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How Have Pattern Recognition Algorithms Been Used to Identify and Support Conjectures in Number Theory?

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Abstract
The discipline of this research paper revolves around recent LLM benchmarks that have mainly pivoted around general mathematical problems. Artificial Intelligence can be used to identify structure in data trends, images, numbers, and signals. Researchers have been using AI modules to discover and prove conjectures in number theory patterns. One of the most celebrated conjectures as an example is the Birch–Swinnerton–Dyer (BSD) conjecture, which detects logical patterns supporting the varying ranks in different elliptic curves, utilizing databases such as the Cremona database within a high-dimensional point cloud. Despite substantial progress over the past few decades, researchers and data scientists are yet to prove this conjecture.
In contemporary mathematical research, pattern recognition and machine learning techniques are primarily employed to **empirically support** conjectures by uncovering statistical correlations and latent structure in large datasets, rather than to generate formal proofs.
Therefore, this research will feature key mathematical patterns in prime numbers, such as the Hermitian symmetric matrix and analytic functions such as the Riemann–Zeta function, to systematically examine patterns and behaviours that emerge from the seemingly chaotic distribution of prime numbers.
By synthesising insights from analytic number theory and modern data-driven methodologies, this paper explores how computational tools help reveal order within mathematical systems traditionally regarded as stochastic or incompressible.
This research demonstrates the scope of AI in pattern recognition, along with the role of number theory as a rigorous and mathematically grounded benchmark for evaluating the capabilities and limitations of artificial intelligence systems.
Ultimately, the paper highlights the growing interdisciplinary relationship between mathematics and artificial intelligence, illustrating how empirical pattern detection complements theoretical reasoning in the investigation of long-standing mathematical conjectures.

Keywords: Artificial Intelligence, Large Language Models, Pattern Recognition, Machine Learning in Mathematics, Number Theory, Mathematical Conjectures, Birch–Swinnerton–Dyer Conjecture, Analytic Number Theory, Prime Number Distribution, Riemann–Zeta Function, Elliptic Curves, Data-Driven Mathematical Discovery

Introduction

The Riemann Zeta hypothesis has posed a great challenge to mathematicians and data scientists alike due the unending structure and irregularity of prime numbers. According to this hypothesis, there is a deeper, complex structure hidden beneath the stochastic partitions of prime numbers, as all real parts of non-trivial zeros from the Riemann Zeta function, $\zeta(s)$, are hypothesized to lie on the line a = 0.5 where s is equal to a + bi. To date, all numerical verification has supported the Riemann Zeta hypothesis with extensively strong empirical evidence.

Numerical verification in this context refers to large-scale computational checks of non-trivial zeros of the Riemann zeta function, which have consistently aligned with the critical line but do not constitute a formal mathematical proof.

However, the conclusive proving of this hypothesis has become a great challenge for modern day researchers, earning this problem its place on the Millennium Prize Problem, denoting that solving this question would earn the recipient a million US dollars.

This distinction between overwhelming empirical evidence and the absence of a deductive proof highlights a central tension in modern mathematics: the gap between computational verification and formal logical certainty.

The issue faced in proving the Riemann-Zeta hypothesis is the infinitismal nature of prime numbers. Data Scientists have proven this conjecture to hold true until the largest verifiable prime number 2^{136},279,841 − 1. This problem holds a lot of value, as prime partitions are largely used in encryptions and cryptography.

The existence and verification of extremely large prime numbers, such as Mersenne primes, is mathematically independent of the Riemann Hypothesis, which instead concerns the distribution of zeros of the zeta function rather than bounds on prime magnitude.

Therefore, further research in this field could permit the fortification of encryption keys in money transactions. While the Riemann Hypothesis does not directly enable stronger encryption schemes, deeper understanding of prime distributions contributes to theoretical foundations underlying modern cryptographic systems. A handful of data scientists have attempted to prove this conjecture with the help of pattern recognition models and LLMs. In practice, these computational and machine learning approaches do not aim to produce formal proofs, but rather to detect statistical regularities and structural patterns that may support or inform theoretical conjectures.

