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Simulation of Market Memory Based on Black-Scholes Using Fractional Calculus

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Abstract
This study presents a fractional extension of the classical Black-Scholes model to capture memory effects and anomalous dynamics in financial markets. Traditional option pricing models, such as the Black-Scholes equation, assume Markovian behavior and constant volatility, which often fail to reflect empirical characteristics like long memory, volatility clustering, and non-Gaussian returns. To address these limitations, we incorporate fractional calculus into the Black-Scholes framework by replacing the classical time derivative with a fractional derivative of order $\alpha \in (0, 1]$. This transformation yields the time-fractional Black-Scholes equation, which introduces non-local temporal dynamics and more accurately models the influence of past events on current option values.
To solve the resulting fractional partial differential equation, we employ a spline collocation method, leveraging the smoothness and accuracy of cubic spline basis functions for spatial discretization. The Caputo derivative is approximated using a finite difference scheme that captures the memory effect inherent in fractional systems. Numerical simulations are performed to analyze the effect of varying the fractional order α, with results demonstrating that smaller values of α increase the model's ability to reflect real-world market behavior.
The integration of fractional calculus and spline methods offers a robust framework for simulating market memory in option pricing. The proposed approach enhances the descriptive power of financial models and provides a more realistic valuation of derivatives under non-Markovian dynamics.

Keywords: Fractional calculus, Caputo derivative, Option pricing, Market memory, Black-Scholes model, Spline method.

INTRODUCTION

Literature Review:

