

Study of LCDM: Assumptions of FLRW

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ABSTRACT:

The expansion rate of the Universe has been a debatable discussion among all minds and with the establishment of the homogeneity model and retrieval of CMB data, several equations and metrics have been derived, which compute the expansion rate of the Universe with an assumption of homogeneity and isotropy throughout. However, it is also notable that the scalability of the homogeneity cannot be validated overall on a small scale. Hence, concepts have been used to establish metrics that help in computing the expansion rate of the Universe by assuming heterogeneous frames and large scaling. A thorough review of isotropy and its deduction to homogeneity is done in this further to check on the scalability of the expansion rate of the Universe in the homogeneity scale. Besides, an assumption that if the Copernican assumption is denied for a case and the Universe is considered to be heterogeneous, what would be the expansion rate of the Universe then and if it is valid then what changes do we need to make in the current cosmological model LCDM are noted and mentioned. We currently have a semi-proven cosmological model which has proven its existence observationally to a huge extent but needs enough mathematical proof to confirm the observations. We take note of the assumptions and based on them we proceed further in concluding all the research areas that require more work to make this a mathematically stable model completely. The main focus of this paper will stick to the assumptions needed for the FLRW metric and further works on the same will come up.

KEYWORDS: Modern Cosmology, Expansion rate, Cosmological Principle, Relativity, Isotropy, Homogeneity

1. INTRODUCTION:

The Standard Cosmological Model (LCDM) that holds the backbone of Modern Cosmology has certain assumptions which have been considered to retain the properties of dark matter and dark energy fields, FLRW geometric isotropy and validation of the Cosmological Principle. However, a significant limitation of this model is that it is suitable for large-scale observations. This model, relying on Einstein's General Theory of Relativity, led to the establishment of the Friedmann-Robertson-Walker model that justifies the Big Bang theory. This further supports the Friedmann-Lemaître (FL) equation that relates the expansion rate of the Universe to its density and curvature. This particular equation is valid for scales larger than the order of 100Mpc. It is represented as:

$$H^2 = \frac{8\pi G \rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (1.1)$$

Here, H is the Hubble parameter denoted by $\frac{\dot{a}}{a}$ where $a(t)$ is the scale factor, ρ_m is average mass density, k is the curvature term and Λ is the cosmological constant representing the energy in the vacuum. $a(t)$ has

different proportions with time in different scale-dominations. In the matter-domination scale, $a(t) \propto t^{\frac{2}{3}}$ and in the radiation-domination scale, $a(t) \propto t^{\frac{1}{2}}$. Since this model supports the Big Bang theory, the mentioned scales are believed to be equal at matter-radiation equality which was approximately 60,000 years after the Big Bang. At the present epoch, the expansion rate is given at a value of H which is specified as H_0 and is known as the Hubble constant. L is a value which comes from experiments that are independent of the Λ CDM model. The origin of the cosmological constant is a link from the circular property of a metric in Euclidean space-time. It was added in GR by Einstein to break the frame of a static Universe as in the former model, $a(t)$ was constant and the age of the Universe was infinite. In this particular assumption, to make the density $\rho_0 > 0$, k must be $+1$. This in turn made the pressure $p_0 < 0$ and concluded the Universe to be contracting. Due to this, Einstein's equation was modified that introduce the Einstein tensor G_{mn} and supported a static Universe. The modified equation had an addition of a constant known as the Lorentz invariant.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (1.2)$$

This modification was considered a “blunder” by Einstein as this established the Universe to be static. Hubble, however, showed the Universe is expanding. The cosmological constant was henceforth used to modify the classical Newtonian gravitational constant G and used in other relatable energy terms such as Vacuum energy. However, the current expression L comes from manipulation of (1.2) where it appears as equivalent mass-energy density ρ_L and is represented by a dimensionless parameter W_L which is defined by

$$\rho_\Lambda \equiv \Omega_\Lambda \rho_c, \quad \Omega_\Lambda = \frac{c^2 \Lambda}{8\pi G \rho_c} \quad (1.3)$$