This paper denotes relevant developments regarding this field of study alongside the various patterns identified to support the Riemann Zeta conjecture via computational mathematics.

Emphasis is placed on empirical pattern detection, numerical experimentation, and data-driven insights rather than traditional deductive proof techniques. This paper synthesizes various sources of research and findings to present the role of number theory in large language models, with a keen focus on prime numbers and computational mathematics. Number Theory has consistently been credited as a vital subject of study in the past, and today plays a tangible role in the digital era.

In recent years, number theory has also emerged as a rigorous benchmark domain for evaluating artificial intelligence systems, owing to its deterministic structure and deep theoretical complexity. Through analysis of empirical experimentation, computational approaches to data-driven tasks, and identification of structure in mathematical patterns, this paper will present how large language models test and explore popular conjectures in the field of mathematics. In doing so, this literature review will denote the capabilities and the shortcomings of modern day pattern recognition algorithms in tackling complex mathematical problems as well.

By explicitly contrasting the strengths of AI-driven empirical analysis with the limitations of automated reasoning, this paper situates large language models as supportive tools rather than replacements for human mathematical insight.

Review Design

This literature review will discuss the role of pattern recognition algorithms in a narrative framework, alluding to its abilities and shortcomings. Furthermore, this paper will also allude to multiple factors affecting experimental simulations, underlying the unsolvability of this field of research.

The narrative review approach is adopted to allow synthesis of theoretical, computational, and empirical perspectives, rather than to conduct a quantitative meta-analysis, which is not well-suited to exploratory mathematical research.

Particular attention is given to the limitations imposed by mathematical incompleteness, computational complexity, and the distinction between empirical validation and formal proof.

This review has cited various sources and research papers, including *Machine Learning of the Prime Distribution* by Alexander Kolpakov and Aiden Rocke; L. Alessandretti, A. Baronchelli, and Y.-H. He, *Machine Learning Meets Number Theory: The Data Science of Birch–Swinnerton–Dyer*; Kolpakov, A., & Rocke, A. A. (2024), *Machine Learning of the Prime Distribution*; Woods, K. S., Doss, C. C., Bowyer, K. W., Solka, J. L., Priebe, C. E., & Kegelmeyer, W. P. Jr., *Comparative Evaluation of Pattern Recognition Techniques for Detection of Microcalcifications in Mammography*; Titchmarsh, E. C., and Heath-Brown, D. R., *The Theory of the Riemann Zeta-Function*, and more.

While some cited works originate outside pure number theory, their inclusion provides methodological insight into pattern recognition evaluation, benchmarking strategies, and model validation techniques applicable to mathematical datasets.

By integrating both classical analytic texts and modern machine learning research, this review aims to bridge foundational mathematical theory with contemporary computational approaches. Sources were selected based on relevance to prime number distribution, elliptic curve analysis, zeta-function behaviour, and the application of pattern recognition algorithms to mathematically rigorous datasets.

Methodology of Literature Review

Firstly, it is important to note that one of the main origins of the intrinsic nature of prime numbers in number theory is their almost incompressibility. Its stochastic placing creates an absence of structure within prime partitions, which can be observed through the prime/composite indicator string — a pattern recognition representation of prime numbers on the natural number dataset where prime and composite numbers are represented binary via 1 and 0 respectively.