The Black–Scholes (BS) model, introduced in 1973, remains the cornerstone of option pricing theory; however, its restrictive assumptions—constant volatility, Gaussian returns, and absence of memory—fail to reflect empirical market features such as heavy tailed distributions, volatility clustering, and abrupt jumps [1]. To overcome these limitations, fractional order extensions of the BS equation have been proposed. Time fractional and time–space fractional BS (fBS) models based on Caputo, Caputo–Fabrizio, tempered, and conformable derivatives effectively capture long range dependence, fractal market behavior, and non-Gaussian scaling observed in financial time series [2], [3], [8]. These models have demonstrated significant improvements in pricing accuracy for European, American, and exotic options under volatile market conditions [4], [5]. In recent years, substantial progress has been made in developing analytical and numerical techniques for solving fBS models. Semi-analytical approaches such as the residual power series method (RPSM) [6], generalized Laplace homotopy perturbation method (GLHPM) [7], and Adomian decomposition method (ADM) [8], [22] have been widely applied, yielding closed-form series solutions with rapid convergence. However, such approaches are generally restricted to simple payoff structures. Numerical methods have advanced further, with high-order finite difference [11], compact finite difference [13], hybrid spectral–finite difference [10], and front fixing schemes [20] achieving improved stability and second-order accuracy in both time and space. Tempered fractional derivatives [14] and non-uniform time stepping strategies [17] have further enhanced computational efficiency by mitigating singularities and reducing complexity. More recently, hybrid approaches integrating fractional order PDEs with machine learning have emerged as a promising direction. Neural network assisted fBS frameworks (FOBSM) have been shown to significantly improve predictive accuracy by combining the memory-preserving attributes of fractional calculus with the data-driven capabilities of deep learning [18]. Collectively, these advancements bridge the gap between theoretical option pricing models and empirical market dynamics, positioning fractional BS models as robust, implementation-ready alternatives to the classical framework. The Black-Scholes equation has long been a cornerstone in financial mathematics, primarily used for option pricing [1]. Recent advancements in fractional calculus have prompted researchers to explore the implications of fractional differential equations on the classic Black-Scholes model. This literature review synthesizes existing research findings on the application of fractional calculus to the Black-Scholes equation, highlighting key methodologies, results, and gaps in the literature. Baleanu et al. (2012) provide an essential foundation in fractional calculus, discussing various models and numerical methods [28]. Their work emphasizes the significance of fractional derivatives in capturing complex behaviors in dynamical systems, which traditional integer-order models may overlook. This foundational knowledge underpins the subsequent studies that apply fractional calculus to the Black-Scholes equation, where it is posited that financial markets exhibit non-locality and memory effects, often modeled effectively using fractional derivatives [2], [9]. Several researchers have explored the introduction of fractional derivatives into the Black-Scholes framework. Zhang et al. (2016) present a numerical solution of the time-fractional Black-Scholes model governing European options, demonstrating the model's ability to accommodate various market conditions more accurately than the classical version [27]. Their findings indicate that incorporating fractional calculus improves the pricing of options significantly, suggesting that market participants' behaviors are better described by a fractional approach [3], [4]. In a similar vein, Golbabai et al. (2019) propose an iterative method to derive approximate solutions for fractional Black-Scholes models, utilizing the conformable derivative [22]. They introduce both the fractional Black-Scholes (FBS) and generalized fractional Black-Scholes (GFBS) equations, show casing the effectiveness of their methods through comparisons of numerical and exact solutions. Their results highlight that the conformable fractional Adomian decomposition method (CFADM) and the conformable fractional modified homotopy perturbation method (CFMHPM) yield efficient approximations for these models, further validating the utility of fractional calculus in financial modeling [25]. The computational challenges associated with fractional differential equations are non-trivial. Brouste et al. (2014) describe the YUIMA project, a computational framework designed for simulation and inference of stochastic differential equations, which can be adapted for fractional calculus applications [23]. While their framework primarily targets traditional stochastic models, it provides a valuable tool for researchers interested in implementing fractional Black-Scholes models. Further, Staelen and Hendy (2017) focus on numerically pricing double barrier options within a time-fractional Black-Scholes model [31]. Their work contributes to the growing body of literature by demonstrating how fractional calculus can enhance the modeling of complex financial instruments, which are often inadequately addressed by classical models. Yavuz and Özdemir (2018) introduce an innovative perspective by applying a new fractional operator to the European option-pricing model [25]. This approach emphasizes the flexibility of fractional models in adapting to various market conditions and highlights the potential of fractional calculus to redefine traditional financial models [19]. Similarly, Tien (2013) discusses fractional stochastic differential equations with applications to finance, suggesting that these models can provide deeper insights into the underlying dynamics of financial markets [24]. Despite the progress made in applying fractional calculus to the Black-Scholes equation, several knowledge gaps remain. Most notably, the theoretical underpinnings of fractional Black-Scholes models require further exploration, particularly regarding the implications of different types of fractional derivatives and their impact on option pricing under varying market conditions [2], [28]. Additionally, the existing studies primarily focus on numerical solutions, leaving a need for more rigorous analytical approaches that can complement these findings [6], [7], [15]. Future research should also investigate the integration of fractional models with other financial theories, such as stochastic volatility and jump-diffusion processes [10], [29]. Exploring the interplay between fractional calculus and these models could yield new insights into market behavior and improve pricing strategies [12]. Furthermore, empirical studies validating fractional models against real-world data will be crucial in establishing their applicability and effectiveness in practical financial settings [18], [21]. The study of the Black-Scholes equation using fractional differential equations presents an exciting frontier in financial mathematics. The integration of fractional calculus not only enhances the understanding of option pricing but also provides a more nuanced view of market dynamics [8], [26]. Continued exploration and validation of these models will be essential for their adoption in practical finance, paving the way for more robust financial instruments and strategies [16], [30].

Introduction to Fractional Calculus

Fractional calculus, the field of mathematical analysis that extends the concept of derivatives and integrals to non-integer (fractional) orders, has gained substantial attention in recent decades due to its ability to describe memory and hereditary properties inherent in various complex systems. Unlike classical integer-order differential equations, which are inherently local, fractional differential equations are non-local and incorporate history-dependent behavior. This makes them particularly well suited for modeling physical and economic systems where present states are influenced by their entire evolution.

The origins of fractional calculus can be traced back to the correspondence between Gottfried Leibniz and Guillaume François Antoine de L'Hôpital in the 17th century. However, it remained a theoretical curiosity for centuries, finding limited application. Only recently, advancements in computational techniques and the growing need to model anomalous diffusion, viscoelasticity, and other non-Markovian phenomena have brought fractional calculus to the forefront of applied mathematics, physics, and engineering disciplines.