Along with (1.3), we use L that is derived from Euclidean Resonance w_n and that gives us the formation of cosmological constant which is currently used in the Λ CDM model and is given by:

$$\Omega_\Lambda = \frac{2}{3} \left[\frac{\pi}{x_u(\Omega_\Lambda)} \right]^2 \quad (1.4)$$

The dimensionless function x_u represents the conformal age of the Universe in Hubble time and is given by $\frac{t_u}{t_H}$. These metrics are based on a number of assumptions where there has been a significant need of a modification in the existing models. The aim of this paper is to note the assumptions that exist on the current model of the universe and how is it affecting the FLRW metric. Conclusions will be made on possible areas of incomplete research that can be done to verify the Λ CDM model and incomplete mathematical proofs that can be completed to make this a more stable model.

2. ASSUMPTIONS ON Λ CDM; THE STANDARD COSMOLOGICAL MODEL:

The present-day Lambda Cold Dark Matter Concordance Model assumes that the Big Bang theory is valid. Besides, it is assumed that the Big Bang theory created the Universe with pure energy. The state of the Big Bang is formulated when the time approaches zero and radiation energy density approaches infinity.

This particular singularity is termed as ‘Big Bang’. The Hawking Singularity Theorem explains the Big Bang. According to this theory, we consider a point in the past and consider the longest timelike curve passing through that point to the present time. Setting a lower bound to the length of the curve to the order of the expansion rate of the Universe can fetch an obvious-existing conjugate point, hence, implying singularity. Still, there is no Mathematical proof of the Big Bang theory to date. In contrary, the Planck spectrum of CMB have a black-body form as recorded by the FIRAS of COBE telescope. The CMB temperature through the spectrophotometer was found to be 2.726 ± 0.010 kelvin for a wavelength range of 5×10^{-4} to 5×10^{-3} metres, and in this range, the rated rms value of deviation from a black-body is approximately 0.01%. This particular phenomenon is possible only when there is high enthalpy involved in the creation of the photons and this is the exact model of Big Bang. Focusing on anisotropic condition, there are primary fluctuations that give rise to density inhomogeneities, which become the fundamental structures of the large-scale Universe. The recombination of the photon-baryon fluid takes almost 70,000 years and as the photons and baryons follow decoupling, the number density falls rapidly hence making the rate of Thomson scattering ($G_{T,e}$) below expansion rate (H). This variation is termed as the *primary fluctuation* as termed above. The redshift of photon decoupling z_{dec} is calculated for observing the fluctuations. z_{dec} is dependent on the Saha equation, which assumes $H + \gamma \rightleftharpoons p + e^-$ to be in equilibrium which fails in the case of decoupling where $G_{T,e}$ drops below H and the photon-baryon fluid remains overionized for that particular temperature to hold equilibrium. These primary fluctuations are imprinted on the surface of the last scattering at z_{dec} . During decoupling, the photon-baryon fluid stops oscillating and the baryons start emitting photons. During this point, the pattern of maxima and minima in density is frozen and preserved in the temperature fluctuations. It is seen that, in a dark matter potential well, if photon-baryon fluid is present and undergoes maximum compression at the time of photon decoupling, the energy density (u) of the fluid is higher than average, and the emitted photons have a higher temperature than average. The energy of photons based on blackbody photons is given by

$$u = a(T)^4 \quad (2.1)$$

The mode of extrema of oscillations becomes peak in CMB power spectrum which is known as Doppler Peak. Since this supports the blackbody radiation, this indirectly validates the Singularity theorem and hence is believed to validate the Big Bang. Another factor that validates the Big Bang is the local distribution of galaxies. As per the current model, galaxies are a result of density fluctuations. For this, we need to see the existence of a galaxy at a particular scale of the Universe for a given redshift. Although the scale factor cannot be directly determined, one can relate it to the wavelength of a distant emitted photon. The ratio of the wavelength of the photon at a cosmic time t to the scale factor of the Universe at that time is always equal to the ratio of the wavelength of a reference frame and the scale factor at that particular time of reference. This is given by

