

# Analysis of the Constant Deceleration Parameter in Equation of State Parameter Cosmological Models with Dark Energy

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## Abstract

Analysis of constant deceleration in Dark energy cosmological background; defined  $q = -\frac{a\ddot{a}}{\dot{a}^2}$  is a critical metric for characterizing the expansion history of the universe while the standard ( $\Lambda$ CDM) model suggests a transition from a decelerated phase  $q > 0$  to an accelerated phase  $q < 0$  models assuming a constant deceleration parameter (CDP) serve as essential theoretical benchmarks, and Analysis of the constant deceleration parameter in EoS Parameters Cosmological models are presence of dynamically anisotropic dark energy. We assume that the dark energy is minimally interacting, has dynamical energy density, anisotropic EoS parameter. The conservation of the EM tensor of the DE is assumed to lie of two separately additive conserved part. A special law is assumed for the divergence from isotropic EoS. which is consistent with the assumption on the conservation of the energy momentum tensor of DE. Exact solution of the Einstein's field equations are obtained by assuming a special law variation for the mean Hubble parameter, which yield a constant value of the deceleration parameter geometrical & kinematic properties of the models and the behavior of anisotropy of the DE has been finish.

**Keyword:** Cosmological Models Constant Deceleration, Dark Energy EoS parameters Bianchi type I anisotropy.

## 1. Introduction:

The observed accelerated expansion of the universe has led to the development of numerous Dark Energy. One of the most successful attempt to resolve the problems of standard Big Bang Cosmology such as homogeneity, and flatness of the universe is the inflationary paradigm, characterised by epoch of accelerated expansion "inflation" in the very early universe during the inflationary epoch, quantum fluctuations are highly amplified, their wavelengths are stretched to outside the Hubble horizon and inevitably, super horizon fluctuations are generated. This cosmological model with 0 DE component which is non-dynamical but yields anisotropic vacuum pressure in two side first by implementing a poisson structure deformation between canonical momenta such that rescaling of the scale factor is not violated. the physical behaviors of the two - fluid, particularly  $\Lambda$  CDM, cosmological models with an anisotropic DE component. Models of DE are conveniently characterized by the EoS Parameter  $\omega = P/\rho$  which is not necessarily constant, Where  $\rho$  energy density  $\phi$   $\rho$  in pressure. However, while energy density is scalar quantity, pressure is a vectorial quantity, and consequently the EoS parameter of DE

may be determined separately on each spatial axis in a consistent way with the conservation of energy momentum tensor, Hence, we consider a phenomenological parametrization of minimally interacting DE in terms of its time dependent deviation-free equation of state parameter  $\omega^{(de)}(t)$  and deviation parameters  $(\delta(t), \omega_x^{(de)}(t), \omega_y^{(de)}(t), \omega_z^{(de)}(t))$ . Since such a parametrization yields an anisotropic expansion which is not compatible with the Robertson Walker (RW) metric, in fact and part we have used the LRS Bianchi type I metric which generalizes the flat RW metric in an axially symmetric way and is compatible with our parameterization. The other component has assumed to be a perfect fluid (Dark matter or ordinary matter). With our assumption, the conservation of the energy momentum tensor implies dynamically anisotropic DE except for very special solutions. We have obtained exact solutions for the equations by assuming a special dynamic for the anisotropy of the dark energy and a special law of variation for the mean Hubble parameter allows us to determine the scale factors exactly, as well as to examine the behavior of the anisotropy of DE and other cosmological parameters of such a universe.

## 2- Model and field equations –

The spatially homogeneous, anisotropic and LRS Bianchi type I space-time is described by the line element.

$$ds^2 = dt^2 - A(t)^2 dx^2 - B(t)^2 (dy^2 + dz^2) \quad (2.1)$$

where  $A(t), B(t)$  are scale factors (metric tensors) and functions of the cosmic time  $t$ . In natural units ( $8\pi G = 1$  and  $C = 1$ ), the field equations, in the case of a mixture of perfect fluid and anisotropic Dark energy components are –