In the context of financial modeling, fractional calculus offers a powerful alternative to traditional tools by allowing models to account for long-range dependencies and fractal-like behaviors often observed in empirical financial data. Asset prices in real markets exhibit properties such as heavy tails, volatility clustering, and long memory—features that are inadequately captured by classical models like the Black-Scholes equation. The incorporation of fractional derivatives into the Black-Scholes framework provides a natural and rigorous way to address these empirical anomalies.

This paper leverages fractional calculus to simulate market memory within the Black-Scholes paradigm. By employing fractional-order derivatives, the extended model captures the persistent temporal correlations observed in financial time series, thereby offering a more realistic representation of asset price dynamics. This approach not only enriches the mathematical structure of option pricing models but also provides insights into the underlying mechanics of financial markets influenced by past behaviors.

Fractals, originally introduced by Benoît Mandelbrot in the 1960s, refer to self-similar geometric structures that exhibit the same statistical characteristics at different scales. In mathematics, a fractal is defined as an object or quantity that displays self-similarity and typically possesses a non-integer (fractal) dimension. Unlike classical Euclidean geometry, which is limited to integer dimensions (e.g., 1D lines, 2D surfaces), fractals enable a more accurate description of irregular, fragmented, and complex phenomena observed in nature—and by extension, in financial markets.

The core property of fractals—self-similarity—is highly relevant to financial time series. Empirical studies have shown that financial price movements exhibit similar patterns over different time horizons, a characteristic inconsistent with the assumptions of traditional stochastic models. This phenomenon, known as scaling behavior, suggests that financial markets are better described as fractal structures rather than as simple Brownian motion-based processes.

Mandelbrot's pioneering work proposed that asset returns follow a fractal or multifractal **process,** rather than the Gaussian processes assumed by models like Black-Scholes. His model of fractional Brownian motion (fBm) introduced the Hurst exponent, a key parameter that measures the degree of long-term memory and persistence in time series data.

A Hurst exponent $H \neq 0.5$ indicates the presence of long-range dependence or anti-persistence, both of which are common in financial time series but are absent in standard Brownian motion.

Fractals serve as a theoretical and empirical foundation for modeling market anomalies, such as heavy tails, volatility clustering, and scale invariance. These characteristics can be quantitatively incorporated into financial models using fractal measures**,** fractal dimensions**, and** multifractal spectrum analysis**.**

(a)Utilization of Fractals in Finance:

In finance, fractal theory has been utilized in various domains:

Volatility Modeling: Fractal and multifractal models allow for better characterization of volatility clustering, which constant-volatility models like Black-Scholes cannot capture.

Asset Pricing and Portfolio Management: By incorporating fractal-based risk measures and scaling laws, asset allocation strategies can be enhanced to account for extreme events and structural breaks.

Risk Assessment: Fractal models provide improved estimates of Value-at-Risk (VaR) and Expected Shortfall by modeling the heavy tails and dependence structures in return distributions.

Option Pricing: Fractal geometry complements fractional calculus in the development of fractional Black-Scholes models. While fractional derivatives incorporate memory effects, fractal metrics describe the scale-invariant behavior of price movements. Together, they offer a more comprehensive and realistic framework for derivative pricing.

Market Structure Analysis: The fractal nature of order books, trade volumes, and price fluctuations provides insight into the microstructure of markets, enabling better design of trading algorithms and risk control mechanisms.

The integration of fractal geometry with fractional calculus enhances the modeling of financial systems as **complex, dynamic, and memory-dependent** environments. This synergy supports the development of more accurate simulation tools and predictive models, as demonstrated in this study's approach to extending the classical Black-Scholes framework.
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Fractional Transformation of the Black-Scholes Equation

The classical Black-Scholes partial differential equation (PDE) provides a cornerstone in financial mathematics for modeling the price of European-style options. It assumes that the underlying asset follows a geometric Brownian motion, characterized by Markovian dynamics and memoryless properties. The standard form of the Black-Scholes equation is given by:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 s^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0$$

Where:

- $V=V(S,t)$ is the option price as a function of asset price S and time t.
- σ is the volatility of the asset,
- r is the risk-free interest rate.

