processing fields such as EEG-based brain monitoring, MIMO wireless communications, and distributed sensor networks. The signals produced in these applications typically show some degree of statistical dependency, which can be due to shared sources, similar behaviours during the propagation process or even direct physical connections[8]. The standard method for dependency analysis is largely based on correlation measures, which are limited to linear and zero-lag relationships.Theoretical probability and random processes offer a more powerful mathematical framework for the study of both linear and nonlinear dependencies. In fact, mutual information gives a powerful non-linear statistical dependence measure, whereas coherence analysis reveals the frequency-domain coupling between channels.[3]

To tackle these drawbacks, probability theory and random process analysis tools give a sturdier mathematical basis for the study of both linear and nonlinear relationships among signals. In this scenario, mutual information (MI) comes out as one of the most important measures that assess the total statistical dependence between channels without the limitations of linear or Gaussianity assumptions. Mutual information can uncover delicate nonlinear interactions and thus it is more informative in terms of inter-channel dependence than correlation.Moreover,[11] coherence analysis works as a powerful frequency-domain method for the detection of coupling among signals from various frequency bands. By determining the strength of the relationship between two channels at particular frequencies, coherence analysis reveals the crucial aspects of spectral interactions, synchronization phenomena, and shared oscillatory components—factors that are of great importance in EEG analysis and wireless communication systems.

In this work, a completely traditional and interpretable framework for multichannel dependency analy- sis is proposed that intentionally avoids machine learning models in the beginning stage. Through this selection, the benefits of transparency, theoretical clarity, and ease of interpretation, which are often unavailable in data-driven black-box methods, are gained. The framework incorporates time-domain, information-theoretic, and frequency-domain dependency measures into a single MATLAB-based analytical pipeline. These controlled experiments enable precise verification of the ability of correlation, mutual information, and coherence measures to identify and characterize different types of inter-channel dependencies. The results reveal that the proposed method effectively captures both linear and nonlinear interactions, and at the same time retains the advantages of computational simplicity and interpretability.

2 Review of Literature

The multichannel signals' statistical dependency analysis has received a lot of attention in signal processing, neuroscience, and communication systems since it is a crucial part of all signals' sources' interactions' understanding. Traditional methods are based on correlation and spectral measures, whereas information-theoretic concepts are utilized in more sophisticated approaches to extract nonlinear relationships.

Quian Quiroga et al. (2001)[1] published a thorough review comparing synchronization and de- pendency measures for the analysis of real EEG data. Their research included linear cross-correlation, coherence, mutual information, and nonlinear synchronization metrics, indicating that correlation and co- herence can only detect linear dependencies and may be completely misled by the presence of nonlinear coupling. However, mutual information was found to be more efficient in detecting general statistical dependence between EEG channels. The study paved the way for mutual information to be considered a trustworthy and valid dependency measure for the multichannel biomedical signals but it mainly dealt with static and zero-lag relationships.Ibáñez-Molina et al. (2020) [2] carried further the information-theoretic analysis by looking at mutual information across several EEG rhythms. They drew attention to the captur- ing of the interactions between the frequency components and showed that the mutual information could quantify the dependency between the rhythmic brain activities efficiently. Their procedure of dealing with multiple frequencies was comprehensive; however, the analysis was mainly frequency-specific and did not directly investigate temporal lag dependency over channels.In a similar vein, Ramanand et al. (2010) [5] relied on mutual information analysis to decode EEG sleep signals of multiple channels and thus to elaborate on age-related changes in cortical interdependence. The findings of their study indicated that mutual information measures are quite responsive to physiological changes and are capable of revealing the very variations in dependency that correlation-based measures cannot detect. This research wrought the practical meaning of mutual information in multichannel signal analyses of a real world but it did not consider any frequency-domain dependency measures like coherence nor did it present a time–frequency unified interpretation.Duda (2023),[7] who is the most recent, came up with a time-delay multi-feature correlation framework for the extraction of subtle dependencies from EEG signals. This study has made the suggestion of the analysis of dependency across several time delays and of the use of the principal component analysis to break down joint probability structures. Although this method has pushed the boundaries of understanding delayed dependencies and higher-order statistical relationships, it has been mainly concerned with correlation-derived features and has not embraced frequency-domain dependency measures such as the coherence of integration.