$$\frac{\lambda(t)}{a(t)} = \frac{\lambda(t_0)}{a(t_0)} \quad (2.2)$$

Here t_0 denotes the cosmic time of the reference frame. Redshift z in turn is defined concerning the observed and reference wavelengths as

$$1+z \equiv \frac{\lambda(t)}{\lambda(t_0)} \tag{2.3}$$

From (2.2) and (2.3), we can define the scale factor at cosmic time t as

$$a(t) = \frac{a(t_0)}{1+z} \tag{2.4}$$

However, it is noted that, for convenience, $a(t_0)=1$ is considered, which transforms $a(t)$ to be $\frac{1}{1+z}$ and hence creates an inverse proportion of redshift with scale factor. If we consider the distribution of galaxies, in a particular redshift, multiple galaxies exist but with the expansion of the Universe, the radius of galaxies remains constant, however, the distances between galaxies increase over time which can be computed using Hubble’s Law that provides recession velocity of a galaxy and is given by H_0d where d is the distance between the observer and the galaxy. This is the point that validates the Big Bang model. A fourth factor validating the Big Bang model is the Big Bang Nucleosynthesis (BBN). This helps in tracing back to the earliest times in the Universe (\sim less than 1 second of Big Bang) where matter density was dominated by radiation density and rate of decrease of temperature T is much greater than 1MeV. Within that interval, thermal equilibrium was existent due to a dominating particle interaction rate to the rate at which temperature was reducing. Weak interactions favoured an equilibrium state of neutron to proton (n/p) ratio, given by $e^{\frac{-\Delta m}{T}}$ approaches unity, which led to the stability of free nucleons thermodynamically. Considering the later times where $T \ll 0.01\text{MeV}$ and $t \gg 10^4$ seconds, nuclei are thermodynamically favoured and usually, nuclear reactions are frozen by this time. These two observations of early and late states determine the outcome of BBN which is dependent upon the baryon-to-photon ratio (h) given by a constant value $(6.03 \pm 0.04) \times 10^{-10}$ which can be equated with the fraction of critical density contributed by baryons as $\eta_{10} \equiv \frac{\eta}{10^{-10}} = \frac{274\Omega_B h^2}{T_{2.73}^3}$ where $T_{2.73} \equiv \frac{T_0}{2.73}$ K and $\Omega_B \equiv \frac{\rho_B}{\rho_{crit}}$ is the fraction of the critical density contributed by baryons and critical density where critical density $\rho_{crit} = 1.88h^2 10^{-29}$ g/cc. At $T \sim 1\text{MeV}$, weak interactions freeze out and the n/p ratio freezes in at about $1/7$. This particular phase is a specific intermediate phase where there are active nuclear reactions with no thermal equilibrium. Further, the favour of occurrence of nuclei over free nucleons at a given temperature is given by the Nuclear Statistical Equilibrium which provides the following temperature favourable temperature T_{nuclei} as

$$T_{nuclei} \approx \frac{B_A}{(A-1)(\ln \eta^{-1} + 1.5 \ln(\frac{m}{T}))} \approx 0.25\text{MeV} \left[1 + \frac{\ln \eta_0}{35} \right] \tag{2.5}$$

Here B_A is the binding energy of nucleus A and m is the nucleon mass ($\approx 1\text{GeV}$). Any nucleosynthesis beyond deuterium involves charged-particle on charged-particle reactions and hence the electrostatic repulsion known as the Coulomb barrier holds an important role in such cases. Coulomb barriers suppress the nuclear reaction rate by exponential factors and hence any nuclear reaction beyond deuterium freezes out at a temperature which is estimated by comparing expansion rate $H \sim \frac{T^2}{m_{pl}}$ to the nuclear reaction rate

G . This temperature ($T_{coulomb}$) is given by

$$T_{coulomb} \sim \frac{0.03}{\left[1 + \frac{\ln \eta_{10}}{7} \right]^3} \text{MeV} \tag{2.6}$$

Observing the BBN closely and analysing the favourable conditions for nucleosynthesis, the reaction rates supporting the creation of nucleons from heavier elements require a very high temperature and strong interactions which were favourable at the beginning of the universe if the Big Bang model is considered.

