International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

A Pressure-Based Criterion for the Stability of Self-Gravitating Matter with a Vacuum Tension Interpretation (VTI)

Vijay T. Ingole[1], Anant S. Wadatkar[2]

[1]Independent Researcher, Former Professor, Professor Ram Meghe Institute of Technology & Research, Amravati 444701, Maharashtra, India
[2]Assistant Professor, Department of Physics, Vinayak Vidnyan Mahavidyalaya, Nandgaon Khandeshwar, Amravati, Maharashtra, India

Abstract:

The stability of self-gravitating matter is commonly analyzed using density distributions, equations of state, or geometric trapping conditions derived from general relativity. In this work, we present a pressure-based interpretation of mass stability that applies uniformly across ordinary matter, neutron-dominated equilibrium, and compact trapped configurations. Using standard Newtonian estimates of gravitational self-pressure for ordinary celestial bodies, empirically inferred equilibrium pressure scales for neutron matter, and an effective confinement pressure associated with energy localization at the Schwarzschild radius, we identify a simple and physically transparent stability hierarchy. Ordinary bodies remain stable because their inward gravitational pressure lies many orders of magnitude below the neutron equilibrium pressure scale. Neutron matter is shown to reside near the boundary of pressure-supported equilibrium, representing the highest stable configuration attainable by known forms of matter. For sufficiently compact masses, the confinement pressure required within the trapping region exceeds this equilibrium limit, implying the absence of any pressure-supported static configuration. Within this framework, the Schwarzschild trapping condition admits a direct physical interpretation as the loss of pressure equilibrium, rather than as a divergence of density or force. The analysis provides a unified and physically grounded criterion for mass stability across widely different astrophysical regimes.

Keywords: gravitational stability; neutron equilibrium pressure interpretation VTI; gravitational confinement pressure; Schwarzschild radius; black-hole formation; pressure-based collapse;

1. Introduction:

The stability of self-gravitating matter is a fundamental problem in physics and astrophysics, spanning an enormous range of mass and length scales—from planets and stars to neutron stars and gravitationally trapped objects. Traditionally, stability has been analyzed using density profiles, equations of state, or geometric criteria derived from general relativity. While these approaches have been remarkably successful within their respective domains, they often emphasize descriptive frameworks rather than a single unifying physical quantity governing stability across regimes.

In classical astrophysics, the stability of ordinary celestial bodies is understood through hydrostatic equilibrium, in which inward gravitational attraction is balanced by pressure gradients arising from thermal motion and material properties [1–3]. For compact objects such as white dwarfs and neutron stars, stability is provided by quantum degeneracy pressure and nuclear interactions, leading to well-defined mass and pressure limits beyond which equilibrium cannot be maintained [4–6]. These limits represent some of the most stringent constraints on matter known in nature.

General relativity introduces an additional and conceptually distinct stability criterion through the Schwarzschild solution, which defines a trapping radius for a given mass [7]. This condition is usually interpreted geometrically, as the formation of a causal boundary from which signals cannot escape. While mathematically precise, this description does not by itself provide a direct physical criterion for material stability or indicate whether any pressure-supported equilibrium can exist within the trapping radius.

In the present work, the empirically identified equilibrium pressure scale is also given a phenomenological interpretation in terms of a Vacuum Tension Interpretation (VTI) [8,9]. Within this interpretation, the vacuum is viewed as providing an effective outward stress that opposes extreme gravitational confinement once matter approaches neutron-scale densities. This concept is introduced solely as a macroscopic descriptor of the observed pressure ceiling, not as a microscopic vacuum theory. No modification of general relativity or nuclear physics is assumed; VTI is used only as a physically intuitive label for the limiting pressure scale that repeatedly appears in neutron equilibrium and gravitational confinement analyses.

2. Pressure Criterion across Celestial Mass Scales:

The stability of a self-gravitating system may be characterized by the balance between inward gravitational confinement pressure and the maximum outward pressure response available to matter. In this section, we compare three representative cases entirely in terms of pressure: an ordinary planetary mass, neutron-dominated matter, and a compact trapped configuration.

2.1 Limiting Equilibrium: Neutron Matter [8,9]

Neutron-dominated matter represents the densest and most compact **pressure-supported** configuration known in nature. We denote by (Pv) the characteristic equilibrium pressure associated with neutron matter, inferred empirically from neutron confinement and neutron-star phenomenology.

This pressure scale is introduced here as an **empirical upper bound** for pressure-supported stability of known matter. No assumptions are made regarding its microscopic origin beyond established nuclear physics.

At nuclear densities, matter becomes neutron-dominated, and pressure support arises from a combination of degeneracy effects and short-range nuclear interactions. A characteristic pressure scale may be estimated from an effective nuclear confinement force acting over a neutron length scale.