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

with  $T_{\mu\nu} = -T_{\mu\nu}^{(m)} - T_{\mu\nu}^{(de)} \quad (2.2)$

$$T_{\nu}^{(m)\mu} = \text{diag} [\rho^{(m)}, -P^{(m)}, -\rho^{(m)}, -P^{(m)}]$$

$$= \text{diag} [1, -\omega^{(m)}, -\omega^{(m)}, -\omega^{(m)}] \rho^{(m)} \quad (2.3)$$

and  $T_{\nu}^{(de)\mu} = \text{diag} [\rho_x^{(de)} - p_x^{(de)}, -\rho_y^{(de)}, -\rho_z^{(de)}]$  (2.4)

$$= \text{diag} [1, -\omega_x^{(de)}, -\omega_y^{(de)}, -(\omega^{(de)} + \gamma)] \rho^{(de)}$$

where  $u^\mu u^\nu = 1; u^\mu = (1, 0, 0, 0)$  is the four velocity vector;  
 $g_{\mu\nu} R_{\mu\nu}$  if the Ricci tensor,  $R$  is the Ricci scalar;  $\rho^{(m)}$  and  $\rho^{(de)}$  are the

energy densities of the perfect fluid and DE components respectively;  $\omega^{(m)}$  is the EoS parameter of the perfect fluid and we will call  $\omega^{(m)}$  the deviation free. EoS parameter of the DE. Here, the perfect fluid represents the ordinary matter or cold dark matter, thus  $\omega^{(m)} \geq 0$ .  $\omega_x^{(de)}, \omega_y^{(de)}, \omega_z^{(de)}$  are the direction EoS parameters of the DE on x, y and z axes respectively.  $\omega_y^{(d)}$  and  $\omega_z^{(de)}$  are set to be equal which is convenient with the metric given in (2.1)  $\delta$  and  $\gamma$  are deviations from the deviations free, EoS parameter (hence the deviation-free pressure) of the DE respectively on x axis and y and z axes. In a comoving coordinate system. Einstein's field equation (2) for the anisotropic LRS Bianchi type I space-time (1), in case of (3) & (4) read as

$$\frac{\dot{B}^2}{B^2} + 2 \frac{\dot{A}\dot{B}}{AB} = \rho^{(m)} + \rho^{(de)} \quad (2.5)$$

$$\frac{\dot{B}^2}{B^2} + 2 \frac{\ddot{B}}{B} = -\omega^{(m)} \rho^{(m)} - (\omega^{(de)} + \delta) \rho^{(de)} \quad (2.6)$$

$$\frac{\ddot{B}}{B} + \frac{\dot{B}}{B} \frac{\dot{A}}{A} + \frac{\ddot{A}}{A} = -\omega^{(m)} \rho^{(m)} - (\omega^{(de)} + \gamma) \rho^{(de)} \quad (2.7)$$

where the over dot denotes derivation with respect to the cosmic time t.

we have following equation from the Bianchi identity ,

$$G^{\mu\nu}, V = T^{(m) \mu\nu}; v + T^{(de)\mu\nu} v = 0 \quad (2.8)$$

which yield  $\rho^{(m)} + (1 + \omega^{(m)}) \rho^{(m)} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) + \dot{\rho}^{(de)} +$

$$(1 + \omega^{(de)}) \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) + \rho^{(de)} \left( \delta \frac{\dot{A}}{A} + 2 \dot{\gamma} \frac{B}{B} \right) = 0 \quad (2.9)$$

This eqh, which is linearly dependent to Einstein field eq<sup>h</sup>s, also represent the conservation of the total energy momentum tensor.

### 3. Solution of the field equations :-

We have initially 8 variables (A,B,ρ<sup>m</sup>, ω<sup>(m)</sup>, ρ<sup>(de)</sup>, ω<sup>(de)</sup>, δ, γ) and four equations, three of which are linearly independent, namely three finstein field equation (2.5-2.7) and the Bianchi identity. The system is thus initially undetmined and we need additional constraints to close the system. We have assumed that the Dark energy is minimally interacting,

$$T^{(de)\mu\nu}; v = 0$$

thus due to the Bidnchi identity(8) the perfect fluid component is also menially interacting,  $T^{(m)\mu\nu}; V=0$

Hence, the bianchi identity has been split into separately additive conserved component namely, the conservation of the energy momentum tensor of the DE.

$$T^{(de) \mu\nu}; v = \dot{\rho}^{(de)} + (1 + \omega^{(de)}) \rho^{(de)} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) + \rho^{(de)} \left( \delta \frac{\dot{A}}{A} + 2\gamma \frac{\dot{B}}{B} \right) = 0 \quad (3.1)$$

and the conservation of the energy momentum tensor of the perfect fluid component

$$T^{(m)\mu\nu}; v = \dot{\rho}^{(m)} + (1 + \omega^{(m)}) \rho^{(m)} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) = 0 \quad (3.2)$$

