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A Study on Plithogenic Signed Graphs and its Operators

F. Sherin[1], F. Nirmala Irudayam[2]

[1]Research scholar, Nirmala College for Women
[2]Assistant Professor, Nirmala College for Women

ABSTRACT
"We introduce the concept of plithogenic signed graphs and define several binary operations on them, including union, join and Cartesian product to model the combination of heterogeneous networks. These operations provide a flexible mathematical framework for integrating multi-attribute and polarity-based relationships. The proposed model is applied to analyse interaction dynamics in complex systems. In particular, it indicates that the absence of appropriate precautionary measures may lead to a higher risk of severe outbreaks in the future.

Keywords: Plithogenic sets, Plithogenic Graphs, Plithogenic Signed Graphs

1.INTRODUCTION

Mathematical set's theory involves of crisp set (CS) theory, Zadeh (1996) introduced fuzzy set (FS) theory. Fuzzy systems have been used successfully for many years in problems involving uncer tainty, vagueness, ambiguity, and imprecision of state. Further, Zadeh (1975) extended fuzzy sets (FS) to interval valued fuzzy sets in which values of membership are intervals instead of real numbers between 0 and 1. Interval valued fuzzy sets (IVFS) provide precise results of uncertainty than fuzzy sets (FS). Another researcher Atanassov (Atanassove, 1986) , introduced membership and non membership and gave the idea of the intuitionistic fuzzy set (IFS). It generalized the idea of Zadeh's fuzzy sets. Jun et al. (2010) gave the idea of cubic sets by combining interval-valued fuzzy sets and fuzzy sets. Cubic sets have many applications in many directions (Jun et al. 2011, 2012). Smarandache extended the idea of Atanassov and gave the concept of neutrosophic set (NS) (Smaran dache 1999, 2005). Further Wang et al. (2005) introduced interval valued neutrosophic sets (INS). In 2017, Jun et al. (2017a, b)) presented the idea of neutrosophic cubic sets (NCS) to handle imprecise information. More recently, Smarandache (2017) and Smarandache and Broumi (2018) introduced for the first-time idea of Plithogeny in Philosophy and as its derivatives give the concept of Plithogenic set/logic/probability/statistics in mathematics and engineering. Plithogeny is the origination, formation, development, evolution of new entities and is a connection or combination of theories and ideas in any field. Plithogeny is the dynamics of many opposites, their neutrals and non opposites, and their organic fusion. The Plithogenic set's theory generalizes previous theories of fuzzy sets. Smarandache introduce the plithogenic set (as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets), which is a set whose elements are characterized by many attributes' values. An attribute value v has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance $d(x, v)$ of the element x, to the set P, with respect to some given criteria. Plithogenic set theory is being extensively used in various decision-making problems as well as in many other applied fields These different versions of sets have been used in the theory of graphs.

Graph theory is the mathematical structure used to design pair-wise relations between objects. It is constructive in solving problems of different fields as they give a clear picture of the problem at hand. The concept of graph theory begins with the problem of Konigsberg bridge problem in 1736. In 1973, Kauffman (1973) introduced the idea of the fuzzy graph. Rosenfeld (1975) developed the concept of fuzzy graph obtaining analogs of several graph theoretical concepts. Atanassov (1995) extended his concept of fuzzy sets to intuitionistic fuzzy graphs. Shannon and Atanassov (1994). Bhattacharya (1987), give some remarks on fuzzy graphs. Akram and Dudek (2011) gave the concept of interval-valued fuzzy graph in 2011. For more details of fuzzy graphs, readers are referred to Akram (2012) and Akram et al. (2013) . The idea of neutrosophic graphs was given by Kandasamy et al. (2015) in the book title as neutrosophic graphs. Rashid et al. (2018) give the concept of neutrosophic cubic graphs with real-life applications in industry. For more details see Gulistan et al. (2018, 2019) and Huang et al. (2019). In this study, Smarandache's plithogenic set is used to introduce the idea of plithogenic signed graphs and also established certain binary operations like union, join and Cartesian product, of plithogenic signed graphs, which are helpful when we discuss two different graphs combined. The primary purpose of this article is to develop the new mathematical model of Plithegonic Signed graphs (P_{SG}) and its examples.

1.1 PRELIMINARIES

(Smarandache 2017), Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of con tradictory and/or neutrals and/or non-contradictory multiple old entities. At the same time, plithogenic means what is about plithogeny. A plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance $d(x, v)$ of the element x, to the set P, with respect to some given criteria. In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. However, there are cases when such dominant attribute value may not be taken into consideration or may not exist; therefore, it is considered zero by default, or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established. The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' t-norm and t-conorm. Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neu trosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership)—for the crisp set and fuzzy set, two values (membership, and non-membership)—for intuitionistic fuzzy set, or three values (membership, non-membership, and indeterminacy)—for the neutrosophic set. So we first provide the defintions of fuzzy set, intuitionistic fuzzy set and neutrosophic set

Definition 1 (Zadeh 1996) Let S be a universe of discourse, then the set

$$F_S = \{\langle x, \lambda_T(x)\rangle : x \in S\} \qquad (1)$$

is called the **fuzzy set**, where $\lambda_T : S \to [0,1]$is the truth (membership value) such that $0 \le \lambda_T(x) \le 1$.

Definition 2 (Atanassov, 1986) Let Sbe a universe of discourse, then the set

$$I_S = \{\langle x, \lambda_T(x), \lambda_F(x)\rangle : x \in S\} \qquad (2)$$

is called the **intuitionistic fuzzy set**, where $\lambda_T, \lambda_F: S \to [0,1]$ are the truth and falsity membership degrees respectively, and

$$0 \le \lambda_T(x) + \lambda_F(x) \le 1.$$

Definition 3 (Smarandache, 1999) Let S be a universe of discourse, then the set

$$N_S = \{\langle x, \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle : x \in S\}$$

is the neutrosophic set, where $\lambda_T, \lambda_I, \lambda_F: S \to [0,1]$are respectively the degrees of truth, indeterminacy, and falsity, and $0 \le \lambda_T(x) + \lambda_I(x) + \lambda_F(x) \le 3$. (Smarandache, 2017) Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute (appurtenance): which has one value (membership) for the crisp set and for fuzzy set, two values (membership and non-membership) for intuitionistic fuzzy set, or three values (membership, non-membership and indeterminacy) for neutrosophic set.

Definition 4 (Smarandache, 2017) Let S be a universal set and $P \subseteq S$. A plithogenic set denoted as P_S is defined as

$$P_S = \{P, \chi, P_\chi, p_{df}, p_{cf}\} \qquad (4)$$

where χ is an appurtenance or attribute, P_χ is the range of attribute values, $p_{df}: P \times P_\chi \to [0,1]^s$ is the degree of appurtenance function (DAF) and $p_{cf}: P_\chi \times P_\chi \to [0,1]^t$ is the degree of contradiction function (DCF) which satisfies the axioms:

$$p_{cf}(a,a) = 0 \text{ and } p_{cf}(a,b) = p_{cf}(b,a).$$

Here $s, t \in \{1,2,3\}$. For $s = t = 1$, P_S is called plithogenic fuzzy set and is denoted by P_{FS}; also for $s = 2, t = 1$, P_S is called plithogenic intuitionistic fuzzy set and is denoted by P_{IFS}; and for $s = 3, t = 1$, P_S is called plithogenic neutrosophic set and is denoted by P_{NS}.

Definition 5 (Rosenfeld, 1975) A fuzzy graph with set of vertices V is defined to be a pair $G = (\theta, \delta)$, where θ is a fuzzy function on V and δ is a fuzzy function on $E \subseteq V \times V$, such that

$$\delta(xy) \le \min\{\theta(x), \theta(y)\} \text{ for all } xy \in E. \qquad (5)$$

We call θ the fuzzy vertex function of V, and δ the fuzzy edge function of E, respectively. Note that δ is a symmetric fuzzy relation on V. Thus, $G = (\theta, \delta)$ is a fuzzy graph of $G^* = (V, E)$if $\delta(xy) \le \min\{\theta(x), \theta(y)\}$ for all $xy \in E$.

Definition 6 (Atanassov, 1995) An intuitionistic fuzzy graph is of the form $G_{IF} = (\theta_V, \delta_E)$ where $\theta_V = (\theta_{VT}, \theta_{VF})$is the degree of membership function $\theta_{TV}: V \to [0,1]$ for vertices and degree of non-membership function $\theta_{FV}: V \to [0,1]$ for vertices. $\delta_{TE}: E \to [0,1]$ for vertices, Also $\delta_E = (\delta_{TE}, \delta_{FE})$consists of degree of membership function $\delta_{TE}: E \to [0,1]$ for edges and degree of non-membership function $\delta_{FE}: E \to [0,1]$for edges such that

$$\delta_{TE}(xy) \le \min\{\theta_{TV}(x), \theta_{TV}(y)\}, \qquad (6)$$
$$\delta_{FE}(xy) \ge \max\{\theta_{FV}(x), \theta_{FV}(y)\}, \qquad (7)$$
$$0 \le \delta_{TE}(xy) + \delta_{FE}(xy) \le 1, \text{ for all } xy \in E.$$

Definition 7 (Gulistan et al. 2018) Let $G^* = (V, E)$be a crisp graph. By a neutrosophic graph of G^*, we mean a pair $G_N = (P, Q)$, where $P = (\theta_T, \theta_I, \theta_F)$is neutrosophic set of vertex set V and $Q = (\delta_T, \delta_I, \delta_F)$is neutrosophic set of edge set E, such that

$$\delta_T(xy) \le \min\{\theta_T(x), \theta_T(y)\}, \qquad (8)$$
$$\delta_I(xy) \le \min\{\theta_I(x), \theta_I(y)\}, \qquad (9)$$

$$\delta_F(xy) \geq \max\{\theta_F(x), \theta_F(y)\}, \qquad (10)$$

for all $x, y \in V$ and $xy \in E$.