Neutron equilibrium parameters:

Neutron equilibrium pressure: [8]	$Fn = 10^5$ N
Neutron Energy:	$En = 1.505 \times 10^{10}$ J
Neutron effective Radius =	$Ro = 1.0 \times 10^{-15}$ m
Neutron mass =	1.68×10^{-27} kg

Pn: Neutron Surface confinement pressure in N/m^2, given by:

$$P_n = \frac{F_n}{4 \, x \, \boldsymbol{\pi} \, x \, {R_0}^2} \qquad (1)$$

Substituting values yields

$$P_n = \frac{10^5}{4 \, x \, \boldsymbol{\pi} \;\; x(1.0 \, x \, 10^{-15})^2} \approx 8.07 \, x \, 10^{33} \;\; \mathrm{N/m^2} \qquad = \mathrm{Pv} \qquad (2)$$

where (Pn) denotes the characteristic equilibrium pressure associated with neutron matter.
Within the neutron Confinement Force, Gravity Constant and Vacuum Tension Interpretation (VTI) adopted here, this pressure scale is denoted by (Pv) and represents an empirically realized upper bound for pressure-supported equilibrium. The near equality (Pn ≈ Pv ≈ $|c^4|$) [8,9] indicates that neutron matter resides at the boundary of stability. Any further increase in inward confinement pressure would exceed this equilibrium scale, for which no known pressure-supported static configuration is available.

The neutron density follows from

$$Dn = \frac{m}{\frac{4}{3} x \pi x Ro^3} \qquad (3)$$

Yielding

$$Dn = \frac{1.68 \, x \, 10^{-27}}{\frac{4}{3} x \pi x \; (1.0 \, x \, 10^{-15})^3} = 4 \, x \, 10^{17} \; kg \; m^{-3} \qquad (4)$$

2.2 Ordinary Celestial Mass: Earth

For ordinary celestial bodies, an order-of-magnitude estimate of gravitational confinement self-pressure may be obtained by distributing the inward gravitational confinement force over the surface area:

$$P = \frac{Gravity \; confinement \; Force}{Surface \; Area} \qquad (5)$$

$$P = \left(\frac{G \, x \, M^2}{R^2}\right) \, x \, \frac{1}{4 \, x \, \boldsymbol{\pi} \, x R^2} \qquad (6)$$

$$P \;\approx\; G \frac{M^2}{(4 \, x \, \pi \, x \, R^4)} \qquad (7)$$

For Earth:

- M = 5.97 x 10^{24} kg
- Radius = 6.37 x 10^6 m

Substitution yields

$$P_{eqrth} = \frac{6.67\ x\ 10^{-11}\ x\ \left(5.97\ x\ 10^{24}\right)^2}{4\ x\ \boldsymbol{\pi}\ x\ (6.37\ x10^6)^4} \quad \approx 1.15\ x\ 10^{11}\ \mathrm{N/m^2} \tag{8}$$

which satisfies

$$P_{eqrth} < Pv\ , \tag{9}$$

by more than twenty orders of magnitude. This vast separation explains the robust stability of ordinary celestial bodies against gravitational collapse.

2.3 Stability Criteria for Massive Celestial Bodies

Having identified neutron matter as the limiting pressure-supported equilibrium and demonstrated that ordinary bodies lie far below this limit, we now consider progressively more compact mass configurations. These include neutron stars and gravitationally trapped objects with representative masses of 4, 7, and 15 M ⨀ solar masses.

This comparison examines how the relative ordering of the characteristic matter radius (Rv) and the Schwarzschild radius (Rs) evolves with mass, and evaluates the corresponding confinement pressures at these radii relative to the equilibrium scale (Pv). The analysis is carried out by direct implementation of the governing equations.

2.3.1 Neutron Star:

A typical Neutron star (1.4 M ⨀) :

- Mass= $1.8 \text{ x } 10^{30}$ kg
- Radius =$1.2 \text{ x } 10^{4}$ m

The gravitational self-pressure is

$$\boldsymbol{Pns} = \frac{\boldsymbol{G\ x}\ \left(2.8\ \mathrm{X}\ 10^{30}\right)^{\mathbf{2}}}{\mathbf{4}\boldsymbol{x}\ \boldsymbol{\pi}\ \boldsymbol{x}\ \left(\mathbf{1.2}\ \boldsymbol{x}\ \mathbf{10^4}\right)^{\mathbf{4}}} \approx \mathbf{2.1}\ \boldsymbol{x}\ \mathbf{10^{33}}\ \boldsymbol{N/m2} \tag{10}$$

Which satisfies

$Pns < Pv$

This separation of nearly four orders of magnitude explains the stability of neutron stars against collapse. The corresponding density is

$$Dns = \frac{1.8\ x\ 10^{30}}{\frac{4}{3}x\pi x\ (1.2\ x\ 10^4)^3} \approx 2\ x\ 10^{17}\ kg\ m^{-3} \tag{11}$$

which satisfies

$Dns < Dn$,

which is of the same order, but slightly below, the characteristic neutron density (Dn).