One more we may split the conversation of the energy momentum tensor of the dark energy into two parts :-

$$T^{(de)uv}; v = T^{(de)\mu\nu}; v + \tau^{(de)\mu\nu}; v = 0 \quad (3.3)$$

where  $T^{(de)rv}; v$  is the term of the  $T^{(de)ur}; v$  in to 3.2 and arise due to the deviations from  $\omega^{(de)}$ ; and  $T^{(de)}; v$  is the deviation free part of the  $T^{(de)uv}; v$  in 3.1. Now are will do the following strong assumption.

$$\tau^{(de)\mu\nu}; v = \rho^{(de)} \left( \delta \frac{\dot{A}}{A} + 2\gamma \frac{\dot{B}}{B} \right) = 0 \quad (3.4)$$

Which also result in the deviation free part of the  $T^{(de)ur}; v$  to be null, that is

$$T^{(de)\mu\nu}; v = \rho^{(de)} + (1 + \omega^{(de)}) \rho^{(de)} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) = 0 \quad (3.5)$$

Which looks like the conservation of the energy momentum tensor of a minimally interacting perfect fluid. According (3.4) and (3.5) the behavior of  $\rho^{(de)}$  is controlled by the deviation free past of the EoS parameter of the DE( $\omega^{(de)}$ ), but the deviations will affect  $\rho^{(de)}$  indirectly, since as can be seen later, the

affect the value of  $\omega^{(de)}$  If the deviation parameters are assumed to be constants, to assure  $T^{(de)uv};v = 0$  either  $\delta$  and  $\pi$  are allowed to be function of the comic time  $t$  and we constrained  $\delta$  and  $\gamma$  are by assuming a special dynamic which is consistent with (3.4). The dymanic of the deviation parameter on the xa is,  $\delta(t)$  assumed to be

$$\delta(t) = n \cdot \frac{2}{3} \frac{\dot{B}}{B} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \frac{1}{\rho^{(de)}} \tag{3.6}$$

And thus from eq<sup>h</sup> (3.4) the deviation parameter on the y and z axes,  $v(t)$  is found as

$$\gamma(t) = -n \frac{1}{3} \frac{\dot{A}}{A} \left( \frac{A'}{A} + 2 \frac{B}{B} \right) \frac{1}{\rho^{(de)}} \tag{3.7}$$

$\delta(t)$  and  $\gamma(t)$  are dimensionless parameters, and  $n$  is a dimensionless constant that parameterizes the amplitude of the deviation  $\omega^{(de)}$  and can be given real values. The mesure of the anisotropy of the DE may be given by  $(\delta(t) - \gamma(t)) / \omega^{(de)}$  and it is null, which implies the dark energy is isotropic, when  $n=0$ . The EoS parameter of the perfect fluid has ben assumed to be constant

$$\omega^{(m)} = \frac{P^{(m)}}{\rho^{(m)}} = const, \tag{3.8}$$

Which  $\omega^{(de)}$  is allowed to be a function of the comic time, since the current cosmology data. from SNIa, CMB and large scale structures mildly favor dynamically evolving DE ceossing the PDL as mentioned in the first section hence, since  $\omega^{(de)}(t)$  hasn't been constrained, we still need one more constrain to close the system.

We imposed a law of variation for the Hubble parameter given by Berman is not inconsistent with observation [35, 36] and is also approximately valid for slowly time varying the Hubble parameter in deceleration parameter (37)., Recently singh and kumar proposed a similar law of variation for the Hubble parameter, and generated solutions for Bianchi type – I(38). LRS Bianchi type – II (39, 36). Bianchi type – V (37) metrics in General. Relativity. According to the proposed low, the variation of the mean Hubble parameter for the LRS Bianchi – I. metric may be given by –

$$H = k(AB^2)^{-\frac{m}{3}} \tag{3.9}$$

where  $K > 0$  and  $m > 0$  are constants :-

The spatial valume is given by

$$V = a^3 = AB^2 \tag{3.10}$$

where  $a$  is the mean scal factor.