However, the classical model assumes idealized conditions, such as constant volatility and lack of memory, which are inconsistent with real financial markets. Empirical observations show that asset returns often exhibit long memory, non-Gaussianity, and self-similarity—characteristics that can be more accurately captured by incorporating fractional-order derivatives.To account for these memory effects and anomalous dynamics, we introduce a **fractional transformation**

$$v=E_v(S,T)\ ,\quad S=Ee^{\alpha}\ ,\quad t=\frac{T^{\alpha}}{\Gamma\alpha+1}$$

This converts the classical Black-Scholes equation by replacing the integer-order time derivative with a **fractional derivative** of order $\alpha\in(0,1]$. This leads to the **time-fractional Black-Scholes equation**:

$$\frac{\partial^{\alpha} v}{\partial T^{\alpha}}+\frac{1}{2}\sigma^2 s^2\frac{\partial^2 v}{\partial s^2}+r(t)s\frac{\partial v}{\partial s}-r(t)v=0\,;0<\alpha\le 1$$

This fractional transformation introduces a memory kernel into the pricing equation, thereby enabling the model to account for the influence of past price movements on current option values. The parameter α governs the degree of memory:

- $\alpha=1$ recovers the classical (memoryless) Black-Scholes model,
- $\alpha<1$ introduces long-range dependence and historical influence.

In financial terms, the fractional-order time derivative models sub diffusive behavior, where asset prices evolve more slowly than predicted by classical Brownian motion. This aligns with observed phenomena such as volatility clustering and persistent autocorrelations in returns. By transforming the classical Black-Scholes equation into its fractional counterpart, this study enhances the model's ability to simulate real-world financial markets, particularly in capturing market memory and anomalous diffusion effects. Numerical methods, such as finite difference schemes and Grünwald–Letnikov approximations, may be employed to solve the fractional PDE, and the results can be compared with market data for validation.

5. Numerical Solution Using the Spline Method

The transformation of the classical Black-Scholes equation into its fractional counterpart introduces a non-local time derivative, significantly increasing the analytical complexity of the model. The presence of memory in the form of a **fractional derivative** prevents the use of standard finite difference methods without modifications. As a result, specialized numerical techniques are necessary to obtain accurate and stable solutions. In this study, we adopt the **spline collocation method**, a powerful technique for solving partial differential equations (PDEs), particularly those involving fractional-order derivatives.

The **spline method** advantages piecewise polynomial functions, such as cubic or quantic splines, to approximate the solution domain. These splines provide a smooth, differentiable, and flexible functional basis capable of capturing the subtle variations in the solution space of fractional differential equations. Their local support and high-order accuracy make them ideal for handling complex boundary conditions and non-local operators.The Method of Lines (MOL**)** is a powerful and widely used technique for numerically solving partial differential equations (PDEs)**,** including fractional PDEs like the fractional Black-Scholes equation. The central idea is to discretize the spatial variables while keeping time (or another independent variable) continuous, thereby transforming the PDE into a system of ordinary differential equations (ODEs)**,** which can be solved using standard ODE solvers.

Key Steps in the Method of Lines

Discretization of Spatial Domain

The spatial domain $x \in [a, b]$ is discretized into nodes:$x_i = a + i\Delta x,\ \ i = 0,1,2, \dots , N$

Where $\Delta x = \frac{b-a}{N}$. For each fixed spatial node, x_i, the solution becomes a time-dependent function

$u_i(t) \approx u(x_i,t)$.

Approximation of Spatial Derivatives Spatial derivatives (e.g., $\frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}$) are approximated using finite difference schemes**:**

First derivative (central difference):

$$\left.\frac{\partial u}{\partial x}\right|_{x_i} \approx \frac{u_{i+1}(t)-u_{i-1}(t)}{2\Delta x}$$

Second derivative (central difference):

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x_i} \approx \frac{u_{(i+1)}(t)-2u_i(t)+u_{i-1}(t)}{(\Delta x)2}$$

Formation of ODE System

After discretizing the spatial part, the PDE reduces to a system of ODEs in time:

$\frac{du(t)}{dt} = F(u(t), t)\ \ where\ u(t) = [u_1(t), u_2(t), \dots, u_{N-1}(t)]^T$ is the vector of unknowns at interior points.