Hence, this is a supporting factor of the Big Bang. Other than this, the reduction of temperatures, effects of Coulomb barriers and freezing out of reactions also support the theory of expanding universe and hence validate the Big Bang.

Another assumption of the LCDM model is the mass-energy content of the universe. As per this assumption, there is 10⁻³⁰% radiation, 4% ordinary (baryonic) matter, 23% dark matter and 73% dark energy. This has been an observational collection of the Wilkinson Microwave Anisotropy Probe (WMAP) that measures all the parameters of the Big Bang, the relative density of baryonic matter and non-baryonic matter and energy densities using the CMB fluctuation spectrum. As per the WMAP, the universe is flat, which brings us to the conclusion that the mean mass-energy density is the same as the critical mass-energy density of the universe at the current time. The equivalent mass-energy density was computed to be $(9.9 \pm 0.005) \times 10^{-30}$ g/cc which is equivalent to 5.9 protons/cc. Along with the above-mentioned assumptions, the present model also assumes GR and cosmological principles to be valid. Further, we will see how homogeneity and isotropy is helping the model and how the expansion rate of the Universe gets affected by it.

3. ASSUMPTIONS AND TESTS OF THE SCALE OF HOMOGENEITY:

The current model assumes the universe to be of same composition and behave geometrically same through all solid angles for a significantly large scale of the universe. This is the property of homogeneity and isotropy as stated by the Cosmological Principle. Unlike isotropy, homogeneity cannot be observed directly. We cannot move in cosmic time and distance and hence we cannot observe variations in constant time slices. This means we cannot see whether the universe is homogeneous at a particular time or not. We can only observe a 2-sphere in a constant time slice of a past light cone of here and now and hence, cannot distinguish an evolving homogeneous distribution of matter. Although this is not directly observable, it can be concluded from isotropy. Isotropy can be observed from matter distributions in different forms. Considering validation of the isotropy of one observer, if a fundamental observer measures isotropic angular diameter distances, number counts, bulk velocities and lensing, then space-time and matter are isotropic. If we consider D to be the angular diameter distance, number count to be N , bulk velocity vector component on transverse plane to be V and lensing distortion of images $L(w_0, z, x^r)$, then, by saying these factors to be isotropic for celestial coordinates $x^p(q, f)$, it indicates:

$$\frac{\partial D}{\partial x^p} = \frac{\partial N}{\partial x^p} = V = L = 0 \quad (3.1)$$

Introducing the coordinate system used in (3.1), the observational coordinates $x^\mu = (\omega, y, \theta, \phi)$ is used where $w = w_0$ is a constant and represent the past light cones on the observer worldline $C(y=0)$ which is normalized such that w measures proper time along C and $y=z$ is the redshift that measures distance down the light rays $(w, q, f) = \text{const.}$ and x^r represents the radial coordinates. Isotropy at angular diameter distance means, the observed region potentially looks the same through all solid angles. Isotropic Number count shows a constant number density of matter in the slice and the slice has no relative velocity in a perpendicular plane (constant V), which means that either the observed zone is moving parallel to the observer and no distortion means there is pure magnification in the images. What we need to know further is that, what happens when the CMB is isotropic for this observer (as of (3.1)). To address this, we consider a 3+1 covariant Lagrangian formalism. Looking into the covariant kinetic theory, in this theory, a photon 4-momentum is split in the following way

$$p^\mu = E(u^\mu + e^\mu) \quad (3.2)$$

In which u^m is the 4-velocity field, E is energy given by $-u_m p^m$ and the direction $e^\mu = \frac{p^{(\mu}}{E}$. The relation between direction and rank 1 component of direction and 4-velocity are respectively provided in (3.3).

$$e^m e_m = 1, e^m u_m = 0 \quad (3.3)$$

Based on (3.2) and (3.3), the photon distribution function can be written as

$$f(x, p) = \sum_{l \geq 0} F_{M_l}(x, E) e^{(M_l)} \quad (3.4)$$

Here, each term $F_{M_l}(x, E) e^{(M_l)}$ is a harmonic where $M_l \equiv \prod_{i=1}^l \mu_i$ and $e^{M_l} \equiv \prod_{k=\mu_1}^{M_l} e^k$. Along the observer's worldline, (3.4) is expressed as given in (3.5), in which we can see that, all covariant multipoles of the distribution function beyond the monopole and their time derivatives must vanish along C .