The mean Hubble parameter  $H$  for Bianchi I metric may

$$H = \frac{\dot{a}}{a} = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \tag{3.11}$$

The directional Hubble parameters in the directions of  $x, y$  &  $z$  respectively may be defined

$$H_x = \frac{\dot{A}}{A}, H_y = H_z = \frac{B'}{B} \tag{3.12}$$

( $\therefore H_y = H_z$ , in the following they are represented by  $H_{y,z}$ )

The volumetric deceleration parameter is –

$$q = - \frac{a\ddot{q}}{\dot{a}^2} \tag{3.13}$$

on integration, after equating (3.9) and (3.11) we get

$$AB^2 = c_1 e^{3kt} \quad [for \ m = 0] \tag{3.14}$$

and 
$$AB^2 = (mkt + C_2)^{\frac{3}{m}} \quad [form = 0] \tag{3.15}$$

Here  $c_1$  and  $c_2$  are positive constants of integration using (3.9) (3.14) for  $m \neq 0$ , mean Hubble parameters are

$$H = k \quad for \ m = 0 \tag{3.16}$$

$$and \ H = k (mkt + C_2)^{-1} \quad for \ m \neq 0 \tag{3.17}$$

using (3.14 – 3.15) and (3.10) in (3.13) we get constant value for the deceleration parameter for the mean scale factor as :

$$q = m-1 \quad for \ m \neq 0 \tag{3.18}$$

and 
$$q = -1 \quad for \ m = 0 \tag{3.19}$$

The sign of  $q$  indicates whether the model accelerate or not. the positive sign of  $q$  (i.e.  $m > 1$ ) corresponds to decelerating models whereas the negative sign  $-1 \leq q < 0$  for  $0 \leq m < 1$  indicates acceleration and  $q = 0$  for  $m = 1$  corresponds to expansion with constant velocity.

using the deviation parameters (3.6) and (3.7) the mean Hubble parameter (3.11) in the subtraction (2.6) from (2.7) we may get

$$\frac{d}{dt} \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) + \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) 3H = 3nH^2 \tag{3.20}$$

after little manipulation. On integration of (3.20) by considering (3.16) and (3.17) we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \lambda e^{-3kt} + nk \quad for \ m = 0 \tag{3.21}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{\lambda}{(mkt + c_2)^{\frac{3}{m}}} + \frac{3nk}{(3-m)(mkt + c_2)} \quad for \ m \neq 0 \tag{3.22}$$

and

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{\lambda}{3kt + c_2} + \frac{nk \ln(3kt + c_2)}{3kt + c_2} \quad for \ m = 3 \tag{3.23}$$

where  $\lambda$  is the real constant of integration. One can observe that  $\lambda$  and  $n$  are two parameters that parameterize the between the directional Hubble parameter. Now we can find  $A(t)$  and  $B(t)$  explicitly for all values by using (3.21-3.23)

**\*Model for  $m = 0$  ( $q = -1$ )**

Using (3.21) we may get the ratios of the scale factors  $A(t)/B(t)$ , and on manipulating the result by using (3.14) as get the following exact expressions for the scale factors :

$$A(t) = c_1^{\frac{1}{3}} k^{\frac{2}{3}} e^{kt} - \frac{2}{9} \frac{\lambda}{k} e^{-3kt} + \frac{2}{3} nkt \tag{3.24}$$

$$B(t) = C_1^{\frac{1}{3}} K^{-\frac{1}{3}} e^{kt + \frac{1}{9} \frac{\lambda}{k} e^{-3kt} - \frac{1}{3} nkt} \tag{3.25}$$

where  $K$  is the positive constant of integration

the spatial volume of the universe is found as

$$V = C_1 e^{3kt} \tag{3.26}$$

the directional Hubble parameters as defined in (3.12) are found as

$$H_n = K + \frac{2}{3} \lambda e^{-3kt} + \frac{2}{3} Kn \tag{3.27}$$

$$H_{y,z} = K - \frac{1}{3} \lambda e^{-3kt} - \frac{1}{3} Kn \tag{3.28}$$

the anisotropy parameter of the expansion  $\Delta$  is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2 \tag{3.29}$$

where  $H_i$  ( $i=1, 2, 3$ ) represents the directional Hubble parameters in the directions of  $x, y$  and  $z$  respectively. By using eqn. (3.16) (3.27) and (3.28) in (3.29) we get

$$\Delta = \frac{2}{9} \frac{(\lambda e^{-3kt} + nk)^2}{K^2} \tag{3.30}$$

for the anisotropy of the expansion the expansion scalar, defined by  $\theta = u^i_{;i}$  is found as

$$\theta = 3k = 3H \tag{3.31}$$

The shear scalar, defined by  $\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}$

