International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Algorithmic Strategies of Graph Coloring for Course Timetable Scheduling Problem

Dr. P. Srilakshmi[1], Dr. A. Sri Krishna Chaitanya[2]

[1,2]Department of Mathematics, Malineni Lakshmaiah Women's Engineering College, Guntur, India

ABSTRACT:
The purpose of this paper is to prepare a course time table by introducing a proposed algorithm based on known heuristic graph colouring algorithms, namely the Welch Powel algorithm and the saturation degree ordering algorithm, which found a better and optimal solution. The proposed algorithm will be used to solve the course time table scheduling problem which is formulated by converting the problem into a graph where subjects will be concede as vertices of graph.

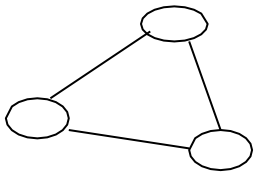
Keywords: Welch Powel, Saturated degree, Graph coloring.

1. INTRODUCTION

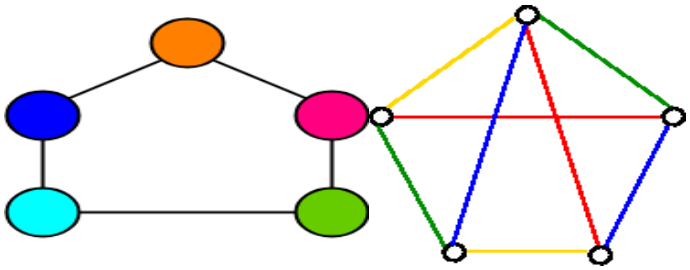
Mathematician Leonhard Euler gave the basic concept of graph theory in 18th century, in which he attempts and solved the famous Konigsberg bridge problem. Graph Theory is ultimately the study of relationships. Given a set of nodes & connections, which can abstract anything like city layouts, scheduling, computer data, graph theory provides a helpful tool to simplify the many real-life problems, studying graphs through a graph provides answers to many problems.

Basic concept of Graph Theory:

An ordered pair G = (V, E) is called graph where V is nonempty set or vertices and E is a set of edges



In graph theory graph coloring is a special case of graph labeling it is an assignment of labels traditionally called "colors" to elements of a graph subject to certain constraints. In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices are of the same color; this is called a vertex coloring. Similarly, an edge coloring assigns a color to each edge so that no two adjacent edges are of the same color, and a face coloring of a planar graph assigns a color to each face or region so that no two faces that share a boundary have the same color.



Graph Coloring

Graph coloring is used to solve many real-life problems, there are many problems whose solution can be easily achieved using graph coloring. Scheduling is a very general problem which is solved using graph coloring. scheduling problem can be defined as allocation of related recourses among several times slot satisfying various types of important preferential constraints acted at creating optimized conflict free scheduling. The following problems are included in the scheduling problem-

Course timetable scheduling
Exam timetable scheduling
School time table scheduling
Aircraft scheduling
Job shop Scheduling
Radio frequency distribution
Register allocation

1.1 DEGREE OF VERTEX: The degree of a vertex v in an undirected graph G is the number of edges incident with (meeting at or ending at) v, A loop connects a vertex to itself so it contributes 2 to the degree of the vertex. Degree of vertex is denoted by deg (v).

1.2 THE DEGREE SEQUENCE: The degree sequence of a graph is the sequence of the degrees of its vertices, usually given in increasing or decreasing order

1.3 ODD DEGREE OF VERTEX: If odd number of edges connected with vertex is called odd degree of vertex

1.4 EVEN DEGREE OF VERTEX: If even number of edges connected with vertex is called odd degree of vertex

1.5 SATURATED DEGREE: The saturation degree of a vertex is the number of different colored vertices connected with that vertex.

Algorithm is a tool to solve any problem systematically. There are many heuristic algorithms present to solve graph coloring problem, but these algorithms also have many limitations which attract researchers towards them and also provide a wide area of research.

2. GRAPH COLORING ALGORITHMS

There are many heuristics technique for coloring graphs in sequential manner.

2.1 GREEDY ALGORITHM: A greedy algorithm is an algorithm that follows a problem-solving heuristic to make a locally optimal choice at each step. The vertices of the graph are colored by a greedy algorithm that considers the vertices of the graph in order and assigns each vertex its first available color. Greedy coloring uses linear time, but generally, they do not try to use the minimum number of colors.

2.2 BACKTRACKING ALGORITHMS: These algorithms use backtracking techniques to solve the problem. These are very easy to implement and provide guarantees of finding the optimal solution, but they have very large exponential computational times.

2.3 GENETIC ALGORITHMS: These algorithms work using genetic operators such as crossover, selection, and mutation to generate schedules. These algorithms are very flexible and can handle complex constraints, but they can be expensive and may not find optimal solutions for problem.