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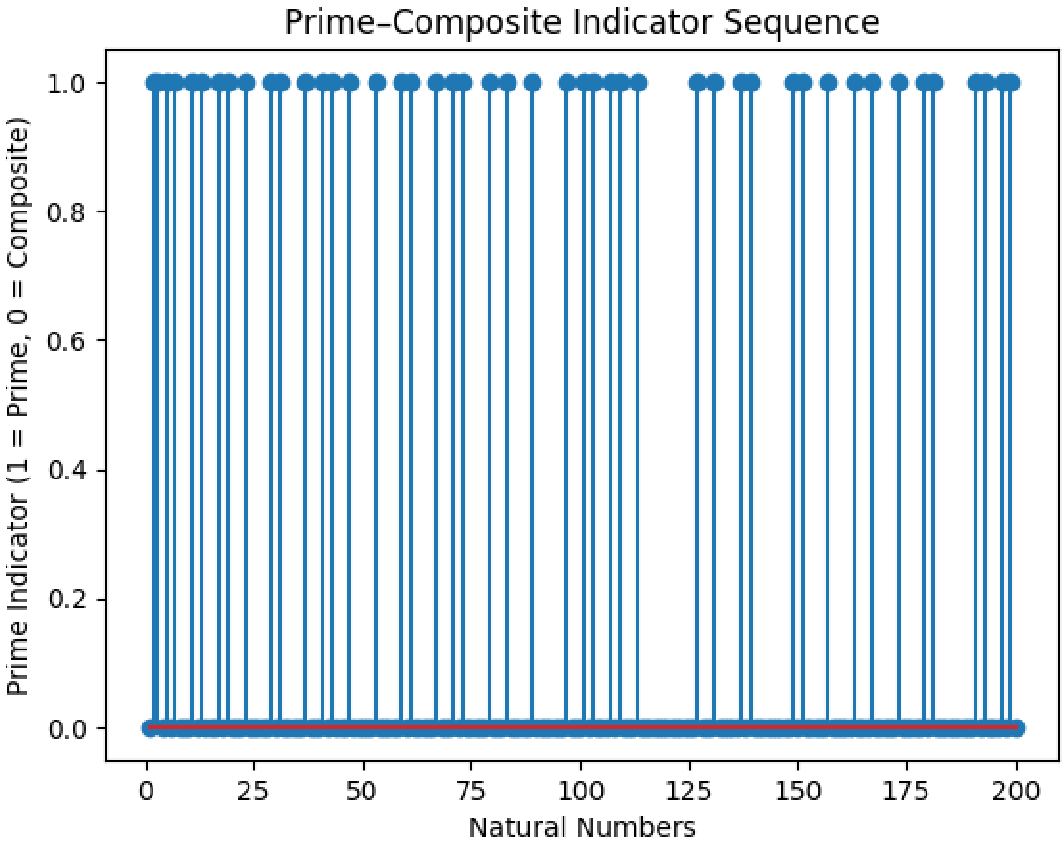


Binary prime–composite indicator sequence illustrating the apparent randomness and incompressibility of prime distributions.

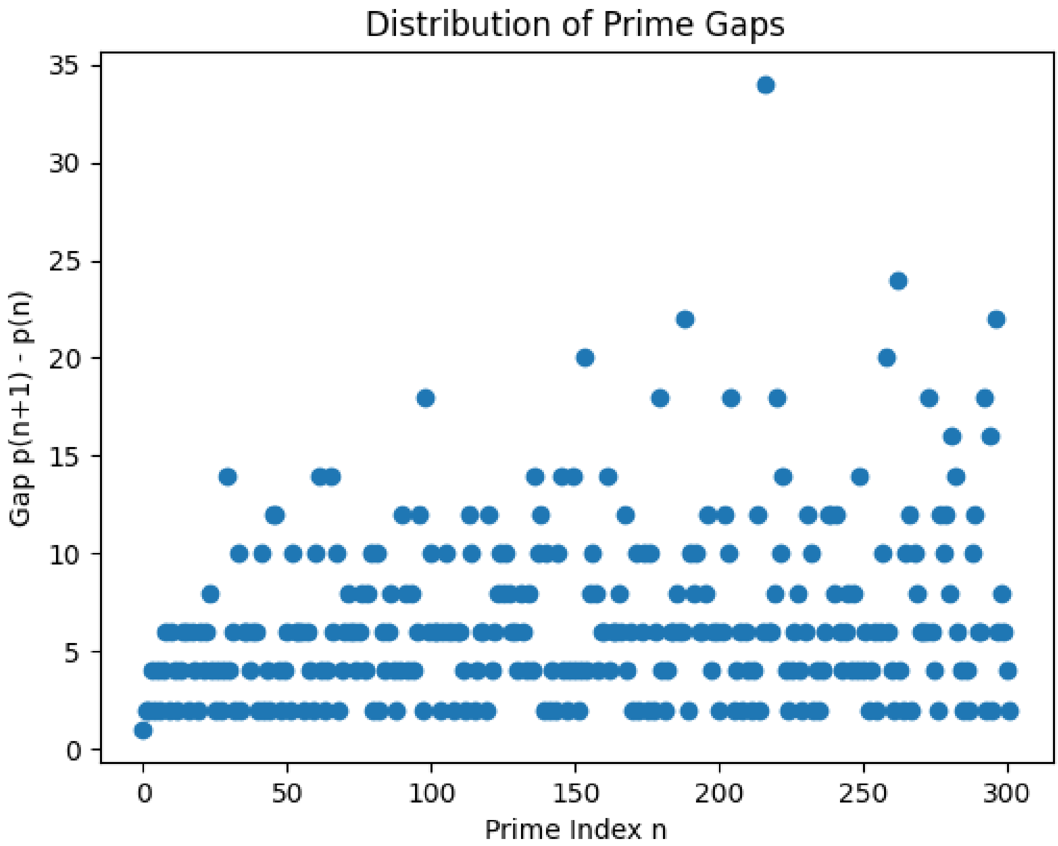
The concept of incompressibility here is used in an information-theoretic sense, reflecting the difficulty of encoding prime distributions using short deterministic descriptions.