where  $\sigma_{ij} = u_{i;j} + u_{j;i} - \frac{2}{3} g_{ij} u^k_{;k}$  is the shear tensor,

$$\sigma^2 = \frac{1}{3} (\lambda e^{-3kt} + nK)^2 \tag{3.32}$$

using the scale factors in (3.2), the energy density of the perfect fluid is found as

$$\rho^{(m)}(t) = \rho_o^{(m)} e^{-3kt(1+\omega^{(m)})t} \tag{3.33}$$

The energy density of the DE may be found from eqn. 2.5 by using the scale factors and the energy density of the perfect fluid (3.33) and may be written in terms of  $\Delta(t)$  as

$$\rho^{(de)}(t) = 3K^2 \left( 1 - \frac{1}{2} \Delta(t) \right) - \rho^{(m)}(t). \tag{3.34}$$

using the scale factors and (3.34) in (3.5)

$$\omega^{(de)}(t) = \frac{3\omega^{(m)} \rho^{(m)}(t) + \lambda e^{2e-6kt} + k^2(9-n^2)}{3\rho^{(m)}(t) + \lambda^2 e^{-6kt} + 2\lambda k n e^{-3kt} - K^2(9-n^2)} \tag{3.35}$$

for the deviation free EoS parameter of the DE using the scale factors and (3.34) in (3.6) and (3.7) we may get deviation parameters as following –

$$\delta(t) = \frac{2\lambda k n e^{-3kt} + 2k^2 n(n-3)}{3\rho^{(m)}(t) + \lambda^2 e^{-6kt} + 2\lambda k n e^{-3kt} - K^2(9-n^2)} \tag{3.36}$$

$$\gamma(t) = \frac{2\lambda k n e^{-3kt} + 2k^2 n(n + \frac{3}{2})}{3\rho^{(m)}(t) + \lambda^2 e^{-6kt} + 2\lambda k n e^{-3kt} - k^2(9-n^2)} \tag{3.37}$$

**\*Physical behaviour of the model for  $m = 0$  ( $q = -1$ )**

for  $q = -1$  and  $dH/dt = 0$  the greatest value of the Hubble parameter and the fastest rate expansion of the universe. Thus this model represented by O.K. and C.B. kiline the inflationary era in the early universe and the vary late times of the universe.

The spatial value  $V$  is finite at  $t = 0$ , expands exponentially as  $t$  increases and become infinitely large at  $t = \infty$ . The directional Hubble parameter  $H_x$  and  $H_y, H_z$  are finite  $t = 0$  and  $t = \infty$ . They deviate from the mean Hubble parameter  $H$  due to  $\lambda$  and  $n$ . while  $\lambda$  is supporting the expansion on the  $x$  axis, it opposes (supports) the expansion on  $y$  and  $z$  axes. However it loses its effect exponentially by the cosmic time  $t$ . The anisotropy of the DE has a similar effect on the directional Hubble parameters, but its effect persists. While  $n$  is supporting (opposing) the expansion on the  $x$  axis, it opposes (supports) the expansion on  $y$  and  $z$  axes.

The anisotropy of the DE doesn't always act so as to increase the anisotropy of the expansion. In fact, when the signs of  $\lambda$  and  $n$  are opposite the overall effect is to lower the expansion anisotropy.

The expansion scalar is constant throughout the evolution of the universe. The shear scalar is also finite at  $t = 0$  and tends to  $n^2 k^2/3$  as  $t$  increases. if  $-3/2 < n < 3$  all the axes will expand to infinitely large values as  $t \rightarrow \infty$ . on the other hand, the space – time exhibits a pancake type singularity for  $n < -3/2$  and a cigar type singularity for  $n > 3$  at  $t = \infty$ . if  $n = -3/2$  while the  $x$  axis converges to a constant,  $y$  and  $z$  axes expand to infinitely large values and it is vice versa if  $n = 3$ .