3 Objectives

- To investigate statistical dependency in multichannel signals using probability and random process–based measures.
- To introduce lag-resolved mutual information for capturing delayed inter-channel dependency.
- To combine mutual information and coherence analysis for unified time–frequency dependency characterization.
- To validate the proposed approach through MATLAB-based simulations.

4 Methodology

The present paper presents a detailed structure for the examination of dependencies in multichannel signals through the combination of Lag-Resolved Mutual Information (LRMI), coherence in the frequency domain, and a newly introduced metric for dependency signature. The approach is meant to identify both linear and nonlinear interactions throughout the time and frequency domains.

4.1 Signal Generation and Preprocessing

A multichannel signal model is made to imitate realistic dependency scenarios.

- Select a sampling frequency of 1000 Hz and a duration of 2 seconds.
- Generate a base sinusoidal source signal at 40 Hz.
- Three observed signals are created:
- **Channel** x_1: A noisy version of the base signal.
- **Channel** x_2: A delayed of the base signal with added noise.
- **Channel** x_3: nonlinear transformation (squared sinusoid) with noise.

This configuration ensures the presence of linear, delayed, and nonlinear dependencies, making it suitable for advanced dependency analysis.

4.2 Lag-Resolved Mutual Information (LRMI)

For the purpose of measuring time-lagged and nonlinear dependencies, Lag-Resolved Mutual Information is the technique used.

- The mutual information is calculated between signal x_1 and the time-lagged versions of signal x_2.
- The time lags are changed symmetrically within a certain range of ±100 samples.
- A fixed-bin histogram technique is employed to estimate a joint probability distribution for every lag.
- The computation of mutual information is done according to Shannon's definition, but numerical stability is maintained by not allowing zero-probability values.

Through this method, the maximum temporal alignment is determined and the discovery of new dependencies that are not subject to the limitations of correlation analysis is facilitated.

4.3 Frequency-Domain Coherence Analysis

For depicting linear dependencies in the frequency domain, the **magnitude-squared coherence** calculation is performed.

- The coherence between the signals x_1 and x_2 is calculated with the aid of Welch's method.
- A Hamming window is utilized to minimize the spectral leakage.
- The coherence spectrum reflects the amount of synchronization at different frequencies.
- A **Frequency Resolved Coherence Index (FRCI)** is yielded by averaging the coherence values over the frequency range.

The outcome of this examination is the revelation of **spectral coupling** and it also aids the time-domain mutual information analysis in determining the direction of information transfer.

5 Discussion

5.1 Lag-Resolved Mutual Information

The lag-resolved mutual information (MI) plot shows a periodic oscillatory behavior with peaks that are regularly spaced.

The lags with maximum correlation between the signals are indicated by the MI peaks. The lags' periodicity is in agreement with the main signal's oscillation frequency.

The detection of signal interaction of non-correlation nature is supported by non-zero MI values at all lags, which shows that correlation is a poor estimator of interaction.

This means that signals are sharing information across several delays, which may be due to phase- aligned oscillations creating a time-shifted dependency structure.