However, the main reason why entropy grows is because of Chebyshev's theorem, where prime numbers are denoted to become rarer as numbers get larger, but never completely disappear, and are said to follow a statistically predictable pattern. Chebyshev's results provide bounds on the distribution of prime numbers; the interpretation of entropy growth in this context is a conceptual analogy rather than a formally defined mathematical consequence of the theorem.

This theorem considers the random integer Z and its prime factorization, and deals with the probability of a prime number P lesser than Z being its factor, which is 1/P. Here, the information cost is log P. Under this theorem, it is seen that as Z increases, the total information required to describe Z via its prime factors increases, causing the increasing behaviour of entropy.

This interpretation aligns with broader information-theoretic perspectives in number theory, where prime factorizations act as a natural encoding scheme for integers.

Secondly, due to the unpredictability of the next prime number, an entropy arises within prime partitions. As the dataset grows larger, the validity of the LLM's predictability grows smaller, leading to yet another form of entropy.



Distribution of gaps between consecutive prime numbers, demonstrating increasing variance as values grow larger.

Large language models are not employed to predict specific future primes, but to analyse aggregate statistical properties and structural features within prime-related datasets.

The previously discussed information cost compiles to add extensive amounts of uncertainty, rendering simple pattern recognition algorithms' work on detecting further evidence unreliable. This limitation becomes especially pronounced when models extrapolate beyond the range of observed data, increasing susceptibility to overfitting and spurious correlations.

These variables accumulate to create a substantial amount of uncertainty within the experiment, introducing sources of errors such as bias and hallucination. In this context, hallucination refers to the generation of mathematically inconsistent or unsupported outputs by language models when faced with high-entropy data.

With all this in mind, the LLM must correspond to Occam's razor with the aim to align with the simplest pattern recognition algorithm that has the maximum entropy distribution via minimal prior assumptions made and consequently minimal KL-divergence.

This principle reflects a modelling preference that favours parsimonious representations rather than a strict optimisation guarantee within deterministic mathematical systems.

This promises minimal hallucination rates and most accurate representation of the given data, permitting the research to remain neutral and unbiased.

Adopting this methodological stance ensures that conclusions drawn from computational analysis remain interpretive and exploratory, rather than overstated as definitive mathematical results.

This highlights the unsolvable nature of prime partitions. For such various reasons, LLMs do not have the computational power to tackle this problem—detecting patterns within the behavior of prime numbers and generating algorithms to accurately predict the placement of the next prime number. The term "unsolvable" is used here in a practical and computational sense, referring to the infeasibility of exact prediction or compression, rather than to a formal statement of mathematical undecidability.