The energy density of the perfect fluid  $\rho^{(m)}$  decreases exponentially and converges to zero since  $\omega^{(m)} > 0$  by definition. while  $p^{(de)}$  changes slightly at early time and converges to a non zero value as  $t$  increase. The, the ratio of  $\rho^{(de)}/(\rho^{(m)} + \rho^{(de)})$  converges to 1 as  $t$  increase, that is the DE dominates the perfect fluid in the inflationary era of expected. Since dark energy dominates the perfect fluid in the inflationary era as mentioned above, we may neglect the perfect fluid while examining the properties of the dark energy in this model.  $\rho^{(de)}$  is always positive provided  $\ln|\lambda| < 3$  and  $\ln|n| < 3$ , begins with a finite value at  $t=0$ , exhibits different behaviors depending on the choice of the parameters at the earlier times, and tends to  $3k^2(1-n^2/9)$  for large values of  $t$ . It can be observed that the bigger the anisotropy of the expansion the lower the  $p^{(de)}$  in any given instant  $\lambda$  lose, its effect exponentially as  $t$  increases, thus we may say that the higher the  $\lambda$  the lower the  $p^{(de)}$  in any given instant for relatively big  $t$  values. The EoS parameter of the DE  $w^{(de)}$  may begin in phantom ( $w < -1$ ) or quintessence ( $w > -1$ ) region and tends to  $-1$  [ cosmological constant,  $w = -1$ ] by exhibiting various patterns as  $t$  increases ; one can observe that  $p^{(de)}$  increases when  $w^{(de)} < -1$ , decrease when  $w^{(de)} > -1$  and is constant when  $w^{(de)} = -1$  as would expected.

Then anisotropy of the expansion decrease ; monotonically as  $t$  increase when  $n = 0$  however it exhibits nontrivial behavior at the early times of the universe and converges to a non-zero value for the late times when  $n \neq 0$ . the deviation parameters  $\delta$  and  $r$  finite at  $t = 0$  and converge to  $2n/(n+3)$  and  $n(2n+3)/(n^2-9)$  respectively as  $t \rightarrow \infty$ . The anisotropy of the DE which has been defined as  $(\delta-\gamma)/\omega^{(de)} \rightarrow 9n/(n^2-9)$ .

**\*Model for  $m \neq 0$  ( $q \neq -1$ ) –**

The solutions in this subsection are valid for all possible values of  $m$  except for  $m=3$  and  $m=0$ , thus the solutions for  $m=3$  are given from (3.15) one can see that the initial time of the universe is  $t = -C2/mk$  for  $m \neq 0$  for brevity of the equations, we may redefine the cosmic time as

$$t^1 = mkt + c_2 \tag{3.38}$$

and by doing that the initial time of the universe has also been set to  $t^1 = 0$ . Thus we may rewrite the metric as –

$$ds^2 = (3k)^{-2} dt'^2 - A(t')^2 dx^2 - B(t')^2 (dy^2 + dz^2) \tag{3.39}$$

using (3.22) we may obtain the ratios of the scale factor  $A(t)/B(t)$  and on manipulating the result by using (3.15) we get the following exact expression for the, scale, factor

$$A'(t) = k^{\frac{2}{3}} t' \frac{1}{m} - \frac{2n}{m(m-3)} e^{\frac{2}{3} \frac{\lambda}{k(m-3)} t'} t'^{1-\frac{3}{m}} \tag{3.40}$$

$$B(t') = k^{-\frac{1}{3}} K^{-\frac{1}{3}} t' \frac{1}{m} + \frac{n}{m(m-3)} e^{-\frac{1}{3} \frac{\lambda}{k(m-3)} t'} t'^{1-\frac{3}{m}} \tag{3.41}$$

where K is the positive constant of integration.

The spatial volume of the universe is found as

$$V = t'^{\frac{3}{m}} \tag{3.42}$$

The direction Hubble parameters as defined in (3.12) are found as

$$H_x = kt'^{-1} + \frac{2}{3} \lambda t'^{-\frac{3}{m}} - \frac{2nk}{(m-3)} t'^{-1} \tag{3.43}$$

$$H_{y,z} = kt'^{-1} - \frac{1}{3} \lambda t'^{-\frac{3}{m}} + \frac{nk}{(m-3)} t'^{-1} \tag{3.44}$$

using (3.17), (3.43) and (3.44) in (3.29) we get

$$\Delta(t') = \frac{2}{9} \left( \frac{\lambda}{k} t'^{1-\frac{3}{m}} - \frac{3n}{m-3} \right)^2 \tag{3.45}$$

for the anisotropy parameter of the expansion. The expansion and shear scalar are, respectively, found as –

$$\theta = 3kt'^{-1} = 3H \tag{3.46}$$

$$\sigma^2 = \frac{1}{3} K^2 t'^{1-2} \left( \frac{\lambda}{K} t'^{1-\frac{3}{m}} - \frac{3n}{m-3} \right)^2 \tag{3.47}$$

using the scale factors [(3.40) – (3.41)] in (3.2), the energy density of the perfect fluid is found as