Definition 8 (Harary 1953) A signed graph (G, σ) is a graph G along with a function $\sigma : E(G) \to \{+1, -1\}$ called the signature of (G, σ), where $\sigma(e)$ is the sign of the edge $e \in E(G)$. The edges in $\sigma^{-1}(+1)$ are the positive edges and the edges in $\sigma^{-1}(-1)$ are the negative edges of (G, σ).

Definition 9 (F.Sultana 2022) Let $G^* = (V, E)$ be a crisp graph. A plithogenic graph denoted as P_G is defined as $P_G = (P_M, P_N)$, where $P_M = (M, \mu, M_\mu, \alpha_{df}, \alpha_{cf})$ is plithogenic set for vertices; where $M \subset V$, μ is an attribute, M_μ is the corresponding range of attribute values such that $\alpha_{df}: M \times M_\mu \to [0,1]^s$ is the degree of appurtenance function (DAF) for vertices defined as $\alpha_{df}(x, a) \in [0,1]^s$, and $\alpha_{cf}: M_\mu \times M_\mu \to [0,1]^t$ is degree of contradiction function (DCF) for vertices. Also $P_N = (N, \nu, N_\nu, \beta_{df}, \beta_{cf})$ is plithogenic set for edges, where $N \subset E$, ν is some attribute, N_ν is the corresponding range of attribute values such that (M_μ, N_ν) is a graph with vertices M_μ and edges N_ν. Also $\beta_{df}: N \times N_\nu \to [0,1]^s$ is the degree of appurtenance function for edges and $\beta_{cf}: N_\nu \times N_\nu \to [0,1]^t$ is degree of contradiction function for edges. Then P_G is plithogenic graph if for all (x, a) and $(y, b) \in M \times M_\mu$,

$$\beta_{df}\big((x, a), (y, b)\big) \leq \min\{\alpha_{df}(x, a), \alpha_{df}(y, b)\}, \qquad (11)$$

$$\beta_{cf}\big((a, b), (c, d)\big) \leq \min\{\alpha_{cf}(a, b), \alpha_{cf}(c, d)\}, \qquad (12)$$

for all $\big((a, b), (c, d)\big) \in N_\nu \times N_\nu$, where $\beta_{cf}\big((a, b), (a, b)\big) = 0$ as $\alpha_{cf}\big((a, a)\big) = 0 = \alpha_{cf}\big((b, b)\big)$. Here $s, t \in \{1,2,3\}$.

Here we discuss a subclass of plithogenic graphs known as plithogenic signed graphs.

2. PLITHOGENIC SIGNED GRAPH (P_{SG}):

In this section, we define a more general class of signed graphs known as plithogenic signed graphs.We also discuss plithogenic signed graphs and their basic operations like union, join and cartesian product of graphs.

Definition 10 (Plithogenic Signed Graph) Let $G^* = (V, E, \sigma)$ be a crisp signed graph, where $\sigma: E \to \{+1, -1\}$. A plithogenic signed graph denoted as P_{SG} is defined as $P_{SG} = (P_M, P_N, \sigma)$, where $P_M = (M, \mu, M_\mu, \alpha^{\pm}_{df}, \alpha^{\pm}_{cf})$ is a plithogenic signed set for vertices; where $M \subset V$, μ is an attribute, M_μ is the corresponding range of attribute values such that $\alpha^{\pm}_{df}: M \times M_\mu \to [-1,1]$ is the degree of appurtenance function (SDAF) for vertices defined as $\alpha^{\pm}_{df}(x, a) \in [-1,1]$, and $\alpha^{\pm}_{cf}: M_\mu \times M_\mu \to [-1,1]$ is the degree of contradiction function (SDCF) for vertices. Also $P_N = (N, \nu, N_\nu, \beta^{\pm}_{df}, \beta^{\pm}_{cf})$ is a plithogenic signed set for edges, where $N \subset E$, ν is some attribute, N_ν is the corresponding range of attribute values such that (M_μ, N_ν) is a signed graph with vertices M_μ and edges N_ν. Also $\beta^{\pm}_{df}: N \times N_\nu \to [-1,1]$ is the degree of appurtenance function for edges and $\beta^{\pm}_{cf}: N_\nu \times N_\nu \to [-1,1]$ is the degree of contradiction function for edges. Then P_{SG} is called a **plithogenic signed graph** if for all $(x, a), (y, b) \in M \times M_\mu$,

$$\beta^{\pm}_{df}\big((x, a), (y, b)\big) \leq \min\big\{\alpha^{\pm}_{df}(x, a), \alpha^{\pm}_{df}(y, b)\big\}, \qquad (13)$$

and for all $\big((a, b), (c, d)\big) \in N_\nu \times N_\nu$,

$$\beta^{\pm}_{cf}\big((a, b), (c, d)\big) \leq \min\big\{\alpha^{\pm}_{cf}(a, b), \alpha^{\pm}_{cf}(c, d)\big\}, \qquad (14)$$

where $\beta_{cf}^{\pm}((a,b),(a,b)) = 0, \alpha_{cf}^{\pm}((a,a)) = 0 = \alpha_{cf}^{\pm}((b,b))$. Each edge $e \in N$ is assigned a sign $\sigma(e) \in \{+1,-1\}$.

Example 1: Let $G^* = (V, E, \sigma)$ be a crisp signed graph, where V is the set f major transport hubs in a country, Eis the set of direct connections, and $\sigma: E \to \{+1,-1\}$ is the sign function on edges. Let $M = \{x,y,z\} \subset V$ be three hubs under consideration and $M_\mu = \{a,b,c,d\} = \{\text{traffic volume,accident rate,weather impact,infrastructure quality}\}$ be the range of values for the attribute μ =“risk factors”.Let $N = \{xy,yz,zx\}$ be the direct connections among the hubs and $N_\nu = \{ab,bc,cd,ad,ac\}$ be the relations among risk factors. Define the signed degree of appurtenance for vertices as $\alpha_{df}^{\pm}: M \times M_\mu \to [-1,1]$ and the signed degree of contradiction function of vertices as $\alpha_{cf}^{\pm}: M_\mu \times M_\mu \to [-1,1]$ is SDAF AND SDCF for the vertices in Table 1a and 1b. Define the signed degree of appurtenance for edge $\beta_{df}^{\pm}: N \times N_\nu \to [-1,1]$ and the signed degree of contradiction of edges as $\beta_{cf}^{\pm}: N_\nu \times N_\nu \to [-1,1]$ in table 2a and 2b.

Table 1(a): Signed Degree of Appurtenance for Vertices

$$\alpha_{df}^{\pm}: M \times M_\mu \to [-1,1]$$

$\alpha_{df}^{\pm}$	x	y	z
a	+0.3	+0.5	+0.2
b	+0.4	+0.1	+0.3
c	+0.2	+0.2	+0.4
d	−0.1	−0.3	−0.1

Table 1(b): Signed Degree of Contradiction for Vertices

$$\alpha_{cf}^{\pm}: M_\mu \times M_\mu \to [-1,1]$$

$\alpha_{cf}^{\pm}$	a	b	c	d
a	0	+0.5	+0.6	−0.5
b	+0.5	0	+0.4	−0.4
c	+0.6	+0.4	0	−0.5
d	−0.5	−0.4	−0.5	0

Table 2(a): Signed Degree of Appurtenance for Edges

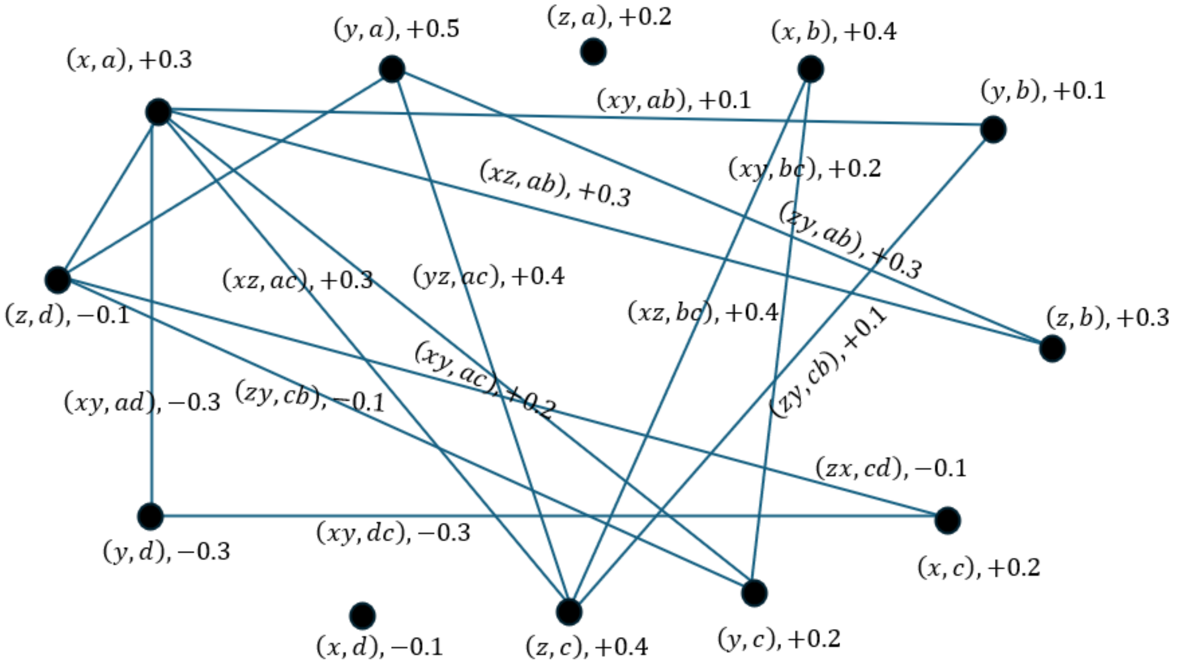
$\beta_{df}^{\pm}$	ab	bc	cd	ad	ac
xy	+0.1	+0.2	−0.3	−0.3	0.2
yz	0.3	0.1	−0.1	−0.1	+0.4
zx	+0.2	+0.2	−0.1	−0.1	+0.2