2.3.2 Objects with Mass (≥ 4 M ⨀)

For non-rotating masses, the event horizon coincides with the Schwarzschild radius

$$Rs = \frac{2\,x\,G\,x\,M}{c^2} \tag{12}$$

The event horizon represents a causal boundary, not a material surface; stability considerations therefore pertain to interior regions and associated pressure balances.

A compact configuration is considered in which the characteristic matter radius (Rd) satisfies

$$\mathrm{Rd} \leq Rs \tag{13}$$

with mean density in the range

$$10^{17} \leq Dn \leq 10^{19}\ \mathrm{kg\ m^{-3}} \tag{14}$$

2.3.3 Four-Solar-Mass Case

$$M = 4\,x\,\mathrm{M}\odot = 7.9\ \mathrm{X}\ 10^{30}\ \mathrm{kg} \tag{15}$$

$$R_s = \frac{2\,x\,(6.67\,x\,10^{-11})\,x\,(7.96\,x\,10^{30})}{(3\,x\,10^{8})^2} \approx 1.18\,x\,10^4\,m \tag{16}$$

The confinement pressure at (Rs) is

$$\boldsymbol{P_{bh}} = \frac{\boldsymbol{G\,x\,(7.9\,\mathrm{X}\,10^{30})^2}}{\boldsymbol{4\,x\,\pi\,x\,(1.18\,x\,10^4)^4}} \approx \boldsymbol{1.75\,x\,10^{34}\,N/m2} \tag{17}$$

which satisfies

$$P_{bh} > P_v \quad \{18\}$$

indicating that no pressure-supported equilibrium exists at the trapping radius.
The mean density at (Rs) is

$$D_n = \frac{7.9\,x\,10^{30}}{\frac{4}{3}x\,\pi\,x\,(1.18\,x\,10^4)^3} \approx 1.2\,x10^{18}\ \mathrm{kg\ m^{-3}} \tag{19}$$

exceeding neutron density by a factor of about three. Observationally, black holes below seven solar masses are relatively rare, suggesting that such lower-mass cases require more extreme compression conditions.

2.3.4 Seven-Solar-Mass Case:

Stellar-mass black holes are most frequently observed near seven solar masses.

$$For\ M = 7\ x\ \mathrm{M}\odot = 1.4\ \mathrm{X}\ 10^{31}\ \mathrm{kg} \tag{20}$$

$$R_s = \frac{2\ x\ (6.67\ x\ 10^{-11})\ x\ (1.4\ x\ 10^{31})}{(3\ x\ 10^8)^2} \approx 2.06\ x\ 10^4\ m \tag{21}$$

The confinement pressure at (Rs) is

$$\boldsymbol{P_{bh}} = \frac{\boldsymbol{G\ x\ (1.4\ \mathrm{X}\ 10^{31})^2}}{\boldsymbol{4\ x\ \pi\ x\ (2.06\ x\ 10^4)^4}} \approx \boldsymbol{9.9\ x\ 10^{33}\ N/m2} \tag{22}$$

Yielding

$$P_{bh} \approx P_v \tag{23}$$

This near equality indicates that a seven-solar-mass object lies at the boundary between pressure-supported neutron equilibrium and pressure-unsupported gravitational trapping, providing a natural explanation for the observed clustering of black holes near this mass.

2.3.5 Fifteen-Solar-Mass Case

For M = 15 M ⊙

$$M = 15\ x\ \mathrm{M}\odot = 3\ \mathrm{X}\ 10^{31}\ \mathrm{kg} \tag{24}$$

$$R_s = \frac{2\ x\ (6.67\ x\ 10^{-11})\ x\ (3\ x\ 10^{31})}{(3\ x\ 10^8)^2} \approx 4.45\ x\ 10^4\ m \tag{25}$$

Assuming neutron-density matter, the characteristic confinement radius is

$$R_d = \left(\frac{3\ x\ 10^{11}}{\frac{4}{3} x\ \pi\ x\ 4\ x\ 10^{17}}\right)^{1/3} = 2.62\ x\ 10^4\ m \tag{26}$$

The confinement pressure at (Rd) is

$$\boldsymbol{P_{bh}\ (Rd)} = \frac{\boldsymbol{G\ x\ (3\ \mathrm{X}\ 10^{31})^2}}{\boldsymbol{4\ x\ \pi\ x\ (2.62\ x\ 10^4)^4}} \approx \boldsymbol{1.02\ x\ 10^{34}\ N/m2} \tag{27}$$

while at the horizon

$$\boldsymbol{P_{bh}\,(Rs) = \frac{G\,x\,(3\,X\,10^{31})^2}{4\,x\,\pi\,x\,(4.45\,x\,10^4)^4} \approx 1.22\,x\,10^{33}\ N/m2} \qquad (28)$$

Satisfying

$$P_{bh}\,(Rs) < Pv < P_{bh}\,(Rd), \qquad (29)$$

This demonstrates that gravitational trapping occurs once confinement pressures exceed the neutron equilibrium scale.