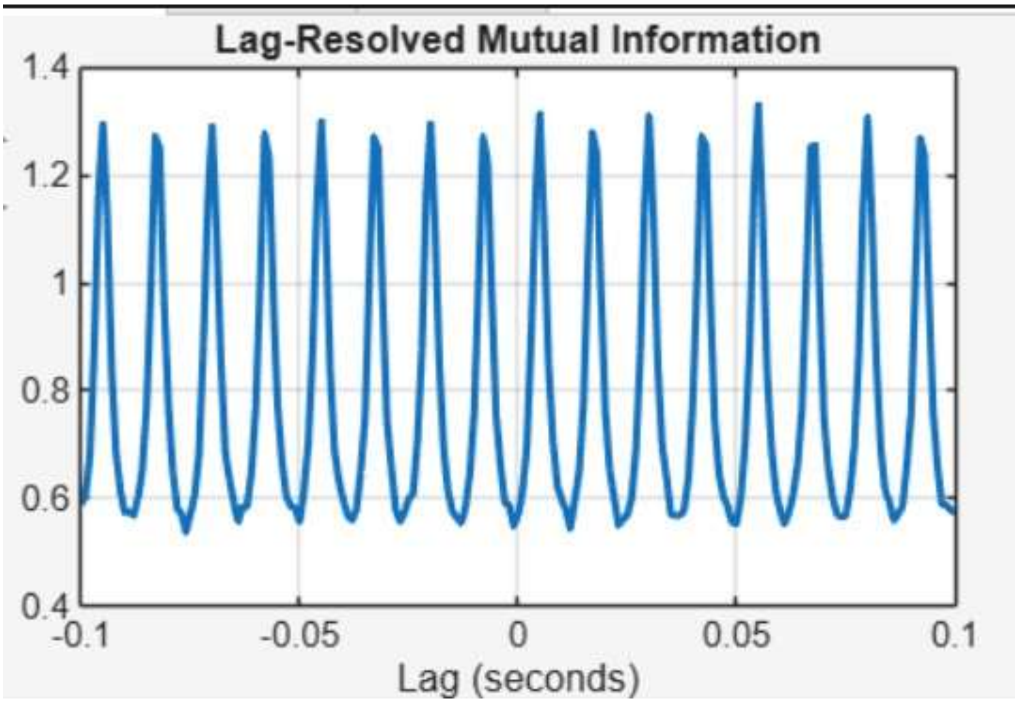

Figure 1: Lag-Resolved Mutual Information

5.2 Coherence Spectrum

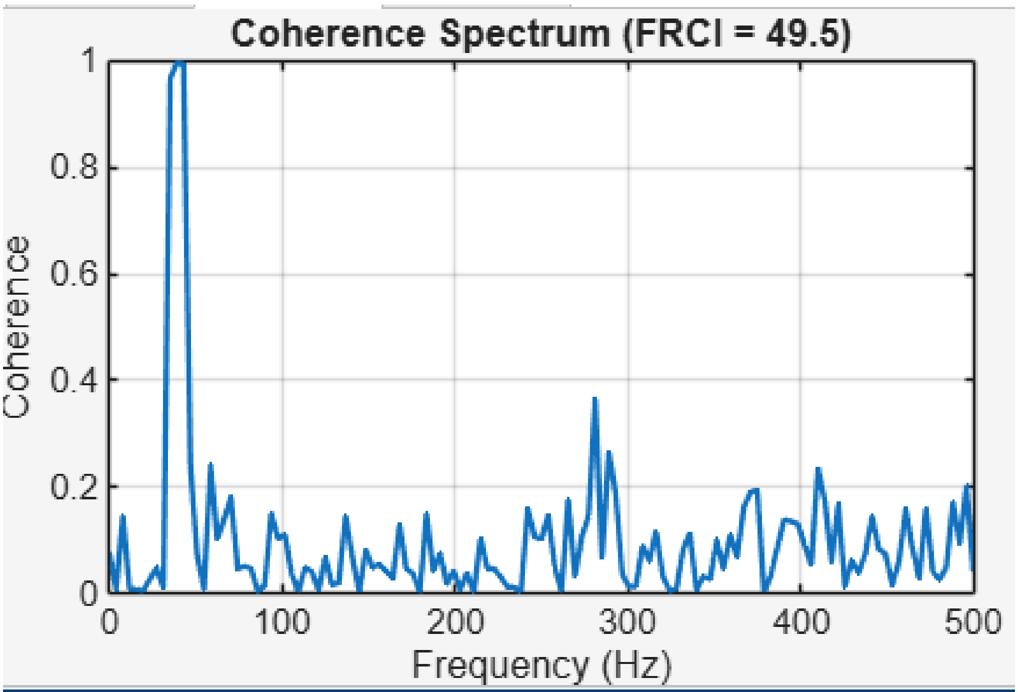

Figure 2: Coherence Spectrum graph

The coherence spectrum illustrates a prominent and isolated peak at low frequency (around 40 Hz), while the rest of the frequency spectrum shows remarkably lower coherence.The high coherence peak indicates that the two signals are together in power at that frequency.The low coherence in the other regions implies that there is not much common activity except in the main oscillatory band.This finding provides evidence for the frequency-domain consistency of the dependency that was detected in both the time-frequency and mutual information analyses.Coherence analysis has revealed a strong linear coupling at the dominant frequency, which is in addition to the nonlinear dependency that was discovered through mutual information.