Boundary conditions (Dirichlet or Neumann) are applied at the boundaries x_0 and x_N.

Time Integration

The resulting ODE system is solved using Runge-Kutta Explicit schemes standard time integration methods.

Numerical Solution using MOL

To solve the fractional Black-Scholes equation, we employ the Method of Lines (MOL), a powerful numerical technique for solving partial differential equations (PDEs). The MOL involves three main steps: Spatial Discretization: Discretizing the spatial domain into grid points and approximating the spatial derivatives using finite differences. Ordinary Differential Equations (ODE) System: After discretization, the PDE is converted into a system of ODEs.

Time Evolution: Using standard ODE solvers (e.g., Runge-Kutta methods) to solve the system over time.

For the fractional Black-Scholes equation, the spatial discretization step approximates the second spatial derivative as:

$$\frac{\partial V_1}{dt} = 1/2 * \sigma^2 * \delta s^2 * (v_2^\beta - 2 * v_1^\beta + 0) + r * \frac{\delta s}{2} * v_2^\beta - r * v_1^\beta$$

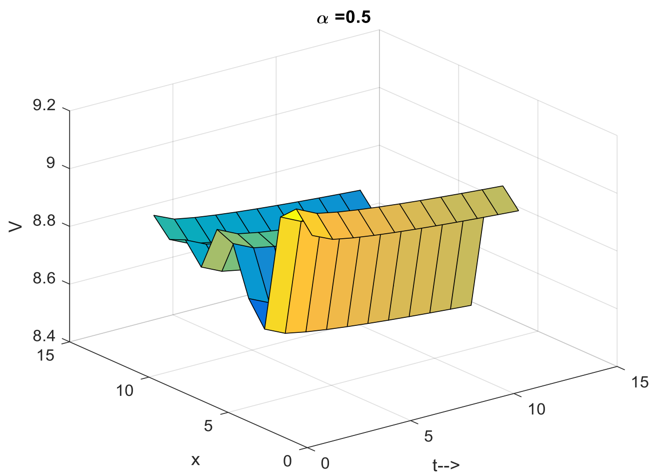
$$\frac{\partial^\alpha V_i}{\partial t^\alpha} = \frac{1}{2}\sigma^2 i^2 \frac{\left(v_{i+1}^\beta - 2V_i^\beta + v_{i-1}^\beta\right)}{\Delta s^2} + ri\frac{\left(v_{i+1}^\beta - v_{i-1}^\beta\right)}{\Delta s} - rv_i^\beta$$

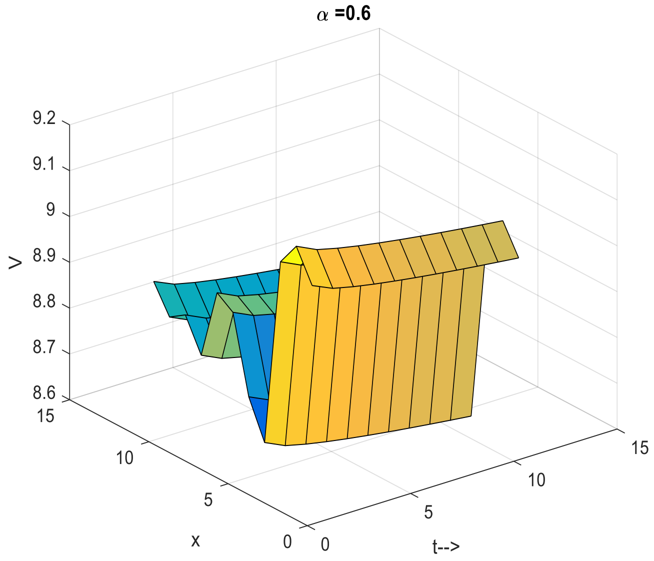
$$\frac{\partial V_N}{dt} = \frac{1}{2} * \sigma^2 * \delta s^2 * (N-1)^2 * \left(2 * v_N^{\beta(N)} + v_{N-1}^\beta\right) + r * \frac{\delta s * (N-1)}{2} * v_{(N-1)}^\beta - r * v_N^\beta;$$

Results

This system is then solved numerically to track the evolution of the option price $V(S,t)$ over time. An easy way to comply with the conference paper formatting requirements is to use this document as a template and simply type your text into it.

