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QUANTUM AMPLITUDE ESTIMATION FOR EXPECTED LOSS COMPUTATION UNDER A DISCRETIZED LATENT-FACTOR MODEL

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Abstract:
We present a quantum algorithm for estimating the expected loss of a credit portfolio driven by a latent risk factor. The method discretizes a continuous latent variable, encodes its probability distribution into quantum amplitudes, embeds the loss function via controlled rotations on an ancilla qubit, and applies Grover-style amplitude amplification. The resulting state admits a two-subspace decomposition enabling estimation of the expected loss using Quantum Amplitude Estimation (QAE) or a maximum-likelihood estimator (MLE) based on Grover power measurements. The approach achieves a quadratic speedup over classical Monte Carlo methods.

Index Terms: Quantum amplitude estimation, Grover operator, credit risk, expected loss, latent factor models.

I. INTRODUCTION

Estimating expected loss (EL) is a central problem in credit risk management and financial engineering. Classical Monte Carlo methods approximate expectations with sampling complexity $O(\varepsilon^{-2})$. Quantum Amplitude Estimation (QAE), introduced by Brassard et al. [1], enables a quadratic speedup, reducing complexity to $O(\varepsilon^{-1})$. In this paper, we formulate expected loss computation under a latent-factor model as an amplitude estimation problem using Grover-style operators.

II. PROBLEM SETTING

Let $Z \sim N(\mu,\sigma^2)$ be a latent systematic risk factor. Define the loss

$$L(Z) = \mathrm{EAD} \cdot \mathrm{LGD} \cdot \mathrm{PD}(Z), \quad (1)$$

and the target quantity

$$\mathrm{E}[L] = \int L(z)\, d\mathrm{P}(z). \quad (2)$$

III. DISCRETIZATION OF THE LATENT VARIABLE

Using n qubits, we define $N = 2^n$ grid points over

$$[\mu - 3\sigma, \mu + 3\sigma]. \quad (3)$$

Let

$$\Delta = \frac{6\sigma}{N-1}, \qquad i = 0, \ldots, N-1.$$

$$z_i = \mu - 3\sigma + i\Delta, \quad (4)$$

Discrete probabilities p_i approximate $\mathrm{P}(Z = z_i)$ and satisfy $^{\mathrm{P}}i\ p_i = 1$.

IV. UNCERTAINTY STATE PREPARATION

Define the uncertainty state

$$|\psi_Z\rangle = \sum_{i=0}^{N-1} \sqrt{p_i}\,|i\rangle. \quad (5)$$

A unitary U prepares $|\psi_Z\rangle$ from $|0\rangle^{\otimes n}$:

$$U\,|0\rangle^{\otimes n} = |\psi_Z\rangle. \quad (6)$$

V. LOSS ENCODING VIA CONTROLLED ROTATIONS

Let

$$L(i) = L(z_i), \qquad f(i) = \frac{L(i)}{L_{\max}} \in [0,1]. \quad (7)$$

Introduce an ancilla qubit and define

$$U_f(|i\rangle\,|0\rangle) = |i\rangle\left(\sqrt{1-f(i)}\,|0\rangle + \sqrt{f(i)}\,|1\rangle\right). \quad (8)$$

This is implemented as

$$U_f = \sum_{i=0}^{N-1} |i\rangle\langle i| \otimes R_y(\theta_i), \qquad \theta_i = 2\arcsin\left(\sqrt{f(i)}\right). \quad (9)$$

Define

$$A = U_f(U \otimes I). \quad (10)$$

VI. AMPLITUDE INTERPRETATION

Applying A to $|0\rangle^{\otimes n}\,|0\rangle$ yields

$$|\Psi\rangle = \sum_{i=0}^{N-1} \sqrt{p_i}\,|i\rangle\left(\sqrt{1-f(i)}\,|0\rangle + \sqrt{f(i)}\,|1\rangle\right). \quad (11)$$

Define

$$a = \sum_{i=0}^{N-1} p_i f(i) = \frac{\mathbb{E}[L]}{L_{\max}}. \quad (12)$$

Then

$$|\Psi\rangle = \sqrt{1-a}\,|\Psi_0\rangle + \sqrt{a}\,|\Psi_1\rangle \quad (13)$$

where $|\Psi_1\rangle$ corresponds to ancilla = 1.