Table 2(b): Signed Degree of Contradiction for Edges

$\beta_{cf}^{\pm}$	ab	bc	cd	ad	ac
ab	0	+0.4	−0.4	−0.4	+0.4
bc	+0.4	0	−0.5	−0.5	+0.5

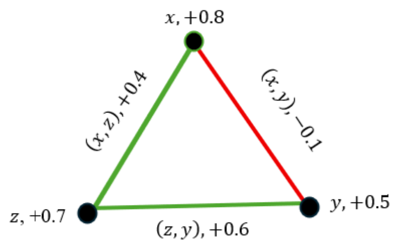
cd	−0.4	−0.5	0	−0.3	+0.5
ad	−0.4	−0.5	−0.3	0	−0.4
ac	+0.4	+0.5	+0.5	−0.4	0

Here second column of SDAF in Table 1a shows that 30% increase risk in traffic volume , 40% increase risk in accident rate, 20% increase in weather impact and 10% decrease risk in infrastructure quality from the transport hub x. Similarly, we have observations for transportation hubs y and z also their SDAF and SDCF for edges is also defined. Now, compute the net influence



Compute:

- $P_{SG}(xy) = 0.1 + 0.2 - 0.3 - 0.3 + 0.2 = -0.1 \Rightarrow \sigma(xy) = -1$
- $P_{SG}(yz) = 0.3 + 0.1 - 0.1 - 0.1 + 0.4 = +0.6 \Rightarrow \sigma(yz) = +1$
- $P_{SG}(zx) = 0.2 + 0.2 - 0.1 - 0.1 + 0.2 = +0.4 \Rightarrow \sigma(zx) = +1$ Finally, we obtained the Plithogenic Signed Graph is to be $P_{SG} = (\{x, y, z\}, \{xy, yz, zx\}, \sigma)$.



Plithogenic Signed Graph

Lemma 1

A plithogenic signed graph $P_{SG} = (P_M, P_N, \sigma)$is said to be a positive plithogenic signed graph if every even length cycle in the induced signed graph (V, E, σ) contains an even number of negative edges

Postive Plithogenic Signed Graph

Proof:

Let C be any even length cycle in (V, E, σ). If C contains an even number of negative edges, then the product of the signs of the edges in $C$is positive, since $(-1)^{\text{even}} = +1$. Hence, the sign of every even cycle is positive. Therefore, the plithogenic signed graph P_{SG} is positive.

Corollary 1

An odd length cycle in a plithogenic signed graph $P_{SG} = (P_M, P_N, \sigma)$having all edges negative is always a negative plithogenic signed graph.

Definition 11 (Balanced Plithogenic Signed Graph**)** A plithogenic signed graph $P_{SG} = (P_M, P_N, \sigma)$is said to be balanced if every cycle in its induced signed graph (V, E, σ) has an even number of negative edges, or equivalently, if the sign of every cycle is positive.

Proposition 1

An odd length cycle in a plithogenic signed graph is balanced if and only if it contains at least one positive edge or an odd number of positive edges.

Proof.

Let C be an odd cycle.

- If C contains at least one positive edge, then the number of negative edges is less than the length of the cycle and can be even, making the product of signs positive.
- Conversely, if all edges are negative, then C has an odd number of negative edges and hence is unbalanced.

Thus, an odd cycle is balanced if and only if it contains at least one positive edge or odd number of positive edges.

Definition 12 (Plithogenic Signed Frustration Number) The plithogenic signed frustration numbe**r** of a plithogenic signed graph $P_{SG} = (P_M, P_N, \sigma)$is defined as the minimum number of edges that must be removed from the induced signed graph (V, E, σ)so that the resulting plithogenic signed graph becomes balanced. In a plithogenic signed graph, the frustration number is computed by selecting and deleting a set of negative edges such that all cycles in the induced signed graph become balanced.

Algorithm as follows:

•Collect all negative edges using the induced sign function σ.

• From these edges, select the edge having minimum signed degree of appurtenance $\beta^{\pm}_{df}$and maximum

signed degree of contradiction $\beta^{\pm}_{cf}$.
•Remove the selected edge and update the graph.
• Repeat the process until all cycles in the graph are balanced.

Definition 13 (Complement of a Plithogenic Signed Graph)

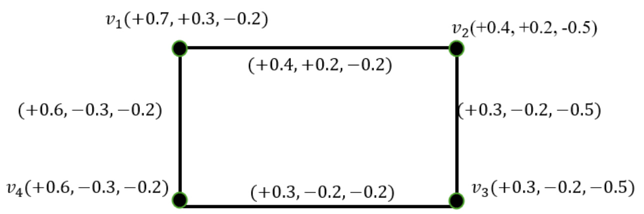
Let $P_{SG} = (P_M, P_N, \sigma)$ be a plithogenic signed graph, where $P_M = \left(M, \mu, M_\mu, \alpha^{\pm}_{df}, \alpha^{\pm}_{cf}\right)$is the plithogenic set of vertices and $P_N = \left(N, \nu, N_\nu, \beta^{\pm}_{df}, \beta^{\pm}_{cf}\right)$is the plithogenic set of edges . The complement of P_{SG}, denoted by P^c_{SG}, is defined as $P^c_{SG} = (P^c_M, P^c_N, \sigma^c)$,where

1. $P^c_M = (M, \mu, M_\mu, (\alpha^{\pm}_{df})^c, , (\alpha^{\pm}_{cf})^c)$with
$$(\alpha^{\pm}_{df})^c(x, a) = -\alpha^{\pm}_{df}(x, a), \forall (x, a) \in M \times M_\mu;$$

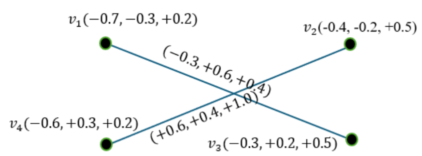
2. $P^c_N = (N^c, \nu, N_\nu, (\beta^{\pm}_{df})^c, (\beta^{\pm}_{cf})^c)$, where $N^c = \{uv \in V \times V : uv \notin N\}$, and
$$(\beta^{\pm}_{df})^c(x, a) = min\{\alpha^{\pm}_{df}(x, a), \alpha^{\pm}_{df}(y, b)\} - \beta^{\pm}_{df}(x, a), \forall (x, a) \in N \times N_\nu;$$

3. The sign function is complemented by
$$\sigma^c(e) = -\sigma(e), \forall e \in E.$$

Thus, the complement of a plithogenic signed graph is obtained by reversing the polarity of all signed appurtenance values and edge signs while preserving the underlying attribute and contradiction structures.



Example 2: The example of complement of a Plithogenic Signed Graph



Complement of a Plithogenic Signed Graph

Theorem 1

The complement of a balanced plithogenic signed graph is always positive if it contains an odd length cycle.

Proof.

Let $P_{SG} = (P_M, P_N, \sigma)$be a balanced plithogenic signed graph and let (V, E, σ)be its induced signed graph. Suppose that $P^c_{SG}$is the complement of P_{SG}.

In the complement graph, every odd length cycle has an odd number of vertices and hence an even number of incident edges per vertex. Therefore, the parity of negative edges in such a cycle is even. Since the sign of a cycle is the product of the signs of its edges, and since an even number of negative edges yields a positive product, it follows that every odd length cycle in P_{SG}^{c} is positive.

Hence, the complement of a balanced plithogenic signed graph is always positive if it contains an odd length cycle.

2.1 UNION OF PLITHOGENIC SIGNED GRAPHS

Definition 10 Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be any two crisp graphs. Also suppose that P_{SG_1} and P_{SG_2} be any two plithogenic signed graphs such that $P_{SG_1} = (P_{SM_1}, P_{SN_1}, \sigma_1)$, where $P_{SM_1} = (M_1, \mu_1, M_{\mu_1}, \alpha_{1df}^{\pm}, \alpha_{1cf}^{\pm})$ and $P_{SN_1} = (N_1, \nu_1, N_{\nu_1}, \beta_{1df}^{\pm}, \beta_{1cf}^{\pm})$. Also $P_{SG_2} = (P_{SM_2}, P_{SN_2}, \sigma_2)$, where $P_{SM_2} = (M_2, \mu_2, M_{\mu_2}, \alpha_{2df}^{\pm}, \alpha_{2cf}^{\pm})$ and $P_{SN_2} = (N_2, \nu_2, N_{\nu_2}, \beta_{2df}^{\pm}, \beta_{2cf}^{\pm})$.