2.4 CONSTRAINT-BASED ALGORITHMS: These algorithms use constraint satisfaction techniques to prepare schedules that satisfy a set of predefined restrictions. These algorithms are highly specialized and can handle complex constraints, but they can be expensive and may not perform well for solving large-scale problems.

2.5 LOCAL SEARCH ALGORITHMS: These are heuristics algorithm to search for the locally optimal schedule for a problem. They are faster than constraint-based and genetic algorithms, but they cannot find an optimal solution.

3. DEGREE BASED ORDERING:

It provides a better systematic way

For coloring a graph. It uses a certain selection normative discipline for choosing the vertex to be colored. Some selection normative discipline for selecting the next vertex to be colored have been select such as

3.1 LARGEST DEGREE ORDERING (LDO): Cipta,H. et al (2023) defined LDO In this algorithm it first selects the vertex which has the maximum degree. During each step it select vertex with maximum degree. The heuristic can be implemented to run in O (n2). Saturation degree algorithm the saturation degree of a vertex is defined as the number of differently colored vertices the vertex is adjacent to. Intuitively, this heuristic provides a more efficiently coloring than largest degree ordering as it can be executed to run in O (n3)

3.2 INCIDENCE DEGREE ORDERING (IDO): Cipta,H. et al (2023) defined IDO as variation of the saturation degree ordering heuristic is the incidence degree ordering. Number of adjacent colored vertices of the vertex is called incidence degree of a vertex. This heuristic can be executed to run in O (n2).

4. WELSH POWEL ALGORITHM: Welsh Powell algorithm is a modified algorithm in which the vertices are arranged in descending order and then the vertices are selected sequentially to provide colors.

4.1 SATURATION DEGREE ORDERING: This algorithm first assigns color to the vertex whose saturation degree is maximum. The saturation degree of a vertex is the number of different colored vertices connected with that vertex.

RELATED WORK

Welch,D. & Powel, M.(1967) gave an algorithm which establish the connection between scheduling or timetabling problem with the well-known problem of coloring the vertices of a graph and also gave new easily determined bound for the number of colors required. Mansuri, A. et al (2010) prepare tow new heuristic graph coloring programs which are based on known heuristic algorithms, have already been introduced. First one is adaptation of the Largest Degree Ordering algorithm, and second one is a adaptation of the Saturation Degree Ordering algorithm. Dahlan, A. (2019) concluded that the Welch Powell Graph Coloring Algorithm can solve properly the problem of scheduling lectures, in their case there are no clashes between scheduled components. Peck, J. et al (1966) work on examination scheduling problem. Wood, D.et al (1969) gave technique for solving large scale time table problem. Johnson, A. et al (1991) applied three different ways of graph coloring with a simulated annealing technique, observing that simulated annealing algorithms may give the best results, but it takes a sufficiently large run time. Kiaer, L.et al (1992) in their paper illustrate a heuristic algorithm using graph coloring approach to find

approximate results for a university course timetabling problem. The algorithm applying a weighted graph to model the problem aimed at finding a least cost k-coloring of the graph (k being number of available timeslots) while minimizing conflicts. Researchers proposed various algorithm to solve the graph coloring problem Mendez, D (2008) gave a cutting algorithm for graph coloring. Lucet, C. et al (2006) gave an exact method to graph coloring. Malviya, A. et al (2022) gave description on various type of Algorithm for graph coloring. Segundo, P. et al (2012) invent new technique using set covering formulation of the given graph and on DASTUR algorithm. The researcher has also given a new technique, Shukla, A et al (2019) the author used a linked list of data structures in his algorithm, which stores the information about color of the vertex and the address of the next vertex to be colored. Swapnil Biswas, S.et al(2023) show that the Welsh-Powell algorithm and the DSATUR algorithm are the most effective in generating optimal schedules. The study also provides perception into the advantages and limitations of using graph coloring in timetable scheduling. Sunanto, U et al (2022) construct modifies python program for scheduling problems and provides an alternative solution to the scheduling problem by using the concept of graph theory applying graph coloring.

5. WELCH POWEL ALGORITHM

Welsh Powell algorithms adopt a greedy strategy of assigning colors by favoring to vertices with higher degree of vertex using heuristic approach. This algorithm specifically assigns a color to the vertices of the graph by giving priority to their maximum degree, then it iterates based on the descending order of the degree of the vertices and assigns a suitable color to each vertex. Since this algorithm works based on the highest degree of vertices, it reduces the chances of conflicts and increases the chances of optimal solution.

5.1. WELSH POWELL ALGORITHM CONSISTS OF FOLLOWING STEPS:

1. Find out the degree of each vertex
2. List the vertices in ascending order by degree.
3. Color the top first vertex with color 1.
4. Go further down the list and color all vertices that are not connected to the colored vertex with the same color.
5. Repeat step 4 with the new color on all uncolored vertices in descending order of degree until all vertices are completely colored.