VII. GROVER OPERATOR

Define reflections

$$S_\chi = I - 2(I \otimes |1\rangle\langle 1|), \quad (14)$$

$$S_0 = I - 2|0\cdots 0\rangle\langle 0\cdots 0|. \quad (15)$$

The Grover operator is

$$Q = AS_0A^\dagger S_\chi. \quad (16)$$

Let $a = \sin^2\theta$. Then after m Grover iterations,

$$\Pr(\text{ancilla} = 1 \mid Q^m) = \sin^2((2m+1)\theta). \quad (17)$$

VIII. ESTIMATION METHODS

A. Quantum Amplitude Estimation

Quantum phase estimation on Q yields eigen phases $\pm 2\theta$ and thus $a = \sin^2\theta$ with complexity $O(\varepsilon^{-1})$ [1].

B. Maximum Likelihood Estimation

Repeated measurements after Q^m yield binomial samples

$$k_m \sim \text{Binomial}(N_m,\ \sin^2((2m+1)\theta)). \quad (18)$$

Let

$p_m(a) = \sin^2\big((2m+1)\arcsin(\sqrt{a})\big).$ (19)

The MLE is

$\hat{a} = \arg\max_{a\in(0,1)} \sum \Big[k_m \log p_m(a) + (N_m - k_m)\log(1 - p_m(a))\Big].$

m

(20)

IX. COMPLEXITY AND RESOURCES

Classical Monte Carlo: Sampling complexity $O(\varepsilon^{-2})$.

Quantum method: QAE achieves $O(\varepsilon^{-1})$ queries to the oracle A [1], [3]. The total number of qubits is $n + 1$ plus auxiliary work qubits. Circuit depth is dominated by controlled rotations and Grover iterations.

X. NUMERICAL RESULTS (SYNTHETIC STUDY)

A. *Experimental Setup*

We consider a synthetic latent-factor model

$Z \sim N(0,1),$ (21)

with probability of default

$PD(Z) = \Phi(\alpha Z + \beta),$ (22)

where Φ is the standard normal CDF, $\alpha = 0.8$, and $\beta = -0.3$. We set EAD = LGD = 1, so $L(Z) = PD(Z)$. The true expected loss is computed via high-precision numerical quadrature.

TABLE I- RMSE COMPARISON BETWEEN CLASSICAL MONTE CARLO AND QUANTUM AMPLITUDE ESTIMATION FOR SYNTHETIC EXPECTED LOSS ESTIMATION.

Samples / Queries (N)	MC RMSE	QAE RMSE
10^2	2.545×10^{-2}	9.047×10^{-3}
3×10^2	1.389×10^{-2}	3.042×10^{-3}
10^3	7.295×10^{-3}	8.917×10^{-4}
3×10^3	4.548×10^{-3}	2.933×10^{-4}
10^4	2.380×10^{-3}	8.882×10^{-5}
3×10^4	1.535×10^{-3}	2.865×10^{-5}

B. *Error Scaling*

Classical Monte Carlo estimation is performed using N independent samples. Quantum amplitude estimation is simulated by ideal Grover queries and exact amplitude extraction. Figure 1 reports the RMSE as a function of the sample size or oracle queries on a log–log scale.

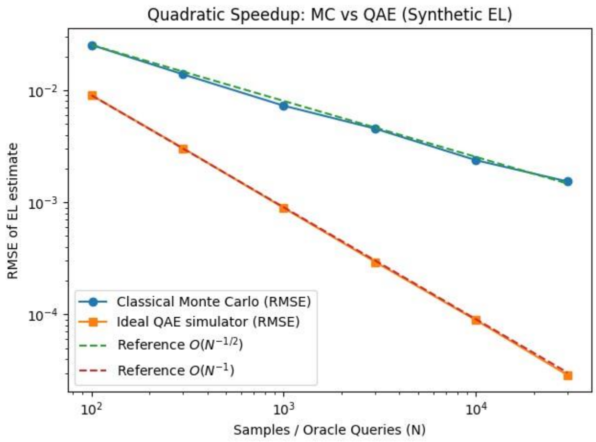

Fig. 1. Root-mean-square error (RMSE) of expected loss estimation versus number of samples (classical Monte Carlo) and oracle queries (quantum amplitude estimation) on a log–log scale. Classical Monte Carlo exhibits $O(N^{-1/2})$ convergence, while quantum amplitude estimation achieves $O(N^{-1})$ scaling, demonstrating a quadratic speedup.

XI. CONCLUSION

We formulated expected loss estimation under a discretized latent-factor model as a quantum amplitude estimation problem. Synthetic numerical experiments confirm the theoretically predicted quadratic speedup over classical Monte Carlo methods. This framework provides a principled foundation for quantum credit risk applications as quantum hardware matures.

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