Then their union is defined as $P_{SG_1} \cup P_{SG_2} = (P_{SM_1} \cup P_{SM_2}, P_{SN_1} \cup P_{SN_2}, \sigma)$,where $P_{SM_1} \cup P_{SM_2} = (M_1 \cup M_2, \mu_1 \cup \mu_2, M_{\mu_1} \cup M_{\mu_2}, \alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm}, \alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm})$ is the union of plithogenic signed sets for vertices. Here $(M_1 \cup M_2) \subset (V_1 \cup V_2)$, $\mu_1 \cup \mu_2$ is an attribute, $M_{\mu_1} \cup M_{\mu_2}$ is the corresponding range of attribute values and $\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm}: (M_1 \cup M_2) \times (M_{\mu_1} \cup M_{\mu_2}) \to [-1,1]$ is the signed degree of appurtenance (SDAF) for vertices such that

1. $(\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm})(x, a) = \alpha_{1df}^{\pm}(x, a)$ if $(x, a) \in (M_1 \times M_{\mu_1}) \setminus (M_2 \times M_{\mu_2})$,
2. $(\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm})(x, a) = \alpha_{2df}^{\pm}(x, a)$ if $(x, a) \in (M_2 \times M_{\mu_2}) \setminus (M_1 \times M_{\mu_1})$,
3. $(\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm})(x, a) = max\{\alpha_{1df}^{\pm}(x, a), \alpha_{2df}^{\pm}(x, a)\}$ if $(x, a) \in (M_1 \times M_{\mu_1}) \cap (M_2 \times M_{\mu_2})$.

Also $\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm}: (M_{\mu_1} \cup M_{\mu_2}) \times (M_{\mu_1} \cup M_{\mu_2}) \to [-1,1]$ is the signed degree of contradiction (SDCF)for vertices such that $(\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm})(a, a) = 0$ and $(\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm})(a, b) = (\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm})(b, a)$.

Similarly $P_{SN_1} \cup P_{SN_2} = (N_1 \cup N_2, \nu_1 \cup \nu_2, N_{\nu_1} \cup N_{\nu_2}, \beta_{1df}^{\pm} \cup \beta_{2df}^{\pm}, \beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm})$ is the union of plithogenic signed sets for edges, where $(\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm}): (N_1 \cup N_2) \times (N_{\nu_1} \cup N_{\nu_2}) \to [-1,1]$ is the signed degree of appurtenance for edges defined and $(\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm}): (N_{\nu_1} \cup N_{\nu_2}) \times (N_{\nu_1} \cup N_{\nu_2})[-1,1]$ is the signed degree of contradiction for edges and is defined as,

4. $(\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm})((x_1, a), (x_2, b)) = \beta_{1df}^{\pm}((x_1, a), (x_2, b))$ if $((x_1, a), (x_2, b)) \in (N_1 \times N_{\nu_1}) \setminus (N_2 \times N_{\nu_2})$,
5. $(\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm})((x_1, a), (x_2, b)) = \beta_{2df}^{\pm}((x_1, a), (x_2, b))$ if $((x_1, a), (x_2, b)) \in (N_2 \times N_{\nu_2}) \setminus (N_1 \times N_{\nu_1})$,
6. $(\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm})((x_1, a), (x_2, b) = max\{\beta_{1df}^{\pm}((x_1, a), (x_2, b)), \beta_{2df}^{\pm}((x_1, a), (x_2, b))\}$ if $((x_1, x_{\nu_1}), (x_2, x_{\nu_2})) \in (N_1 \times N_{\nu_1}) \cap (N_2 \times N_{\nu_2})$ also we have SDCF for edges $(\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm}): (N_{\nu_1} \cup N_{\nu_2}) \times (N_{\nu_1} \cup N_{\nu_2})[-1,1]$ such that $(\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm})((a, b), (a, b)) = 0$ and $(\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm})((a, b), (c, d)) = (\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm})((c, d), (a, b))$ for all $((a, b), (c, d)) \in (N_{\nu_1} \cup N_{\nu_2}) \times (N_{\nu_1} \cup N_{\nu_2})$.Then $P_{SG_1 \cup SG_2} = (P_{SM_1 \cup M_2}, P_{SN_1 \cup N_2}, \sigma)$ is a plithogenic signed graph

If $\left(\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm}\right)\left((x,a),(y,b)\right) \leq min\begin{Bmatrix}\left(\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm}\right)(x,a), \\ \left(\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm}\right)(y,b)\end{Bmatrix}$ (15)

for all $\left((x,a),(y,b)\right) \in (N_1 \cup N_2) \times (N_{v_1} \cup N_{v_2})$;

$$\left(\left(\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm}\right)\left((a,b),(c,d)\right)\right) \leq min\begin{Bmatrix}\left(\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm}\right)(a,b), \\ \left(\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm}\right)(c,d)\end{Bmatrix} \quad (16)$$

for all $\left((a,b),(c,d)\right) \in \left(N_{v_1} \cup N_{v_2}\right)$.

Example 3

Consider any two plithogenic signed graphs $P_{SG_1} = \left(P_{SV_1}, P_{SE_1}, \sigma_1\right)$and $P_{SG_2} = \left(P_{SV_2}, P_{SE_2}, \sigma_2\right)$of crisp graphs$G_1^* = (V_1, E_1)$and $G_2^* = (V_2, E_2)$, where $P_{SV_1} = \left(P_1, \mu_1, P_{\mu_1}, \alpha_{1df}^{\pm}, \alpha_{1cf}^{\pm}\right)$and $P_{SE_1} = \left(Q_1, v_1, Q_{v_1}, \beta_{1df}^{\pm}, \beta_{1cf}^{\pm}\right)$ such that $P_1 \subseteq V_1$is a graph with vertices $P_1 = \{x, y, z\}$and edges $Q_1 = \{xy, yz, zx\}$, $(\mu_1, v_1)$is an attribute, $\left(P_{\mu_1}, Q_{v_1}\right)$be a graph with vertices $P_{\mu_1} = \{a, b, d\}$and edges $Q_{v_1} = \{ab, bd\}$. Also let $\alpha_{1df}^{\pm}: P_1 \times P_{\mu_1} \to [-1,1]$and $\alpha_{1cf}^{\pm}: P_{\mu_1} \times P_{\mu_1} \to [-1,1]$ be the signed degree of appurtenance and degree of contradiction for vertices defined as Also $\beta_{1df}^{\pm}: Q_1 \times Q_{v_1} \to [-1,1]$and $\beta_{1cf}^{\pm}: Q_{v_1} \times Q_{v_1} \to [-1,1]$ be the signed degree of appurtenance and degree of contradiction for edges defined. have a plithogenic signed graph P_{SG_2}; with $P_{SV_2} = \left(P_2, \mu_2, P_{\mu_2}, \alpha_{2df}^{\pm}, \alpha_{2cf}^{\pm}\right)$& $Q_{SE_2} = \left(Q_2, v_2, Q_{v_2}, \beta_{2df}^{\pm}, \beta_{2cf}^{\pm}\right)$
such that $(P_2, Q_2)$is a graph with vertices $P_2 = \{x, z, r\}$and edges $Q_2 = \{xz, zr, xr\}$, (μ_2, v_2)be an attribute, $\left(P_{\mu_2}, Q_{v_2}\right)$is a graph with vertices $P_{\mu_2} = \{a, c, d\}$and edges $Q_{v_2} = \{ac, cd\}$. Also let $\alpha_{2df}^{\pm}: P_2 \times P_{\mu_2} \to [-1,1]$and $\alpha_{2cf}^{\pm}: P_{\mu_2} \times P_{\mu_2} \to [-1,1]$ be the signed degree of appurtenance and degree of contradiction for vertices defined as Also $\beta_{2df}^{\pm}: Q_2 \times Q_{v_2} \to [-1,1]$ and $\beta_{2cf}^{\pm}: Q_{v_2} \times Q_{v_2} \to [-1,1]$ be the signed degree of appurtenance and degree of contradiction for edges defined. Then their union is defined as $P_{SG_1} \cup P_{SG_2} = \left(P_{SV_1} \cup P_{SV_2}, Q_{SE_1} \cup Q_{SE_2}, \sigma\right)$ where $P_{SV_1} \cup P_{SV_2} = (P_1 \cup P_2, \mu_1 \cup \mu_2, P_{\mu_1} \cup P_{\mu_2}, (\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm}), (\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm}))$ and $Q_{SE_1} \cup Q_{SE_2} = (Q_1 \cup Q_2, v_1 \cup v_2, Q_{v_1} \cup Q_{v_2}, (\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm}), (\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm}))$.Here we have $P_1 \cup P_2 = \{x, y, z, r\}$, $Q_1 \cup Q_2 = \{xy, yz, zx, zr, xr\}$ such that $(P_1 \cup P_2, Q_1 \cup Q_2)$is a graph, $(\mu_1 \cup \mu_2, v_1 \cup v_2)$is an attribute, $P_{\mu_1} \cup P_{\mu_2} = \{a, b, c, d\}$ is the range of attribute for vertices and $Q_{v_1} \cup Q_{v_2} = \{ab, bd, ac, cd\}$ is range of attribute for edges so that $\left(P_{\mu_1} \cup P_{\mu_2}, Q_{v_1} \cup Q_{v_2}\right)$ is a graph.