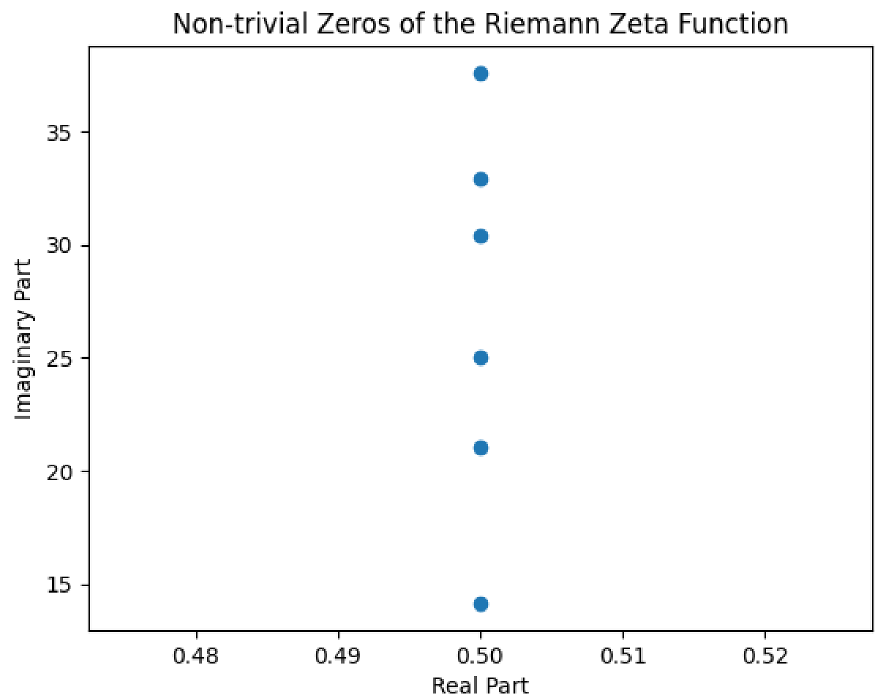
Large language models are not designed to predict individual primes; their limitations arise from the intrinsic high entropy and irregularity of prime distributions rather than from insufficient model scale alone. On the contrary, the prime number database consequently becomes optimal for running simulations and training. It is mathematically rigorous, ensuring that LLM simulations reliably denote the benchmarking of algorithms even though this database is deterministic.

This determinism is a significant advantage, as it allows reproducible experimentation and objective evaluation of model behaviour without ambiguity introduced by noisy or incomplete data. Indeed, prime numbers seem to follow an irregular structure with random placing, which is true to an extent. However, this is not entirely the case.

Analytic number theory provides evidence that apparent randomness in prime distribution coexists with deep underlying structure. The Riemann Zeta function encodes information about all natural numbers. Since natural numbers can be factorized into prime numbers, this function can be said to heavily rely upon prime numbers. Through its Euler product representation, the zeta function explicitly links prime numbers to the analytic properties of $\zeta(s)$.

Upon incorporating complex numbers in this function, mathematicians came across non-trivial zeros, which contain information of the irregularity about prime numbers from their predicted behavioural patterns. These non-trivial zeros are understood to govern fluctuations in prime-counting functions and deviations from average distribution trends.

According to the Riemann Hypothesis, all non-trivial zero values lie on the line where the real part is equal to 0.5, revealing order and symmetry in the distribution of prime numbers.