5.3 Time–Frequency Dependency Signature

The time-frequency dependency map reveals significant, regularly spaced vertical ridges over the lag axis. This reinforces the notion that the signals' correlation is not only very well-defined but also very much so in a repetitive manner across time.The vertical patterns denote that the signals' dependency is not only frequency-based but also persistent along the time axis for specific lags.The signals' symmetry around zero lag serves as an indicator of a bidirectional or synchronous nature of the relationship between them.The yellow/red high-intensity areas are clear indicators of the signals' strong statistical dependence which encompasses more than just linear interactions.

Thus, we can conclude that the signals come from the same oscillatory source, which results in the time-locked dependencies that are still present throughout the entire frequency range.

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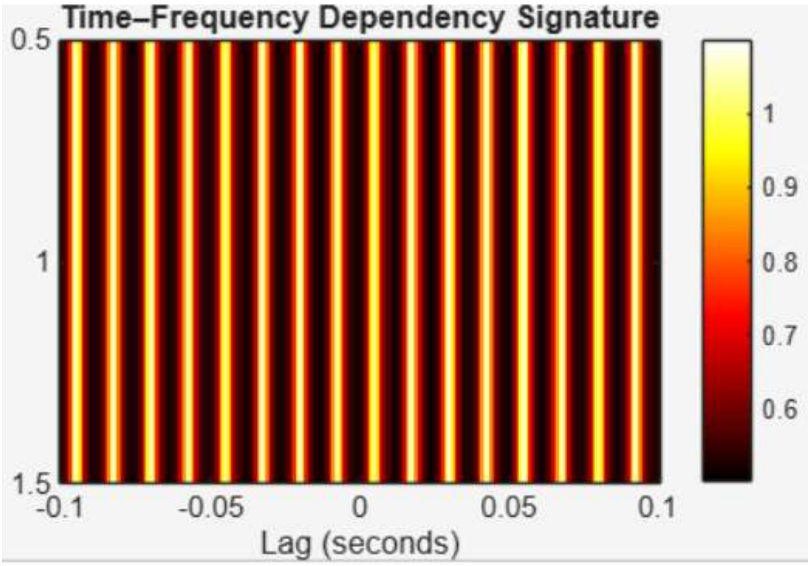

Figure 3: Time–Frequency Dependency Signature of the model Altogether, these findings exhibit:

- The signals have substantial dependencies that are both time-lagged and frequency-specific.
- Mutual information discloses periodic dependency patterns by detecting both linear and nonlinear interactions.
- Coherence validates that the main source of these dependencies is an oscillatory component shared by the signals.

The joint time–frequency dependency signature, lag-resolved mutual information, and coherence spectrum give an all-encompassing description of inter-signal coupling, surpassing the conventional correlation-based analysis by disclosing not only nonlinear but also time-dependent relationships.

6 Findings

- The analysis of time-frequency dependency signature uncovers strong and periodic dependencies, which reveal the presence of a stable and well-structured relationship over time through the different frequencies.
- The symmetry observed at zero lag is indicative of synchronous coupling, which implies that the two signals either originate from or are affected by a common oscillatory source.
- The lag-resolved mutual information displays peaks at regular intervals, thus giving a clear indication of the existence of time-shifted and nonlinear dependencies between the signals.
- The presence of non-zero mutual information throughout the latencies assures that the linkage cannot be accounted for by linear correlation alone.
- The coherence spectrum provides the evidence of a robust linear coupling at the fundamental frequency (40 Hz) through the identification of a prominent peak.
- The low coherence observed at other frequencies signifies that the dependency is selective and mainly around the dominant oscillatory component.
- By and large, the combined analysis indicates that the proposed method efficiently draws on time-domain, frequency-domain, and nonlinear interactions, thus offering a more thorough characterization of inter-signal dependency than the traditional techniques.