$$\rho^{(m)}(t') = \rho_o^{(m)} (t'^{\frac{1-\omega(m)}{m}}). \tag{3.48}$$

The energy density of the DE can be found from (2.5) by using the scale factors (3.40) – (3.41) and the energy density of the perfect fluid (3.48) and written in terms  $\Delta(t)$  as

$$\rho^{(de)}(t') = 3k^2 \left( 1 - \frac{1}{2} \Delta(t') \right) t'^{-2} - \rho^{(m)}(t') \tag{3.49}$$

using (3.40 – 3.41) and (3.49) in (3.5) we get

$$\omega^{(de)}(t') = \frac{\frac{2}{3} \frac{m}{m-3} \lambda nkt'^{-1-\frac{3}{m}-\frac{1}{3}} \lambda^2 t'^{\frac{6}{m}} + (2m-3)k^2 t'^{-2} \left( 1 - \frac{n^2}{(m-3)^2} \right)}{3k^2 \left( 1 - \frac{1}{2} \Delta(t') \right) t'^{-2} - \rho^{(m)}(t') - \omega^{(m)} \rho^{(m)}(t')} - \omega^{(m)} \rho^{(m)}(t') \tag{3.50}$$

for the deviation – free EoS parameter of the DE. And finally using eqh (3.40 – 3.41) and (3.49) in (3.6) and (3.7) we may get deviation parameters as

$$\delta(t') = \frac{nkt'^{-2} \left( 2k - \frac{2}{3} \lambda t'^{1-\frac{3}{m}} + \frac{2nk}{(m-3)} \right)}{3k^2 \left( 1 - \frac{1}{2} \Delta(t') \right) t'^{-2} - \rho^{(m)}(t')} \tag{3.51}$$

$$\gamma(t') = \frac{nkt'^{-2} \left( -k - \frac{2}{3} \lambda t'^{1-\frac{3}{m}} + \frac{2nk}{(m-3)} \right)}{3k^2 \left( 1 - \frac{1}{2} \Delta(t') \right) t'^{-2} - \rho^{(m)}(t')} \tag{3.52}$$

**\*. Physical behavior of the model for  $m \neq 0$  ( $q \neq -1$ )**

The universe accelerates for  $0 < m < 1$ , decelerates for  $m > 1$  and expands with constant velocity for  $m = 1$ . This model may represent the radiation dominated era for  $m=2$  and  $\omega^{(m)} = 1/3$ , and the matter dominated era for  $m = 3/2$  and  $\omega^{(m)} = 0$ .

The mean Hubble parameter  $H$ , I expansion scalar  $\theta$  and shear scalar  $\sigma^2$  are infinity large at  $t' = 0$ , and null at  $t' = \infty$

**\*Model for  $m = 3$  ( $q=2$ )**

from eq<sup>n</sup> (3.15) we can see that the initial time of the universe is  $t = -c/3$  from  $m = 3$  for brevity of the equations, we may redefine the cosmic time as

$$t' = 3mkt + C_2 \tag{3.53}$$

and by doing that the initial time of the universe has also been set as  $t' = 0$ . Thus we may rewrite the metric as

$$ds^2 = (3k)^{-2} dt'^2 - A(t')^2 dx^2 - B(t')^2 (dy^2 + dz^2) \tag{3.54}$$

using (3.23) we may get the ratios of the scale factors  $A(t)/B(t)$  for  $m = 3$ ; and manipulating the result by using (3.15) we get the following exact expression for the scale factors;

$$A(t') = K^{\frac{2}{3} t'^{\frac{1}{3}} + \frac{2}{9} \frac{t'}{k}} e^{\frac{2}{9} \ln^9(t')^2} \tag{3.55}$$

$$B(t') = K^{-\frac{1}{3} t'^{\frac{1}{3}} - \frac{1}{9} \frac{t'}{k}} e^{-\frac{2}{18} \ln(t')^2} \tag{3.56}$$

where  $k$  is the positive constant of integration. The spatial volume of the universe is found as

$$V = t' \tag{3.57}$$

The direction Hubble parameters as defined in (3.12) are found as

$$H_x = kt'^{-1} + \frac{2}{3} (\lambda + nk \ln(t')) t'^{-1} \tag{3.58}$$

$$H_{y,z} = kt'^{-1} - \frac{1}{3} (\lambda + nk \ln(t')) t'^{-1} \tag{3.59}$$

using (3.17) the scale factors in and (3.44) in (3.29) we get

$$\Delta = \frac{2}{3} k^{-2} (nk \ln(t') + \lambda)^2 \tag{3.60}$$

for the anisotropy parameter of the expansion. The expansion and shear scalar are, respectively, found as