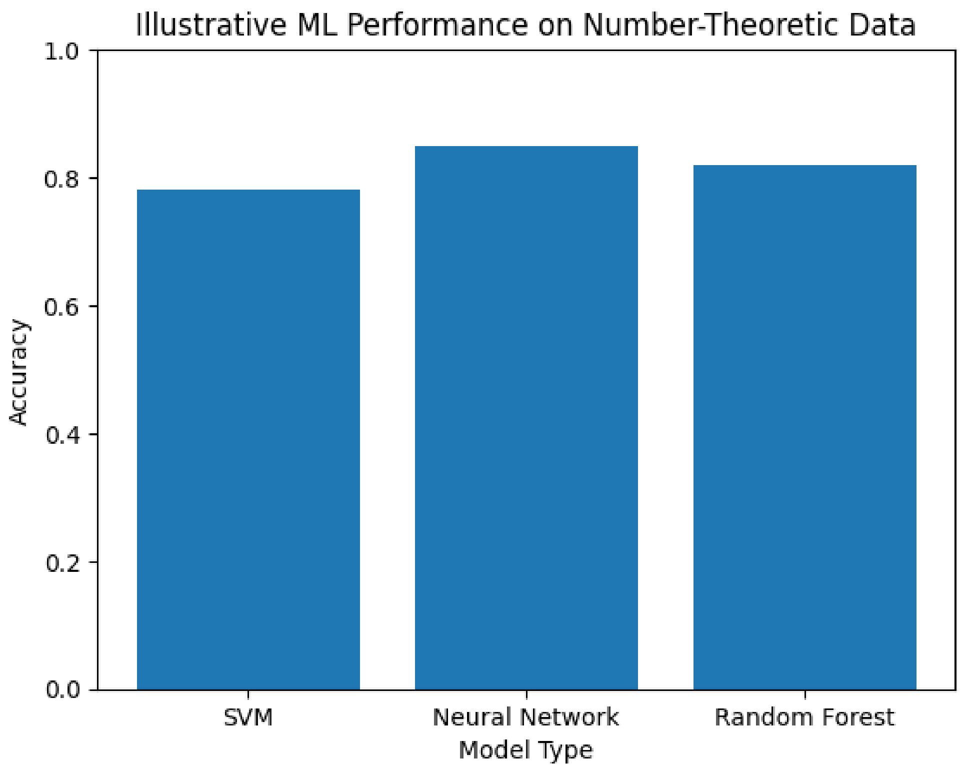


Computed non-trivial zeros of the Riemann zeta function, showing empirical alignment along the critical line Re(s) = 0.5.
This symmetry suggests tight bounds on error terms in prime distribution estimates, though it does not provide explicit formulas for individual primes.
This thereby denotes that prime numbers obey an orderly, complex pattern that is yet to be fully understood. The coexistence of apparent randomness and hidden structure makes prime numbers an ideal subject for exploratory computational analysis. This is where pattern recognition algorithms are commonly introduced.
Such algorithms are employed to analyse numerical data related to prime distributions and zeta-function zeros, offering empirical insights that complement classical theoretical approaches without substituting formal proof.
Pattern recognition algorithms and LLMs are the closest forms of verification researchers have discovered to prove this conjecture. These tools do not constitute formal verification or proof, but rather provide empirical support by identifying numerical regularities and consistency across large datasets. Currently, the Riemann Zeta conjecture is supported with only empirical findings because prime numbers are neverending, which was famously proven by Euclid 2000 years ago. He used proof by contradiction to do so.
Euclid's proof establishes the infinitude of prime numbers, which contributes to the difficulty of exhaustively verifying conjectures related to their global distribution.
Assuming that prime numbers are finite, consider the number N where
N = (p1*p2*p3*...*px) + 1, where _px is the largest prime number.
If N were to be divided by any of the prime numbers in the prime number dataset, there would be a remainder of 1. This indicates that N is either a prime number or that it has a prime factor that is not mentioned in the list. Either way, there is a contradiction, proving that there will always be a larger prime number.
This classical argument highlights the inherent unboundedness of prime numbers, reinforcing why purely computational enumeration can never substitute for deductive proof.
Circling back to empirical findings from LLMs and pattern recognition algorithms, these digitalized methods have taken over experimentation in this field of mathematics, as LLMs are able to conduct verification of proofs and conjectures much more efficiently than humans.
In practice, LLMs assist with checking logical consistency, symbolic manipulation, and numerical verification of known results, rather than independently verifying conjectures.
However, creative simplification of problems to achieve results in a simpler form of solvation is beyond it. The formulation of novel proof strategies and conceptual reframing of problems remains a fundamentally human-driven process.
Indeed, there have been various extensions to achieve such behaviour in LLMs, notably chain-of-thought prompting, encouraging the algorithm to obey a step-by-step process in solvation, which has shown impressive improvements in large language models' ability to perform complex reasoning.
These prompting techniques improve transparency and intermediate reasoning quality but do not grant genuine mathematical insight or originality. However, it has not yet approached the human mind's creative thought process, imbuing potential applications from varied fields of research to reach intuitive outcomes. Human mathematicians excel at transferring intuition across domains, identifying analogies, and introducing abstract frameworks that are not directly inferable from data alone. At that point, LLMs