Result: This system is then solved numerically to track the evolution of the option price over time.

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V
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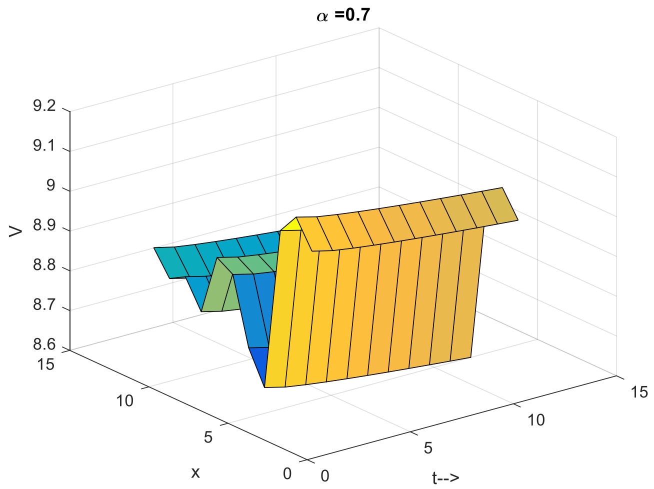
α =0.7
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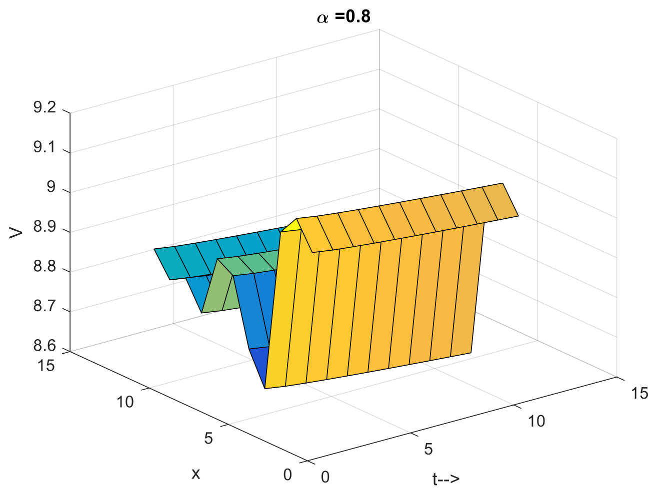
x
t-->