$$f(x_C, p) = F(x_C, E) \text{ and } F_{\mu_1 \dots \mu_j | C} = \dot{F}_{\mu_1 \dots \mu_j | C} = 0 \quad \forall j \geq 1 \quad (3.5)$$

Now that we know the variation of factors in isotropic conditions for a single observer, we can focus on the scenario of multiple observers. In this case, if all observers in an expanding space-time with L see isotropy of four matter observables (addressed in (3.1)), then we have geometrical isotropy along all worldlines. This form of isotropy implies an equivalent mass-energy distribution from all angles and hence validates homogeneity. Such a condition is referred to as FLRW; where the universe is homogeneous, isotropic and expanding. There are statements that uses Cosmological Principle in order to validate homogeneity and uses the parameter of angular distance diameter only. As per this, if all observers observe $\frac{\partial D}{\partial x^p} = 0$ up to the third order in a redshift series expansion (i.e. up to $O(z^3)$) and space-time is provided to be expanding, then, space-time in that region is FLRW. Focusing on testing these theories mathematically, there are two observational tests for testing homogeneity. One of them uses galaxy clusters as probes of the anisotropy found in CMB at the cosmological distances down the past light cones. The CMB photons that are scattered into our line of sight induce spectral distortions in the CMB temperature that we observe. The distortion varies with the blackbody temperature of the CMB photons that arrive at clusters from our light cone. As the blackbody temperature variation is significant, the distortion is significant. Such a significant distortion indicates anisotropy and a breakpoint of homogeneity. Another breakpoint of homogeneity is determined by the kinetic SZ signal induced by the bulk radial motion of the cluster gas. A significantly large SZ temperature (or large non-perturbed thermal effect) implies a non-FLRW Universe. Another way of testing homogeneity uses consistency relations that hold FLRW. This is considered a more reliable and mathematically feasible way to test homogeneity. For this, the anomalies in distance measurements are considered. There are two geometric effects on distance measurements, one being the bend of geodesics due to curvature and the other being the expansion changes in the radial distances. The FLRW combines these two effects into one mathematical equation, which uses the Hubble rate and distance data to find the curvature at the present cosmological time. This is considered the most stable equation as it is independent of any model of the Universe, even does not require matter distribution

specifications and is overall independent of any cosmological parameters. The equation derived in the FLRW model and the relation to find the curvature are given below:

$$D_L(z) = \frac{(1+z)}{H_0 \sqrt{-\Omega_{K0}}} \sin \left(\sqrt{-\Omega_{K0}} \int_0^z \frac{H_0}{H(z')} dz' \right), \Omega_{K0} = \frac{H^2(z) [(1+z)D_L'(z) - D_L(z)]^2 - (1+z)^4}{H_0^2 (1+z)^2 D_L^2(z)} \quad (3.6)$$

To test statistical homogeneity, we use the chi-square test where we can differentiate to get the consistency relation $k = 0$ by using the fact that Ω_{K0} is constant. $k = 0$ is a valid independent test for any FLRW geometry. A maximum variation of the order of 10^{-5} can be expected from the test, although, a significant change in the result can imply a break in homogeneity. Focusing on the existence of homogeneity in different scales of the current cosmological model, a conclusion made on the data of the D12 BOSS CMASS galaxy sample of the Sloan Digital Sky Survey shows homogeneity on scales greater than $a(t) = 64.3 \pm 1.6h^{-1}$ Mpc and z ranging from 0.538 to 0.592. Through all the equations and proof of homogeneity, we have come across to the topic that assumes all these factors to conclude metrics on expansion and geometric isotropy, namely the FLRW.