5.2. SATURATION DEGREE ORDERING: Klotz, W. et al (2002) Saturated degree ordering is defined as follows. Suppose that vertices $V_1, V_2,\ldots,V_{k-1}$ have been chosen and colored. Then at step k, vertex V_k with the maximum Saturation degree is selected. The saturation degree of a vertex is defined as the number of differently colored vertices the vertex is adjacent to. For example, if a vertex v has degree equal to 5 where one of its neighbors is uncolored, two of them are colored with color equal to 2, while the last one is colored with color equal to 3, then v has saturation degree equal to 2. When the vertex of maximum saturation degree, is selected the solution to the problem of vertex degree equality is found in favor of the vertex with the largest degree. This heuristic algorithm provides a better coloring solution than Largest Degree Ordering and Welsh Powell since it first. colors vertices most constrained by previous color choices. The heuristic can be implemented to run in O(n2).

6. PROPOSED ALGORITHM

In this paper, a new heuristic graph coloring algorithm based on known algorithms is introduced. In this algorithm, we construct new algorithm by combining Welch Powell algorithm with saturation degree

ordering. The algorithm works as Welch Powell method but when we found that there are two vertices with the same degree, the saturation degree was used to select between them, if the saturation degree is also the same, then those uncolored vertices Take a subgraph of and choose the vertex with maximum degree among them, in any case the degree in the subgraph is even then choose one vertex from them at random. There are two criteria for coloring a vertex:

- The number of vertices connected to a vertex with the largest degree ordering.
- Number of vertices of different colors associated with vertex saturation degree. placing order.

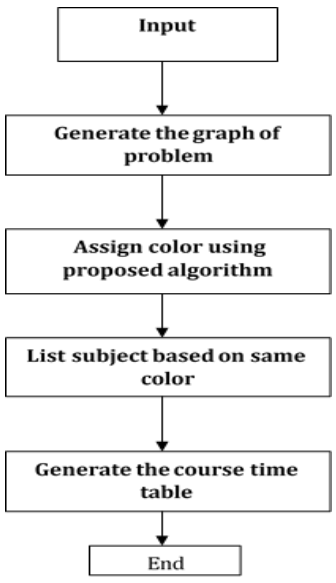
Modified algorithm consists of following steps:

1. Find out the degree of each vertex
2. List the vertices in descending order by degree.
3. Color the top first vertex with color 1.
4. Proceed through the list of vertices, selecting the next vertex with the highest degree. If two or more vertices have the same degree, prioritize the saturation degree of the vertex, and assign a color to the vertex with the highest saturation degree, otherwise color the vertex using Welsh Powel algorithm.
5. Proceed through the list and color all vertices that are not connected to a previously colored vertex with the same color.
6. Repeat step 4 to 5 on all uncolored vertices with a new color, in descending order of degrees until all the vertices are colored.

7. METHODOLOGY

In this paper, vertex coloring approach is used to solve the course time table scheduling problem. The problem is first transformed into a graph where courses are represented as vertices, with edges between vertices drawn according to the type of graph created. The scheduling problem can be represented using vertex coloring in such a way that an optimal solution can be obtained. The proposed algorithm can find more optimal and unique solutions. Two step method is used to solve problem, in first step problem is formulate into graph coloring problem and in the second step proposed algorithm is used to color the graph. overall work flow of methodology given in following diagram .

Work flow diagram of methodology



6. PROBLEM OF SCHEDULING COURSE TIME TABLE

Colleges offer a variety of subject combinations to students under new education policy.

Streams like B.A, B.Com. or B.Sc., students can take one subject as Major and other subjects as Minor/Elective papers and vocational. Also, to conduct such courses smoothly, minimum time slots need to be scheduled in such manner that teacher of respective subjects as per their availability without any conflict. In the following subsection, we have presented the course time table of B.Com. first year branch of Shri Neelkanth Eshwar Govt. PG College Khandwa using proposed modified algorithm of graph coloring.

7. CONSTRAINT

Constraints are the most integral part of any scheduling problem. In other words, we can say that a scheduling problem has arisen only if there is a constraint. These are several constraints involved in creating a schedule. Constraints are mainly classified into hard and soft constraints; these can be accepted or rejected depending on the satisfaction of the schedule.

7.1 HARD CONSTRAINTS -Hard constraints are necessary conditions that must be satisfied to produce ideal scheduling. If a hard constraint cannot be successfully satisfied by a schedule, such a schedule cannot be accepted. For example, subjects that have the usual number of students cannot be scheduled in the same time-slot, courses cannot be assigned more than the available time-slot or the maximum number of lectures. In scheduling data collection, which involves resources such as teachers and classes, no courses can be scheduled at the same time-slot in the same class, and there cannot be more than one course taught by the same teacher during the week. Can be taught in a single day Can be determined. It is possible that the time cannot be specified.