$\alpha_{1df}^{\pm}$	x	y	z	$\alpha_{1cf}^{\pm}$	a	b	c
a	+0.2	+0.3	+0.4	a	0	+0.4	+0.5
b	−0.5	+0.6	+0.1	b	+0.4	0	+0.3
d	+0.1	−0.5	−0.2	d	−0.5	+0.3	0

Table 3: SDAF and SDCF for vertices of P_{SG_1}

Table 4: SDAF and SDCF for Edges of P_{SG_1}

$\beta^{\pm}_{1df}$	xy	yz	xz	$\beta^{\pm}_{1cf}$	ab	bd
ab	+0.2	+0.3	+0.1	ab	0	+0.4
bd	−0.5	+0.1	-0.2	bd	+0.4	0

Table 5: SDAF and SDCF for vertices of P_{SG_2}

$\alpha^{\pm}_{2df}$	x	z	r	$\alpha^{\pm}_{2cf}$	a	c	d
a	+0.2	+0.4	+0.3	a	0	+0.5	+0.4
c	−0.6	+0.1	+0.3	c	+0.5	0	+0.6
d	+0.4	−0.2	−0.1	d	−0.4	+0.6	0

Table 6: SDAF and SDCF for Edges of P_{SG_2}

$\beta^{\pm}_{2df}$	xz	zr	xr	$\beta^{\pm}_{2cf}$	ac	cd
ac	-0.1	+0.3	+0.2	ac	0	+0.5
cd	−0.6	−0.1	−0.6	cd	+0.5	0

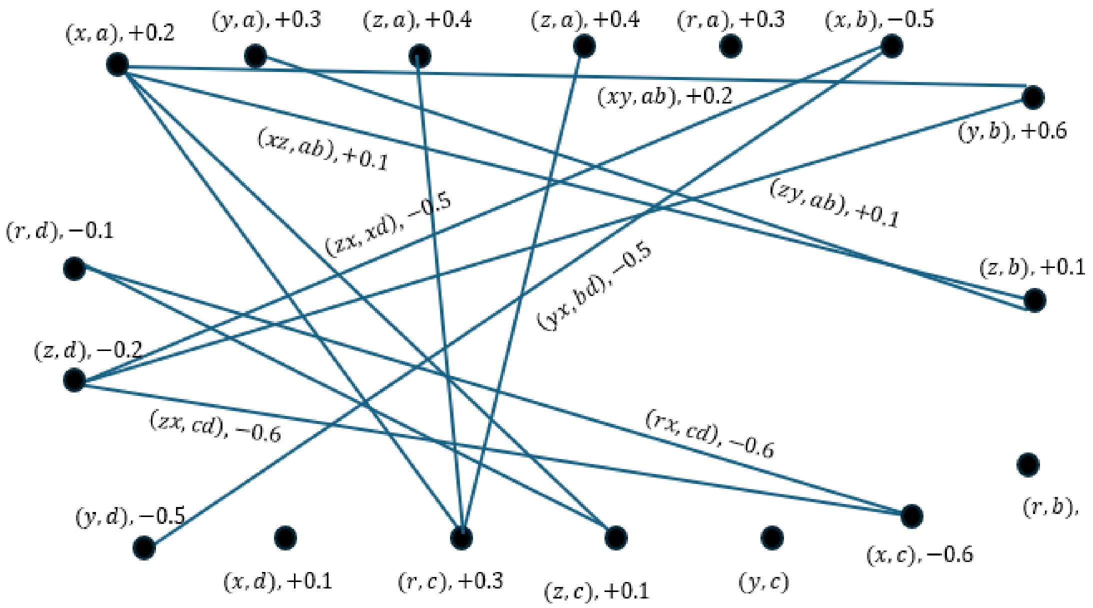
Table 7: SDAF and SDCF for vertices of $P_{SG_1 \cup SG_2}$

$\alpha^{\pm}_{1df} \cup \alpha^{\pm}_{2df}$	x	y	z	r	$\alpha^{\pm}_{1cf} \cup \alpha^{\pm}_{2cf}$	a	b	c	d
a	+0.2	+0.3	+0.4	+0.3	a	0	+0.4	+0.5	+0.4
b	-0.5	+0.6	+0.1	---	b	+0.4	0	+0.3	+0.6
c	-0.6	---	+0.1	+0.3	c	+0.5	+0.6	0	+0.6
d	+0.1	-0.5	-0.2	-0.1	d	-0.4	+0.3	+0.6	0

Table 8: SDAF and SDCF for edges of $P_{SG_1 \cup SG_2}$

$\beta^{\pm}_{1df} \cup \beta^{\pm}_{2df}$	xy	yz	xz	rz	xr	$\beta^{\pm}_{1cf} \cup \beta^{\pm}_{2cf}$	ab	bd	ac	cd
ab	+0.2	+0.3	+0.1	---	---	ab	0	+0.4	-0.1	+0.5
bd	−0.5	+0.1	-0.2	---	---	bd	+0.4	0	+0.3	+0.4
ac	---	---	-0.1	+0.3	+0.2	ac	+0.5	-0.2	0	+0.5

cd	---	---	-0.6	-0.1	-0.6	cd	-0.1	+0.4	-0.3	0



Union of Plithogenic Signed Graph

2.2 JOIN OF PLITHOGENIC SIGNED GRAPHS

Definition 11 Consider any two plithogenic signed graphs $P_{SG_1} = (P_{SV_1}, Q_{SE_1}, \sigma_1)$and $P_{SG_2} = (P_{SV_2}, Q_{SE_2}, \sigma_2)$as given in Definition 10 of crisp signed graphs $G_1^* = (V_1, E_1, \sigma_1)$and $G_2^* = (V_2, E_2, \sigma_2)$, We define their join as $P_{SG_1} + P_{SG_2} = (P_{SV_1} + P_{SV_2}, Q_{SE_1} + Q_{SE_2}, \sigma)$ where $P_{SV_1} + P_{SV_2} = (P_1 \cup P_2, \mu_1 \cup \mu_2, P_{\mu_1} \cup P_{\mu_2}, (\alpha_{1df}^{\pm} \cup \alpha_{2df}^{\pm}), (\alpha_{1cf}^{\pm} \cup \alpha_{2cf}^{\pm}))$ and $Q_{SE_1} + Q_{SE_2} = (Q_1 \cup Q_2 \cup Q', \nu_1 \cup \nu_2, Q_{\nu_1} \cup Q_{\nu_2}, (\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm}), (\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm}))$ and $Q' = P_1 \times P_2$. Here we have $P_1 \cup P_2 \subseteq V_1 \cup V_2$,$(P_1 \cup P_2, Q_1 \cup Q_2 \cup Q')$is a graph, $(\mu_1 \cup \mu_2, \nu_1 \cup \nu_2)$is an attribute, and $P_{\mu_1} \cup P_{\mu_2}$is the range of attribute. The sign function σon the join is defined by

$$\sigma(e) = \begin{cases} \sigma_1(e), & e \in Q_1 \\ \sigma_2(e), & e \in Q_2 \\ +1, & e \in Q' \end{cases}$$

attributes for vertices and $Q_{\nu_1} \cup Q_{\nu_2}$is range of attributes for edges so that $(P_{\mu_1} \cup P_{\mu_2}, Q_{\nu_1} \cup Q_{\nu_2})$ is a graph. Here the signed degree of appurtenance for vertices of $P_{SG_1 + SG_2}$is defined as

$$(\alpha_{1df}^{\pm} + \alpha_{2df}^{\pm}) : (P_1 \cup P_2) \times (P_{\mu_1} \cup P_{\mu_2}) \to [-1,1]$$

such that

(i) $(\alpha_{1df}^{\pm} + \alpha_{2df}^{\pm})(x, x_\mu) = \alpha_{1df}^{\pm}(x, x_\mu)$if $(x, x_\mu) \in (P_1 \times P_{\mu_1}) \setminus (P_2 \times P_{\mu_2})$,

(ii) $(\alpha_{1df}^{\pm} + \alpha_{2df}^{\pm})(x, x_\mu) = \alpha_{2df}^{\pm}(x, x_\mu)$if $(x, x_\mu) \in (P_2 \times P_{\mu_2}) \setminus (P_1 \times P_{\mu_1})$,

(iii) $(\alpha_{1df}^{\pm}+\alpha_{2df}^{\pm})(x,x_\mu)=max\,\{\alpha_{1df}^{\pm}(x,x_\mu),\alpha_{2df}^{\pm}(x,x_\mu)\}$if $(x,x_\mu)\in(P_1\times P_{\mu_1})\cap(P_2\times P_{\mu_2})$.
The degree of contradiction for vertices is
$(\alpha_{1cf}^{\pm}+\alpha_{2cf}^{\pm}):(P_{\mu_1}\cup P_{\mu_2})\times(P_{\mu_1}\cup P_{\mu_2})\to[-1,1]$ such that $(\alpha_{1cf}^{\pm}+\alpha_{2cf}^{\pm})(a,a)=0$for all $(a,a)$and$(\alpha_{1cf}^{\pm}+\alpha_{2cf}^{\pm})(a,b)=(\alpha_{1cf}^{\pm}+\alpha_{2cf}^{\pm})(b,a)$ for all (a,b).
Also the signed degree of appurtenance for edges of $P_{SG_1+SG_2}$, i.e. for $Q_1\cup Q_2\cup Q'$, where Q'stands for the set of all edges joining the nodes of $P_1$and P_2, is given by
$(\beta_{1df}^{\pm}+\beta_{2df}^{\pm}):(Q_1\cup Q_2\cup Q')\times(Q_{\nu_1}\cup Q_{\nu_2})\to[-1,1]$ and the degree of contradiction for edges is $(\beta_{1cf}^{\pm}+\beta_{2cf}^{\pm}):(Q_{\nu_1}\cup Q_{\nu_2})\times(Q_{\nu_1}\cup Q_{\nu_2})\to[-1,1]$.
(ii)$(\beta_{1df}^{\pm}+\beta_{2df}^{\pm})((x,a),(y,b))=(\beta_{1df}^{\pm}\cup\beta_{2df}^{\pm})((x,a),(y,b))$ if $((x,a),(y,b))\in(Q_1\cup Q_2)\times Q_{\nu_1\cup\nu_2}$.
(iii) $(\beta_{1df}^{\pm}+\beta_{2df}^{\pm})((x,a),(y,b))=min\,\{\alpha_{1cf}^{\pm}(x,a),\alpha_{2cf}^{\pm}(y,b)\}$ and the degree of contradiction for edges is $(\beta_{1cf}^{\pm}+\beta_{2cf}^{\pm}):Q_{\nu_1\cup\nu_2}\times Q_{\nu_1\cup\nu_2}\to[-1,1]$ such that $((\beta_{1cf}^{\pm}+\beta_{2cf}^{\pm})((a,b),(a,b))=0$for all $((a,b),(a,b))\in Q_{\nu_1\cup\nu_2}\times Q_{\nu_1\cup\nu_2}$. Also we have $((\beta_{1cf}^{\pm}+\beta_{2cf}^{\pm})((a,b),(c,d))=((\beta_{1cf}^{\pm}+\beta_{2cf}^{\pm})((c,d),(a,b))$
Then $P_{SG_1+SG_2}=(P_{SV_1+SV_2},Q_{SE_1+SE_2},\sigma)$is a plithogenic signed graph iff