4. COMPUTING EXPANSIONS WITH FLRW METRIC:

The FLRW is a modification of the Robertson-Walker (RW) metric and the Friedmann equations. RW is generally used for determining the geometry of space-time by providing a total interval along the worldline C by using the basic definition of metric in a worldline to define a flat space-time. However, the Friedmann equations derive a metric that uses Einstein's field equations to calculate the geometry of space-time for a given mass-energy distribution. Discussing the RW metric first, it uses the differential distance formula given by $(dl)^2 = \sum (di)^2 \forall i \in \{x, y, z\}$ which is further used to measure interval ds along C which in turn is given by $(ds)^2 = (cdt)^2 - (dl)^2$. Further, the curvature in two-dimensional space (K) is used as given in (4.1)

$$K = \frac{3}{\pi} \lim_{X \rightarrow 0} \frac{2\pi X - C_r}{X^3} \quad (4.1)$$

where C_r is the measured curvature and X is the radius of curvature (or half of angular distance diameter). (4.1) is further used with the spherical coordinate form of distances to get the distance element in three-dimensional Euclidean space as

$$(dl)^2 = \left(\frac{dr}{\sqrt{1 - \frac{r^2}{R^2}}} \right)^2 + (rd\phi)^2 \quad (4.2)$$

Where R is the radius of the sphere, r and ϕ are the polar coordinates (r being the co-moving radial distance) and $Rd\phi$ is given by dX which is written as $\frac{dr}{\sqrt{1 - \frac{r^2}{R^2}}}$ geometrically. This form as derived in (4.2) is used further

in the metric for worldline C and the polar form is used throughout where the curvature is written in forms of scale factor and independent term k which is a time-independent factor, derived from curvature term $K = \frac{k}{a^2(t)}$ to create the RW metric:

$$(ds)^2=(cdt)^2-a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (4.3)$$

For deriving the Friedmann equations, we need to see an unmodified version of (1.2) where the cosmological constant does not add in as a term and this modified equation, wherever used further in this paper will be denoted as modified(1.2). Further, we use the tensor notation to write the RW metric which is $ds^2=g_{mn}dx^m dx^n$ and g_{mn} is the metric tensor. Re-writing the metric tensor as the diagonal matrix of non-vanishing elements of the RW metric, we get,

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{a^2}{1-kr^2} & 0 & 0 \\ 0 & 0 & -a^2r^2 & 0 \\ 0 & 0 & 0 & -a^2r^2 \sin^2\theta \end{bmatrix} \quad (4.4)$$

Where the time-time and space-space components T_{00} and T_{11} of the stress-energy tensor T_{mn} (refer to (1.2)) for a comoving observer in an equivalent mass-energy density and pressure (p_m) are given as $r_m c^2$ and $\frac{p_m a^2}{1-kr^2}$ respectively. Using the stress-energy tensor components as results to find Einstein tensors G_{00} and G_{11} using modified(1.2), we get the following Einstein Tensors

$$G_{00} = \frac{3(a^2+kc^2)}{(ca)^2}, G_{11} = -\frac{(2a\ddot{a}+a^2+k)}{c^2(1-kr^2)} \quad (4.5)$$

Using the results of (4.5) and their corresponding stress-energy tensor values in modified(1.2), we get relations for time evolution on the cosmic scale factor, which form the two Friedmann Equations where the rate factor term $\frac{\dot{a}}{a}$ is denoted as Hubble parameter H and is written as

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3} \rho_m, \quad 2\frac{\ddot{a}}{a} + H^2 + \frac{kc^2}{a^2} = -\frac{8\pi G}{3} p_m \quad (4.6)$$

In the first equation, where the constant kc^2 is substituted as k in (1.1) the Hubble parameter is the only parameter kept on the left-hand side and the cosmological constant term is added to the equation to form the FLRW metric (recall (1.1))

$$H^2 = \frac{8\pi G \rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (4.7)$$

CONCLUSION:

The current model has a huge observational support from different astronomical surveys. However, there is still a lack of Mathematical evidence such as a proper Mathematical proof of the Big Bang and elementary proofs that support the GR. The current model and metric are suitable enough for the large-scale cosmos but a mathematical model that can support the scale ranges below 50Mpc. Further work is going on that helps calculate expansion in the anisotropic model and removes the dependencies of the assumptions.

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