are best at supporting such intuitive outcomes with laborious calculations and probable implications on other realms of study.
This complementary relationship positions LLMs as computational collaborators that enhance efficiency and exploration, while leaving conceptual innovation and proof construction to human researchers.

Synthesis of Findings

To summarize, prime numbers are notorious for their incompressible nature, but despite their irregularity show an indirect, hidden pattern shown by the Riemann Zeta conjecture.
This apparent contradiction between randomness and structure lies at the core of analytic number theory and motivates extensive theoretical and computational investigation. This conjecture has been supported by strong empirical findings, but is yet to be proven due to the intrinsic and infinitismal nature of prime numbers.
The lack of a proof stems not from insufficient data, but from the infinite scope and analytical complexity of the problem.
One of the key difficulties presented by the prime number database is that the information required to find the next largest prime increases as numbers become larger. This growth in informational complexity reflects the increasing sparsity and irregular spacing of primes along the number line. Due to the absence of structure and Chebyshev's theorem, entropy increases as Z increases.
Chebyshev's results describe bounds on prime distribution; the notion of increasing entropy here represents an information-theoretic interpretation rather than a formal mathematical consequence. This makes it more difficult for LLMs to compute large prime number databases and find the next largest prime number. Large language models are not intended to compute or predict individual primes, but instead to analyse statistical properties and structural patterns within prime-related datasets.



Illustrative comparison of machine learning model performance in identifying structural patterns in number-theoretic datasets

This increases uncertainty in the accuracy of an LLM's predictabiliAs datasets grow, model uncertainty increases when attempting extrapolation beyond observed numerical ranges.

Therefore, the LLM must follow with Occam's razor for minimal KL-divergence and maximum accuracy.

This principle reflects a modelling preference for simplicity and neutrality rather than a guarantee of optimal performance in deterministic mathematical systems.

While all of this depicts the unsolvability of prime partitions, it also denotes the significance of utilizing number theory and prime numbers to train LLMs, and to observe its potential and limit. Number theory thus serves a dual role: it exposes the limitations of current AI systems while simultaneously providing a rigorous, well-defined benchmark for evaluating their reasoning, generalisation, and robustness.

Discussion

Due to its complex nature, prime numbers can be rather useful without having to be entirely resolved. For instance, they are heavily used in cryptography for encryption along with digital bank accounts.

The practical utility of prime numbers lies not in their complete theoretical resolution, but in their computational hardness properties. The way these security systems such as RSA work is that they take two large prime numbers, p and q, and multiply them to get N. N is visible to the public, whereas p and q are kept hidden for security purposes. The logic here is that multiplication is feasible and easy, whereas factorization is difficult.

RSA security relies on the assumed computational difficulty of factoring large semiprime numbers using classical algorithms. However, if one were to resolve prime numbers on a large scale, RSA would collapse, leading to worldwide vulnerability of digital security.

More precisely, it is the efficient factorisation of large semiprimes—not the resolution of prime numbers themselves—that would compromise RSA. This goes to prove how much today's world relies upon the unsolvability of prime factorisation, and the devastating effects of being successful at resolving this problem.

Modern digital infrastructure, including financial systems and secure communications, is deeply dependent on this computational asymmetry. That being said, the presented computational challenge can be theoretically resolved by quantum computing, which would in this hypothetical scenario effectively factorise semi-prime numbers, undermining RSA's security system.

Algorithms such as Shor's algorithm demonstrate this theoretical vulnerability, although large-scale, fault-tolerant quantum computers capable of executing such attacks do not yet exist. A shrewd detail about modern day AI is its limits in solving mathematical problems due to varied reasons, such as disregarding of suitable NLP landscapes such as chain-of-thought prompting. While chain-of-thought prompting improves intermediate reasoning, limitations in AI stem primarily from the absence of genuine mathematical understanding rather than prompting strategies alone

On the contrary, its computational power surpasses human capabilities in pure mathematical calculations.