$$(\beta_{1df}^{\pm}+\beta_{2df}^{\pm})((x,a),(y,b))\le min\,\{(\alpha_{1df}^{\pm}+\alpha_{2df}^{\pm})(x,a),(\alpha_{1df}^{\pm}+\alpha_{2df}^{\pm})(y,b)\}\qquad(18)$$

for all $((x,a),(y,b))\in(Q_1\cup Q_2\cup Q')\times Q_{\nu_1\cup\nu_2}$;also

$$((\beta_{1cf}^{\pm}+\beta_{2cf}^{\pm})((a,b),(c,d))\le min\,\{(\alpha_{1cf}^{\pm}+\alpha_{2cf}^{\pm})(a,b),(\alpha_{1cf}^{\pm}+\alpha_{2cf}^{\pm})(c,d)\}\qquad(19)$$

for all $((a,b),(c,d))$.

Example 4: Consider any two plithogenic signed graphs $P_{SG_1}=(P_{SV_1},Q_{SE_1},\sigma_1)$and $P_{SG_2}=(P_{SV_2},Q_{SE_2},\sigma_2)$ of crisp signed graphs $G_1^*=(V_1,E_1,\sigma_1)$and $G_2^*=(V_2,E_2,\sigma_2)$, where $P_{SV_1}=(P_1,\mu_1,P_{\mu_1},\alpha_{1df}^{\pm},\alpha_{1cf}^{\pm})$and $P_{SE_1}=(Q_1,\nu_1,Q_{\nu_1},\beta_{1df}^{\pm},\beta_{1cf}^{\pm})$ such that (P_1,Q_2) be a graph with vertices $P_1=\{x,y,z\}$and edges $Q_1=\{xy,yz,xz\}$, Let (l_1,m_1)be an attribute, and (P_{l_1},Q_{m_1})be a graph with vertices$P_{l_1}=\{a,b\}$ and edges $Q_{m_1}=\{ab\}$. Also let $\alpha_{1df}^{\pm}:P_1\times P_{l_1}\to[-1,1]$ be the signed degree of appurtenance function (SDAF) for vertices, and $\alpha_{1cf}^{\pm}:P_{l_1}\times P_{l_1}\to[-1,1]$be the signed degree of contradiction function (SDCF) for vertices.Also let $\beta_{1df}^{\pm}:Q_1\times Q_{m_1}\to[-1,1]$ be the signed degree of appurtenance function for edges, and $\beta_{1cf}^{\pm}:Q_{m_1}\times Q_{m_1}\to[-1,1]$ be the signed degree of contradiction function for edges. For the second graph P_{SG_2},Let $P_{SV_2}=(P_2,l_2,P_{l_2},\alpha_{2df}^{\pm},\alpha_{2cf}^{\pm})$ and $Q_{SE_2}=(Q_2,m_2,Q_{m_2},\beta_{2df}^{\pm},\beta_{2cf}^{\pm})$, such that $(P_2,Q_2)$is a graph with vertices $P_2=\{x,z,r\}$ and edges $Q_2=\{xz,zr,xr\}$. Let (l_2,m_2)be an attribute, and (P_{l_2},Q_{m_2})be a graph with vertices $P_{l_2}=\{a,c\}$ and edges $Q_{m_2}=\{ac\}$. Also let $\alpha_{2df}^{\pm}:P_2\times P_{l_2}\to[-1,1]$ be the SDAF for vertices, and $\alpha_{2cf}^{\pm}:P_{l_2}\times P_{l_2}\to[-1,1]$ be the SDCF for vertices. Also let $\beta_{2df}^{\pm}:Q_2\times Q_{m_2}\to[-1,1]$ be the SDAF for edges, and $\beta_{2cf}^{\pm}:Q_{m_2}\times Q_{m_2}\to[0,1]$ be the SDCF for edges. We define their join as $P_{SG_1}+P_{SG_2}=(P_{SV_1}+SV_2,\,Q_{SE_1}+SE_2,\,\sigma)$

where $P_{SV_1} + P_{SV_2} = (P_1 \cup P_2,\ l_1 \cup l_2,\ P_{l_1 \cup l_2},\ (\alpha_{1df}^{\pm} + \alpha_{2df}^{\pm}),\ (\alpha_{1cf}^{\pm} + \alpha_{2cf}^{\pm}))$ and $Q_{SE_1} + Q_{SE_2} = (Q_1 \cup Q_2,\ m_1 \cup m_2,\ Q_{m_1 \cup m_2},\ (\beta_{1df}^{\pm} + \beta_{2df}^{\pm}),\ (\beta_{1cf}^{\pm} + \beta_{2cf}^{\pm}))$. Also, $Q_{SE_1} \cup SE_2 = (Q_1 \cup Q_2 \cup Q_0,\ m_1 \cup m_2,\ Q_{m_1 \cup m_2},\ (\beta_{1df}^{\pm} \cup \beta_{2df}^{\pm}),\ (\beta_{1cf}^{\pm} \cup \beta_{2cf}^{\pm}))$. Here we have $P_1 \cup P_2 \subseteq V_1 \cup V_2, Q_1 \cup Q_2 \subseteq E_1 \cup E_2$, such that $(P_1 \cup P_2, Q_1 \cup Q_2)$is a graph, $(l_1 \cup l_2, m_1 \cup m_2)$is an attribute, $P_{l_1 \cup l_2}$is the range of attributes for vertices and $Q_{m_1 \cup m_2}$is the range of attributes for edges so that $\left(P_{l_1 \cup l_2}, Q_{m_1 \cup m_2}\right)$ is a graph.

Here the signed degree of appurtenance function (SDAF) for vertices of $P_{SG_1} + P_{SG_2}$is

$$(\alpha_{1df}^{\pm} + \alpha_{2df}^{\pm}): (P_1 \cup P_2) \times P_{l_1 \cup l_2} \to [-1,1],$$

and the signed degree of contradiction function (SDCF) for vertices is

$\left(\alpha_{1cf}^{\pm} + \alpha_{2cf}^{\pm}\right): P_{l_1 \cup l_2} \times P_{l_1 \cup l_2} \to [-1,1]$, defined as in the corresponding tables.

Also, the signed degree of appurtenance function for edges is

$(\beta_{1df}^{\pm} + \beta_{2df}^{\pm}): (Q_1 \cup Q_2 \cup Q_0) \times Q_{m_1 \cup m_2} \to [-1,1]$, and the signed degree of contradiction function for edges is $(\beta_{1cf}^{\pm} + \beta_{2cf}^{\pm}): Q_{m_1 \cup m_2} \times Q_{m_1 \cup m_2} \to [0,1]$, defined as in the corresponding tables. Each edge $e \in Q_1 \cup Q_2 \cup Q_0$is assigned a sign $\sigma(e) \in \{+1, -1\}$.