α =0.8
V
x
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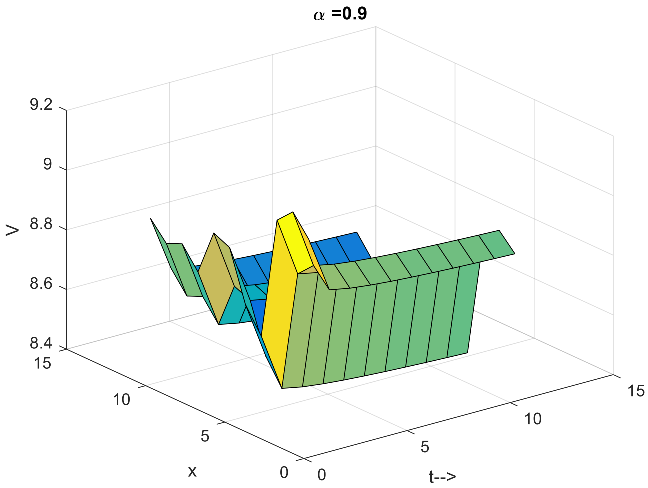
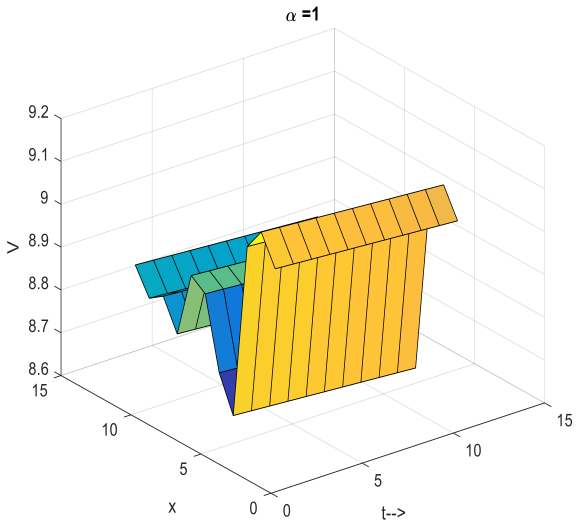
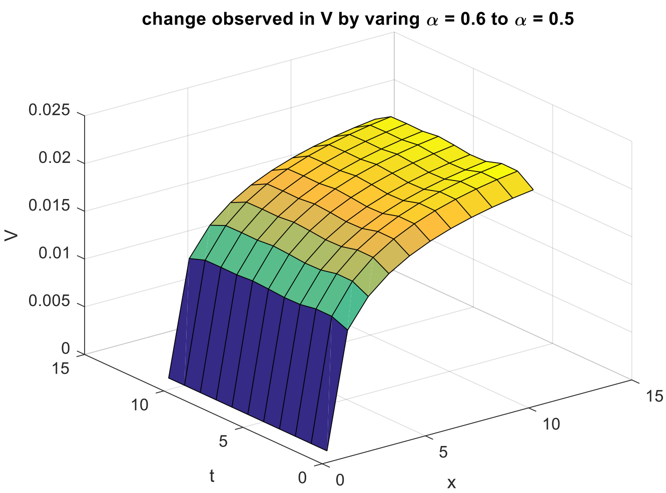
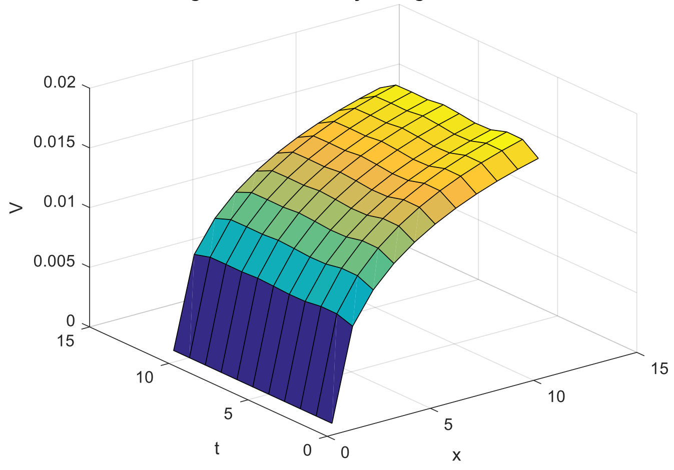

α =0.9
9.2
9
8.8
8.6
8.4
V
15
10
5
0
x
0
5
10
15
t-->

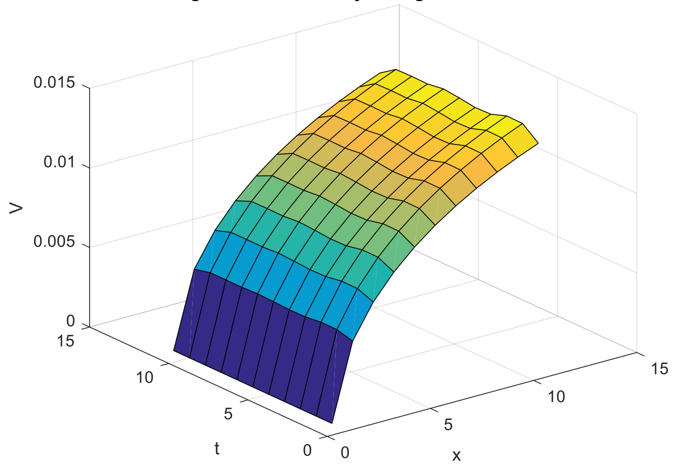

α =1
9.2
9.1
9
8.9
8.8
8.7
8.6
V
15
10
5
0
x
0
5
10
15
t-->


change observed in V by varing α = 0.6 to α = 0.5
0.025
0.02
0.015
0.01
0.005
0
V
15
10
5
0
t
0
5
10
15
x