AI systems excel at large-scale computation, symbolic manipulation, and consistency checking across extensive datasets.

Coupling the logical reasoning of well-trained mathematicians alongside the computational abilities of modern day LLMs is one of the defining reasons for the accomplishments humans have made in this field of study.

This human–AI collaboration reflects a complementary dynamic, where intuition and abstraction are supplied by humans and efficiency and scale are provided by machines.

The future of AI assistance in mathematics promises many things, importantly lesser calculations and labor. Automation of routine verification and computation frees mathematicians to focus on conceptual development and creative problem-solving.

However, a complete takeover is unlikely given that human interpretation and ideas strongly drive modern day findings in mathematics. Mathematical discovery fundamentally depends on insight, abstraction, and cross-domain intuition, which remain human strengths.

Artificial intelligence has reached the stage where it can aid mathematicians in verifying one another's work, permitting widespread collaboration amongst mathematicians. This has accelerated peer verification, reduced human error, and enabled broader participation in complex research efforts.

It has evolved into a cooperative team where different members are allotted different tasks required to be completed for the complex solvation of a conjecture, where AI conducts said allocation for different mathematicians permitting steadfast work.

In this collaborative framework, AI functions as an organisational and computational assistant rather than an autonomous problem-solver. Both mathematicians and LLMs have their own, equal roles in modern day mathematics, hinting towards a future of better cooperation amongst the two bodies. This partnership model reflects a shift toward interdisciplinary, technology-augmented mathematical research. To conclude, the stochastic nature of prime partitions has created one of the most difficult problems in the realm of computational mathematics.

Their resistance to compression and prediction underpins both theoretical depth and practical relevance. With various conjectures and hypotheses, namely the Riemann Zeta Hypothesis, this realm of study has a lot of depth but can seemingly never be completely resolved. The enduring nature of these problems highlights the limits of both human and artificial computation.

This is why it plays a substantial role in cryptography and physics, and will continue to be applicable in many new fields in the near future. As computational tools evolve, number theory will remain a central testing ground for both mathematical insight and the practical limits of artificial intelligence.

Future Use

The continued study of prime numbers and their underlying structure presents significant opportunities for future research, particularly at the intersection of number theory and artificial intelligence.

As computational resources and data availability increase, pattern recognition algorithms can be applied to increasingly large and refined numerical datasets, enabling deeper empirical exploration of longstanding conjectures such as the Riemann Hypothesis and the Birch–Swinnerton–Dyer conjecture.

Future applications of large language models in mathematics are likely to focus on verification, conjecture generation, and hypothesis testing rather than formal proof construction. LLMs may assist mathematicians by proposing plausible conjectural relationships, identifying anomalies in datasets, and cross-referencing results across vast bodies of mathematical literature. In cryptography, continued advancements in number theory will play a crucial role in developing post-quantum encryption schemes.

As quantum computing advances threaten classical cryptographic systems such as RSA, insights from number theory will be essential in designing secure alternatives resistant to quantum attacks. Beyond cryptography, the methods discussed in this paper may find applications in physics, complexity theory, and information science.

The study of spectral properties of mathematical objects, entropy, and randomness has direct parallels in quantum mechanics, statistical physics, and signal processing. From an artificial intelligence perspective, number theory will continue to serve as a rigorous benchmark for evaluating reasoning, generalisation, and robustness in AI systems. Because mathematical datasets are deterministic and free from noise, they provide an ideal testing ground for distinguishing genuine reasoning ability from pattern memorisation in AI models.

Ultimately, future research will likely emphasize deeper human–AI collaboration, where artificial intelligence supports large-scale computation and verification while human researchers guide interpretation, abstraction, and creative insight. Such cooperation promises not only advancements in mathematical understanding but also a clearer definition of the practical and theoretical limits of artificial intelligence itself.

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