Table 9: SDAF and SDCF for vertices of P_{SG_1}

$\alpha_{1df}^{\pm}$	x	y	z	$\alpha_{1cf}^{\pm}$	ab
a	+0.2	+0.3	+0.4	ab	0
b	−0.5	+0.6	+0.1		

Table 10: SDAF and SDCF for Edges of P_{SG_1}

$\beta_{1df}^{\pm}$	xy	yz	xz	$\beta_{1cf}^{\pm}$	ab
ab	+0.2	+0.3	+0.1	ab	0

Table 11: SDAF and SDCF for vertices of P_{SG_2}

$\alpha_{2df}^{\pm}$	x	z	r	$\alpha_{2cf}^{\pm}$	ac
a	+0.2	+0.4	+0.3	ac	0
c	−0.6	+0.1	+0.3		

Table 12: SDAF and SDCF for Edges of P_{SG_2}

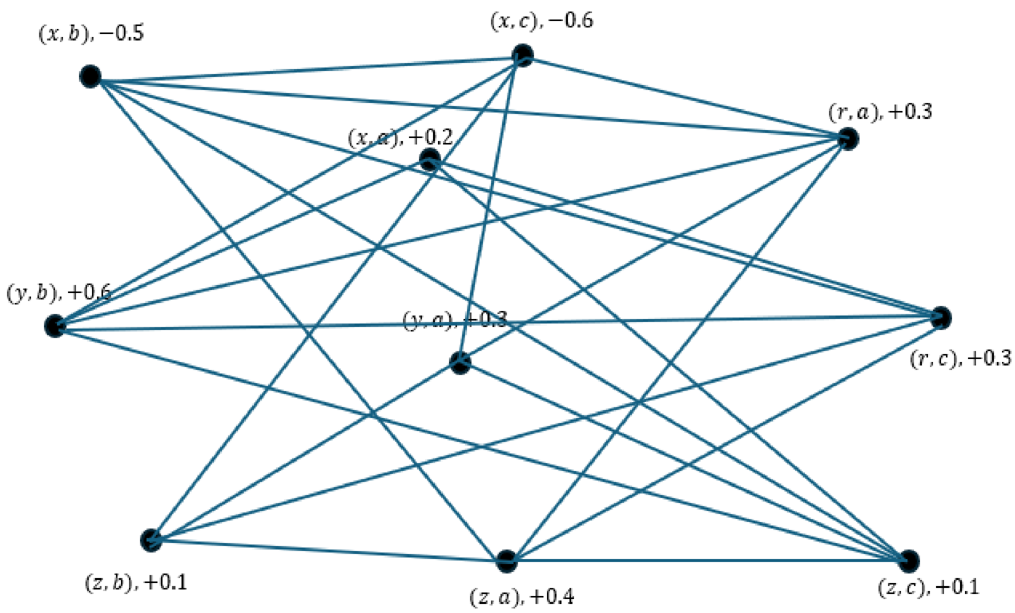
$\beta_{2df}^{\pm}$	xz	zr	xr	$\beta_{2cf}^{\pm}$	ac
ac	+0.1	+0.3	+0.2	ac	0

Table 13: SDAF and SDCF for Edges of$P_{SG_1} + P_{SG_2}$

$\beta_{1df}^{\pm} + \beta_{2df}^{\pm}$	xz	yz	xz	rz	xr	yr	$\beta_{1cf}^{\pm} + \beta_{2cf}^{\pm}$	ab	ac	bc
ab	+0.1	+0.1	+0.1	+0.1	+0.3	+0.2	ab	0	+0.5	-0.2
ac	+0.1	+0.1	+0.1	+0.1	+0.3	+0.2	ac	+0.5	0	+0.1
bc	+0.1	+0.1	+0.1	+0.1	+0.3	+0.2	bc	-0.2	+.0.1	0

Table 14: SDAF and SDCF for vertices of $P_{SG_1} + P_{SG_2}$

$\alpha^{\pm}_{1df} + \alpha^{\pm}_{2df}$	x	y	z	r	$\alpha^{\pm}_{1cf} + \alpha^{\pm}_{2cf}$	ab	ac	bc
a	+0.2	+0.3	+0.4	+0.3	ab	0	+0.5	+0.4
b	-0.5	+0.6	+0.1	-----	ac	+0.5	0	-0.1
c	-0.6	----	+0.1	+0.3	bc	+0.4	-0.1	0



2.3 CARTESIAN PRODUCT OF PLITHOGENIC SIGNED GRAPHS

Definition 12 Consider any two plithogenic signed graphs $P_{SG_1} = (P_{M_1}, P_{N_1}, \sigma_1)$and $P_{SG_2} = (P_{M_2}, P_{N_2}, \sigma_2)$ as defined on crisp signed graphs $G_1^* = (V_1, E_1, \sigma_1)andG_2^* = (V_2, E_2, \sigma_2)$. We define their Cartesian product as $P_{SG_1 \times SG_2} = \left(P_{M_1 \times M_2}, P_{N_1 \times N_2}, \sigma_\times\right)$ where

$$P_{M_1 \times M_2} = \left(M_1 \times M_2, \mu_1 \times \mu_2, (M_{\mu_1} \times M_{\mu_2}), \alpha^{\pm}_{1df} \times \alpha^{\pm}_{2df}, \alpha^{\pm}_{1cf} \times \alpha^{\pm}_{2cf}\right)$$

is the Cartesian product of plithogenic sets for vertices; where $(M_1 \times M_2) \subset (V_1 \times V_2), \mu_1 \times \mu_2$ is an attribute,and $\left(M_{\mu_1} \times M_{\mu_2}\right)$is the corresponding range of attribute values.

Also, $\alpha^{\pm}_{1df} \times \alpha^{\pm}_{2df}: (M_1 \times M_{\mu_1}) \times (M_2 \times M_{\mu_2}) \to [-1,1]$is the Signed Degree of Appurtenance Function (SDAF) for vertices such that $(\alpha^{\pm}_{1df} \times \alpha^{\pm}_{2df})\left((x_1, x_{1\mu}), (x_2, x_{2\mu})\right) \in [-1,1]$, and is defined as $(\alpha^{\pm}_{1df} \times \alpha^{\pm}_{2df})\left((x_1, x_{1\mu}), (x_2, x_{2\mu})\right) = min\left\{\alpha^{\pm}_{1df}(x_1, x_{1\mu}), \alpha^{\pm}_{2df}(x_2, x_{2\mu})\right\}$ for all $\left((x_1, x_{1\mu}), (x_2, x_{2\mu})\right) \in (M_1 \times M_{\mu_1}) \times (M_2 \times M_{\mu_2})$. Let$P_{N_1 \times N_2} = \left(N_1 \times N_2, \nu_1 \times \nu_2, (N_{\nu_1} \times N_{\nu_2}), \beta^{\pm}_{1df} \times \beta^{\pm}_{2df}, \beta^{\pm}_{1cf} \times \beta^{\pm}_{2cf}\right)$ be the Cartesian product of plithogenic sets for edges, where

$(N_1 \times N_2) \subset (E_1 \times E_2), \nu_1 \times \nu_2$is some attribute, and $(N_{\nu_1} \times N_{\nu_2})$is the corresponding range of attribute values. Also, $\beta_{1df}^{\pm} \times \beta_{2df}^{\pm}: (N_1 \times N_{\nu_1}) \times (N_2 \times N_{\nu_2}) \to [-1,1]$ is the Signed Degree of Appurtenance Function (SDAF) for edges defined as:

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})\left(\begin{pmatrix}((x, x_\mu), (x_2, x_{2\mu})), \\ ((x, x_\mu), (x_2', x_{2\mu}'))\end{pmatrix}\right) = min\,\{\alpha_{1df}^{\pm}(x, x_\mu), \beta_{2df}^{\pm}((x_2, x_{2\mu}), (x_2', x_{2\mu}'))\}$$

for all $(((x, x_\mu), (x_2, x_{2\mu})), ((x, x_\mu), (x_2', x_{2\mu}'))) \in (N_1 \times N_{\nu_1}) \times (N_2 \times N_{\nu_2})$.

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})\left(\begin{pmatrix}((x_1, x_{1\mu}), (x, x_\mu)), \\ ((x_1', x_{1\mu}'), (x, x_\mu))\end{pmatrix}\right) = min\,\{\beta_{1df}^{\pm}((x_1, x_{1\mu}), (x_1', x_{1\mu}')), \alpha_{2df}^{\pm}(x, x_\mu)\}$$

for all $(((x_1, x_{1\mu}), (x, x_\mu)), ((x_1', x_{1\mu}'), (x, x_\mu))) \in (N_1 \times N_{\nu_1}) \times (N_2 \times N_{\nu_2})$.Also, $\alpha_{1cf}^{\pm} \times \alpha_{2cf}^{\pm}: (M_{\mu_1} \times M_{\mu_1}) \times (M_{\mu_2} \times M_{\mu_2}) \to [-1,1]$ is the Degree of Contradiction Function for vertices, such that $(\alpha_{1cf}^{\pm} \times \alpha_{2cf}^{\pm})((x_{i\mu}, x_{i\mu}), (x_{i\mu}', x_{i\mu}')) = 0$ for all $((x_{i\mu}, x_{i\mu}), (x_{i\mu}', x_{i\mu}')) \in (M_{\mu_1} \times M_{\mu_1}) \times (M_{\mu_2} \times)$,and symmetry holds:
$(\alpha_{1cf} \times \alpha_{2cf})((x_{i\mu}, x_{j\mu}), (x_{i\mu}', x_{j\mu}')) = (\alpha_{1cf} \times \alpha_{2cf})((x_{j\mu}, x_{i\mu}), (x_{j\mu}', x_{i\mu}'))$.Similarly, $\beta_{1cf}^{\pm} \times \beta_{2cf}^{\pm}: (N_{\nu_1} \times N_{\nu_2}) \to [-1,1]$ is the Degree of Contradiction Function for edges, such hat $(\beta_{1cf}^{\pm} \times \beta_{2cf}^{\pm})\left(\begin{pmatrix}((x_{1\mu}, x_{1\mu}), (x_{1\mu}, x_{1\mu})), \\ ((x_{1\mu}, x_{1\mu}), (x_{1\mu}, x_{1\mu}))\end{pmatrix}\right) = 0$ for all elements in $(N_{\nu_1} \times N_{\nu_2})$, and symmetry holds for distinct pairs.Then $P_{SG_1 \times SG_2} = (P_{M_1 \times M_2}, P_{N_1 \times N_2}, \sigma_\times)$ is a plithogenic signed graph if and only if