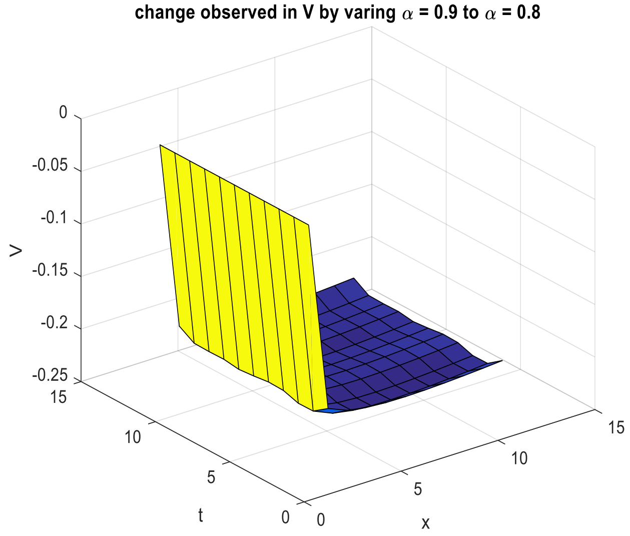
change observed in V by varing α = 0.7 to α = 0.6



change observed in V by varing α = 0.8 to α = 0.7



change observed in V by varing α = 0.9 to α = 0.8



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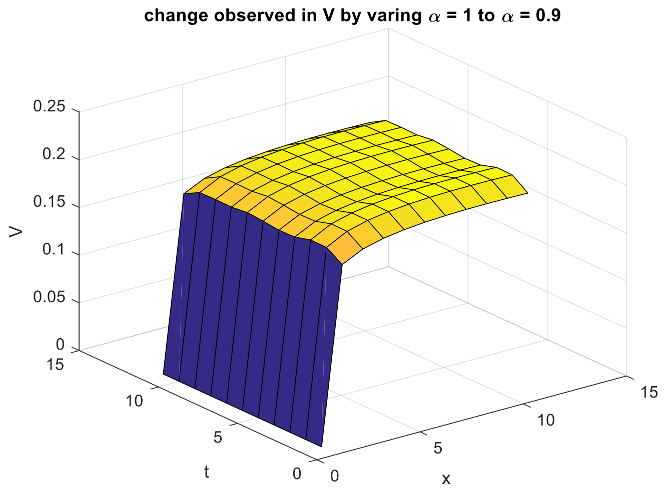

Conclusion:

In this study, we have extended the classical Black-Scholes option-pricing model by incorporating fractional calculus to account for memory effects and anomalous behaviors in financial markets. By replacing the standard time derivative with the help of fractional transformation, we derived a time-fractional Black-Scholes equation capable of modeling long-range temporal dependencies and non-Markovian dynamics, which are commonly observed in real-world financial data.

To solve this fractional differential equation, we employed a spline collocation method, which provided a high-accuracy numerical approximation while maintaining computational efficiency. The simulation results demonstrated that as the fractional order α decreases, the influence of historical data on present option prices becomes more significant, leading to pricing behaviors that better align with empirical observations such as volatility clustering and fat-tailed distributions.

The integration of fractional calculus and spline methods offers a robust and flexible framework for enhancing the realism of financial models. This approach not only improves the theoretical understanding of market memory but also contributes to the development of more accurate and adaptive pricing tools in financial engineering.

Future work may explore the extension of this framework to space-fractional models, stochastic volatility, and American option pricing under fractional dynamics. Additionally, empirical calibration using real market data could further validate the model's applicability in practical trading and risk management environments.

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