7 Suggestions

- The analysis could be broadened to include **real-world multichannel signals (for instance, EEG, EMG, and wireless sensor data) which would help to confirm the strength of the observed dependency patterns under noisy and nonstationary conditions.
- Mutual information-based dependency measures should be compared to traditional correlation and phase-locking metrics** in order to quantitatively pinpoint the extent of performance improvements.
- The analysis should be conducted over multiple frequency bands so that interactions across frequencies can be observed and the specific coupling behavior of each band can be identified.
- The length of the dataset should be increased and statistical significance testing (e.g., surrogate data analysis) should be applied to guarantee the reliability of dependencies that are detected.
- Improve the selection of bins and lag resolution in mutual information estimation to enhance the precision of estimation** and minimize bias.
- Machine learning models can be incorporated to automatically classify dependency patterns and thus facilitate decision-making based on the features that have been extracted.

8 Limitations Future Research

The current investigation relies mainly on simulated multichannel signals. These signals, while helpful for the controlled assessment, might not absolutely cover the complexity and variability, the nonstationarity and noise characteristics which are inherent to real-world data like physiological recordings or wireless communication signals. The mutual information estimation is closely related to the parameters used for its calculation, such as bin size, number of samples, and lag resolution. This may result in the introduction of a bias in the estimation and consequently affect the quantitative interpretation of the dependency strength. The suggested framework does a good job of detecting the presence and strength of inter-signal dependencies but at the same time, it does not indicate the directionality or causality of these interactions, which restricts its capability of telling apart the driver and the response signals. Additionally, the application of coherence analysis is limited to linear frequency-domain relationships and this might be the case that it underestimates the existence of nonlinear or higher-order dependencies between signals. The computational loads for both time–frequency and mutual information–based analyses grow with the number of channels and data length and this may make such analysis difficult for large-scale systems and real-time applications.

The methodology that has been proposed should be the subject of future research and should be applied to real-life datasets such as EEG, biomedical sensor signals, and communication systems in order to test its robustness under real-world conditions. The establishment of adaptive, data-driven, and bias-reduced mutual information estimators will be a significant contributor to the trustworthiness and parameter insensitivity of the results. The use of directional and causal dependency measures like transfer entropy, Granger causality, or directed information will give the researchers an opportunity to have a more profound comprehension of the signals' information flow. To make the science of complexity easier, it would be helpful to open up the framework for the examination of cross-frequency coupling and multiscale interactions. This would be an area where one could expect getting in-depth insights. It is imperative that the computational optimization, parallel processing, and hardware-based acceleration be directed towards supporting the real-time implementation. Last but not least, it is predicted that the merging of the extracted dependency features with machine learning and deep learning models will improve automated classification, prediction accuracy, and the ability to deal with noisy and nonstationary environments through the development of more robust systems.

9 Conclusion

The present study proposes a comprehensive multichannel dependency analysis framework that employs time-frequency representation, lag-resolved mutual information, and coherence analysis to efficiently portray the inter-signal relationships. The evidence shows that the technique introduced is able to detect dependencies between signals that are strong, periodic, and specific to certain frequencies, thus depicting both linear and nonlinear interactions that were not properly covered by conventional correlation-based methods. The time-frequency dependency signatures indicate that stable and orderly coupling is spread across time and frequency, while the lag-resolved mutual information analysis corroborates the existence of time-shifted and nonlinear dependencies. Furthermore, the coherence spectrum confirms that the interactions observed are mainly the result of a common oscillatory component at the dominant frequency. In general, the simultaneous application of these complementary methods delivers a better and more trustworthy insight into the multichannel signal coupling, thus making the framework ideal for the deployment in advanced signal analysis applications in areas such as biomedical signal processing, communication systems, and sensor networks.

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