2.6 Physical Interpretation:

These examples establish a simple pressure hierarchy:

- Ordinary matter: (P < Pv) → stable
- Neutron matter: (P < Pv) → marginal equilibrium
- Trapped configurations**:** (Pgrav > Pv) → no pressure-supported equilibrium

In this framework, the Schwarzschild trapping condition corresponds physically to the exhaustion of material pressure support, rather than to a divergence of density or force.

In this framework, the Schwarzschild trapping condition corresponds physically to the exhaustion of pressure support rather than to a divergence of density or force.

3. Conclusions

In this work, a pressure-based criterion has been developed to interpret the stability of self-gravitating matter across a wide range of astrophysical scales. By expressing stability in terms of inward gravitational confinement pressure and comparing it with an empirically identified equilibrium pressure scale, a unified and physically transparent picture emerges that applies consistently to ordinary matter, neutron-dominated matter, and gravitationally trapped configurations.

Using standard Newtonian estimates, it was shown that ordinary celestial bodies operate at confinement pressures many orders of magnitude below the equilibrium pressure scale Pv, explaining their robust long-term stability. Neutron matter was identified as residing at the boundary of pressure-supported equilibrium, with its characteristic confinement pressure closely matching Pv. This establishes neutron matter as the densest and most compact stable configuration known for pressure-supported matter.

Extending the analysis to progressively more compact stellar masses reveals that, once the gravitational confinement pressure exceeds Pv, no pressure-supported static configuration is available. In this regime, gravitational trapping arises not from a divergence of density or force, but from the exhaustion of all known pressure-support mechanisms. The Schwarzschild trapping condition is thus given a clear physical interpretation as the loss of pressure equilibrium rather than as a purely geometric or singular behavior.

A particularly notable result is that stellar-mass black holes near seven solar masses naturally emerge as a transitional case, where the confinement pressure at the trapping radius is comparable to Pv. This provides a plausible physical explanation for the observed clustering of black-hole masses near this scale, while lower-mass black holes require more extreme compression and are correspondingly less abundant.

Within the present framework, the equilibrium pressure scale Pv is treated strictly as an empirically realized upper bound for pressure-supported stability. Its phenomenological interpretation as a Vacuum Tension Interaction (VTI) is introduced only as a possible physical explanation for this limiting scale, without invoking a specific microscopic or quantum-gravitational mechanism. The numerical proximity of Pv to relativistic stress scales is noted as suggestive but is not assumed to be fundamental.

The model proposed here is not intended as a definitive description of black-hole interiors, but as a physically motivated and internally consistent hypothesis constrained by known pressure and density limits of matter. Its value lies in providing a simple stability criterion that connects neutron equilibrium physics with gravitational trapping conditions, and in offering testable insights into the mass scales at which pressure-supported matter must give way to irreversible confinement.

Future work may explore observational consequences, extensions to rotating systems, and possible microscopic interpretations of the equilibrium pressure scale. As with many foundational ideas in astrophysics, the present framework is offered not as a final answer, but as a stimulus for further theoretical and observational scrutiny.

References:

1. S. Chandrasekhar, An Introduction to the Study of Stellar Structure, Dover Publications, 1957.
2. J. Binney and S. Tremaine, Galactic Dynamics, 2nd ed., Princeton University Press, 2008.
3. C. J. Hansen, S. D. Kawaler, and V. Trimble, Stellar Interiors: Physical Principles, Structure, and Evolution, Springer, 2004.
4. S. Chandrasekhar, “The Maximum Mass of Ideal White Dwarfs,” Astrophysical Journal 74, 81–82 (1931).
5. J. R. Oppenheimer and G. M. Volkoff, “On Massive Neutron Cores,” Physical Review 55, 374–381 (1939).
6. N. K. Glendenning, Compact Stars: Nuclear Physics, Particle Physics, and General Relativity, Springer, 2000.
7. K. Schwarzschild, “On the Gravitational Field of a Mass Point According to Einstein’s Theory,” Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, 189–196 (1916).
8. V. T. Ingole, Anant S. Wadatkar “A Classical Derivation of the Neutron Strong Force from Internally Confined Charge Structure.” Advanced International Journal for Research (AIJFR), Volume 7, Issue 1 (January-February 2026)
9. V. T. Ingole, Anant S. Wadatkar “A Proposed Framework for the Gravitational Constant Based on Neutron Energy-Radius Confinement.” (Accepted)