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})(((x, a), (y, b)), ((z, c), (r, d))) \leq min \begin{Bmatrix} (\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x, a), (y, b)), \\ (\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((z, c), (r, d)) \end{Bmatrix}$$

for all $(((x, a), (y, b)), ((z, c), (r, d))) \in (N_1 \times N_{\nu_1}) \times (N_2 \times N_{\nu_2})$.
Each product edge $(e_1, e_2) \in N_1 \times N_2$is assigned a sign $\sigma_\times(e_1, e_2) = \sigma_1(e_1) \cdot \sigma_2(e_2)$

Example 5 Let $G_1^* = (V_1, E_1, \sigma_1), G_2^* = (V_2, E_2, \sigma_2)$be two crisp signed graphs.
Suppose that $P_{SG_1} = (P_{M_1}, P_{N_1}, \sigma_1), P_{SG_2} = (P_{M_2}, P_{N_2}, \sigma_2)$are two plithogenic signed graphs such that $P_{M_1} = (M_1, \mu_1, M_{\mu_1}, \alpha_{1df}^{\pm}, \alpha_{1cf}^{\pm})$and $P_{N_1} = (N_1, \nu_1, N_{\nu_1}, \beta_{1df}^{\pm}, \beta_{1cf}^{\pm})$.
Let $M_1 = \{x, y\} \subset V_1, M_{\mu_1} = \{a, b\}$be the vertex attribute range.Let $N_1 = \{xy\} \subset E_1, N_{\nu_1} = \{ab\}$be the edge attribute range. Cartesian Product $P_{SG_1} \times P_{SG_2}$ The vertex set is:

$$M_1 \times M_{\mu_1} = \{(x, a), (x, b), (y, a), (y, b)\}$$
$$M_2 \times M_{\mu_2} = \{(x, a), (x, c), (z, a), (z, c)\}$$

Hence the vertex pairs are all ordered pairs from these sets.SDAF for vertices

$$(\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x_1, x_{1\mu}), (x_2, x_{2\mu})) = min\,\{\alpha_{1df}^{\pm}(x_1, x_{1\mu}), \alpha_{2df}^{\pm}(x_2, x_{2\mu})\}.$$

Then $P_{SG_1 \times SG_2}$is a plithogenic signed graph iff:

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})(((x, a), (y, b)), ((z, c), (r, d))) \leq min \begin{Bmatrix} (\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x, a), (y, b)), \\ (\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((z, c), (r, d)) \end{Bmatrix}$$

for all $(((x,a),(y,b)),((z,c),(r,d))) \in (N_1 \times N_{v_1}) \times (N_2 \times N_{v_2})$.

Computations:

1. $$(\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x,a),(x,c)) = min\,\{+0.3,+0.2\} = +0.2$$
2. $$(\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x,b),(x,c)) = min\,\{+0.4,+0.2\} = +0.2$$
3. $$(\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x,b),(z,c)) = min\,\{+0.4,+0.2\} = +0.2$$
4. $$(\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((x,b),(z,a)) = min\,\{+0.4,+0.5\} = +0.4$$
5. $$(\alpha_{1df}^{\pm} \times \alpha_{2df}^{\pm})((y,a),(z,a)) = min\,\{+0.7,+0.5\} = +0.5$$

and so on. Signed DAF for edges

Case 1:

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})(((x,a),(x,c)),((x,a),(z,c))) = min\,\{\alpha_{1df}^{\pm}(x,a), \beta_{2df}^{\pm}((x,c),(z,c))\}$$
$$= min\,\{+0.3,+0.2\} = +0.2$$

Case 2:

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})(((x,a),(z,a)),((x,a),(z,c))) = min\,\{\alpha_{1df}^{\pm}(x,a), \beta_{2df}^{\pm}((z,a),(z,c))\}$$
$$= +0.2$$

Case 3:

$$(\beta_{1df}^{\pm} \times \beta_{2df}^{\pm})(((x,b),(x,a)),((y,b),(x,a))) = min\,\{\beta_{1df}^{\pm}((x,b),(y,b)), \alpha_{2df}^{\pm}(x,a)\}$$
$$= +0.4$$

For every product edge $(e_1,e_2) \in N_1 \times N_2$, $\sigma_{\times}(e_1,e_2) = \sigma_1(e_1) \cdot \sigma_2(e_2) = (+1) \cdot (+1) = +1$.

Table 15 : SDAF and SDCF for vertices of P_{SG_1}

$\alpha_{1df}^{\pm}$	xy		$\beta_{1df}^{\pm}$	xy
a	+0.3	+0.7	ab	0.3
b	+0.4	+0.6		

Table 16: SDAF and SDCF for Edges of P_{SG_1}

$\alpha_{1cf}^{\pm}$	a	b	$\beta_{1cf}^{\pm}$	ab
a	0	+0.3	ab	0
b	+0.3	0		

Table 17: SDAF and SDCF for vertices of P_{SG_2}

$\alpha^{\pm}_{2df}$	xz		$\beta^{\pm}_{2cf}$	xz
a	+0.3	+0.5	ac	0.2
c	+0.6	+0.2		

Table 18: SDAF and SDCF for Edges of P_{SG_2}

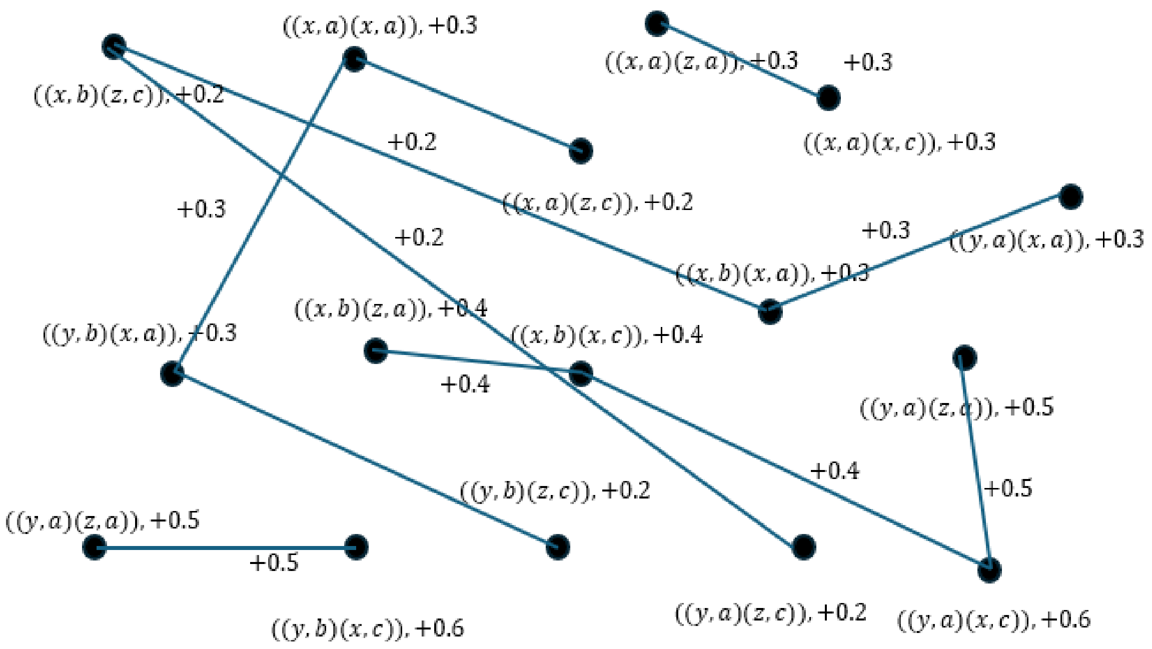
$\alpha^{\pm}_{2cf}$	a	c	$\beta^{\pm}_{2cf}$	xz
a	0	+0.5	ac	0
c	+0.5	0		

Table 19: SDAF and SDCF for vertices of $P_{SG_1} \times P_{SG_2}$

$\alpha^{\pm}_{df}$	a_1	a_2	a_3	a_4	$\alpha^{\pm}_{cf}$	a_1	a_1	a_1	a_1
V_S	+0.4	+0.11	+0.35	+0.3	a_1	0	+0.1	+0.35	+0.2
V_M	+0.34	+0.03	+0.36	+0.4	a_1	+0.1	0	+0.6	+0.4
V_n	+1	+1	+1	+1	a_1	+0.35	+0.6	0	+0.15
					a_1	+0.2	+0.4	+0.15	

Table 20: SDAF and SDCF for vertices of $P_{SG_1} \times P_{SG_2}$

$\alpha^{\pm}_{df}$	a_1	a_2	a_3	a_4	$\alpha^{\pm}_{cf}$	a_1	a_1	a_1	a_1
V_S	+0.4	+0.11	+0.35	+0.3	a_1	0	+0.1	+0.35	+0.2
V_M	+0.34	+0.03	+0.36	+0.4	a_1	+0.1	0	+0.6	+0.4
V_n	+1	+1	+1	+1	a_1	+0.35	+0.6	0	+0.15
					a_1	+0.2	+0.4	+0.15	0



Cartesian Product of Plithogenic Signed graphs

Conclusion

In this article, we have introduced and systematically developed the concept of Plithogenic Signed Graphs (PSG) as an extension of plithogenic graphs by incorporating the notion of edge sign along with degrees of appurtenance and contradiction. This framework provides a more expressive and realistic modeling tool for systems where relationships are not only uncertain but also possess positive or negative influence.We have defined and analyzed fundamental binary operations on plithogenic signed graphs and its operators. Overall, this study enriches the theoretical foundation of plithogenic graph theory and opens new directions for future research, including the study of other graph operations, balance theory in PSGs, and real-world applications in complex systems with both uncertainty and polarity".

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