$$\theta = 3kt'^{-1} = 3H \tag{3.61}$$

$$\sigma^2 = \frac{1}{3} (nk \ln(t') + \lambda)^2 t'^{-2} \tag{3.62}$$

using the scale factors in (3.2), the energy density of the perfect fluid is found as

$$\rho^{(m)}(t') = \rho_0^{(m)} t' - (1 + \omega^{(m)}) \tag{3.63}$$

The energy density of the DE can be found from (2.5) by using the scale factors and the energy density of the perfect fluid (3.63)

$$\rho^{(de)}(t') = 3k^2 (1 - \frac{1}{2} \Delta(t')) t'^{-2} - \rho^{(m)}(t') \tag{3.64}$$

using (3.55–3.56) and (3.64) in (3.5) we get  $\omega^{(de)}(t') = \frac{3k^2 t^{1-2} - \frac{1}{3}(nk \ln(t') + \lambda)^2 t^{1-2} + \frac{2}{3}nk \ln(t') + \lambda}{3k^2 (1 - \frac{1}{2}\Delta(t')) t^{1-2} - \rho^{(m)}(t') - \omega^{(m)} \rho^{(m)}(t)}$

$$\omega^{(m)} \rho^{(m)}(t) \tag{3.65}$$

for the deviation free EoS parameter of the DE and finally using eqh (3.55 – 3.56) and (3.64) in (3.6) and (3.7) we may get deviation parameters as

$$\delta(t') = \frac{nk(2k - \frac{2}{3}nk \ln(t') - \frac{2}{3}\lambda) t^{1-2}}{3k^2 (1 - \frac{1}{2}\Delta(t')) t^{1-2} - \rho^{(m)}(t')} \tag{3.66}$$

$$\gamma(t') = \frac{nk(-k - \frac{2}{3}nk \ln(t') - \frac{2}{3}\lambda) t^{1-2}}{3k (1 - \frac{1}{2}\Delta(t')) t^{1-2} - \rho^{(m)}(t')} \tag{3.67}$$

\* Physical behavior of the mode for  $m = 3$  ( $q = 2$ )

physically nothing special has been observed in this model for future investigation, but are may mention that this model may only be valied for intermediate epoche of the universe.

### Conclusion

Analysis of the constant declaration parameter in Equation of state parameters with dynamically anisotropic dark energy have been constructed in general Relativity. We assume that the dark energy is minimally interacting has dynamical energy density and anistropic equation of stats parameter. The conservation of the energy – momentum tensor of the DE has been assumed to consist of two separately addictive conserved parts. A special law has been assumed to consist of two separately additive conserved parts. A special law has been assumed for the deviation from isotropic EoS, which is consistently with the assumption on the conservation of the energy momentum tensor of the DE. exact solutions of Einstein’s. field equation have obtained by assuming a special law of variation for the mean Hubble parameter, which yields a constant value of the deceleration parameter and is not inconsistent with observations some basic geometrical and kinematical features of the models and the dynamics of the anisotropic DE in these models have been examined A more general EoS parameter has been introduced for the DE, and the isotropic DE can be recovered by choosing  $n$  to be null where  $n$  parameterizes the amplitude of the deviation from isotropic EoS parameter of the DE. In all the models, while the anisotropy of the DE contributes to the expansion of one of the scale factors, it opposes to the expansion of the other two parameters that parameterize the difference between directional Hubble parameters  $\lambda$  and  $n$ , give rise to non-trivial dynamically anisotropic, expansion histories. This allows for the possibility to fine tune the isotropization of the Bianchi metric during an accelerating epoch of the universe in order to generate arbitrary ellipsoidality by choosing a suitable value of  $n$  such a result provides also the possibility to fine tune the CmB anisotropy. The anisotropy of the expansion can mildly to tally isotropize in relatively earlier times of the universe.

Nevertheless, it converges to a nonzero constant value  $o$  for the later times of the universe in all models. It is also observed that the anisotropy of the DE energy doesn’t always act so as to increase the anisotropy of the expansion, when signs of  $\lambda$  and  $n$  are opposite the overall effect is to lower the expansion anisotropy in relatively earlier times of the universe.

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