7.2 SOFT CONSTRAINTS -soft constraints are optional conditions; the scheduling problem can be considered successful without satisfying these conditions, but all hard constraints must be satisfied. Often, it is impossible to include all soft constraints in a schedule. Provided that all hard restrictions are satisfied. For example- a teacher may prefer to take practical classes only in the second half, Honors and Pass classes are preferred to be scheduled in non-overlapping time- slots,

The list of available courses and subjects of the college is given in following table

Subject combination

Branch name	Sub 1 compulsory	Sub2 compulsory	Sub3 compulsory	Sub4 Elective	Sub5 Vocational
B.com plain	F.A/C	BRF	B.O. and comm.	FSI /PD	BAI/BE
B.com com	F.A/C	BRF	B.O. and comm.	DPS/CF	WD/DP
B.com tax	F.A/C	BRF	B.O. and comm.	SP	SS
B.com mar	F.A/C	BRF	B.O. and comm.	GST	BE/BAI

F.A/C-Financial Account, BRF-business regulatory framework, B.O. - Business organization and communication, FSI – finance service and insurance, PD – personality development, BAI – banking and insurance, BE – business economics, DPS – data processing software, CF – computer fundamental, WD-web designing, DP – desktop publishing, SP – sales promotion, SS salesmanship, GST – goods and services tax Hindi and English are compulsory subjects for all branches.

HARD CONSTRAINTS

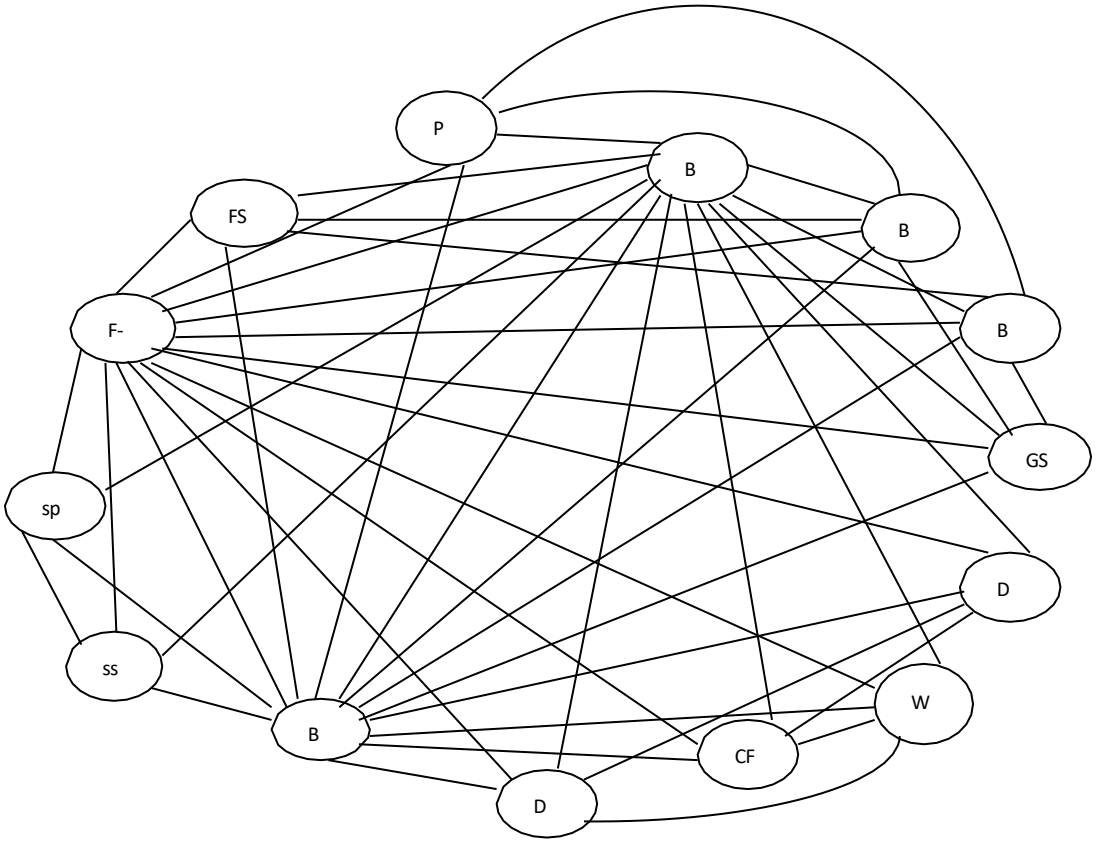
- Courses having common student cannot be given at the same time slot on the same day.
- Total number of available periods is 7 (maximum)

SOFT CONSTRAINTS

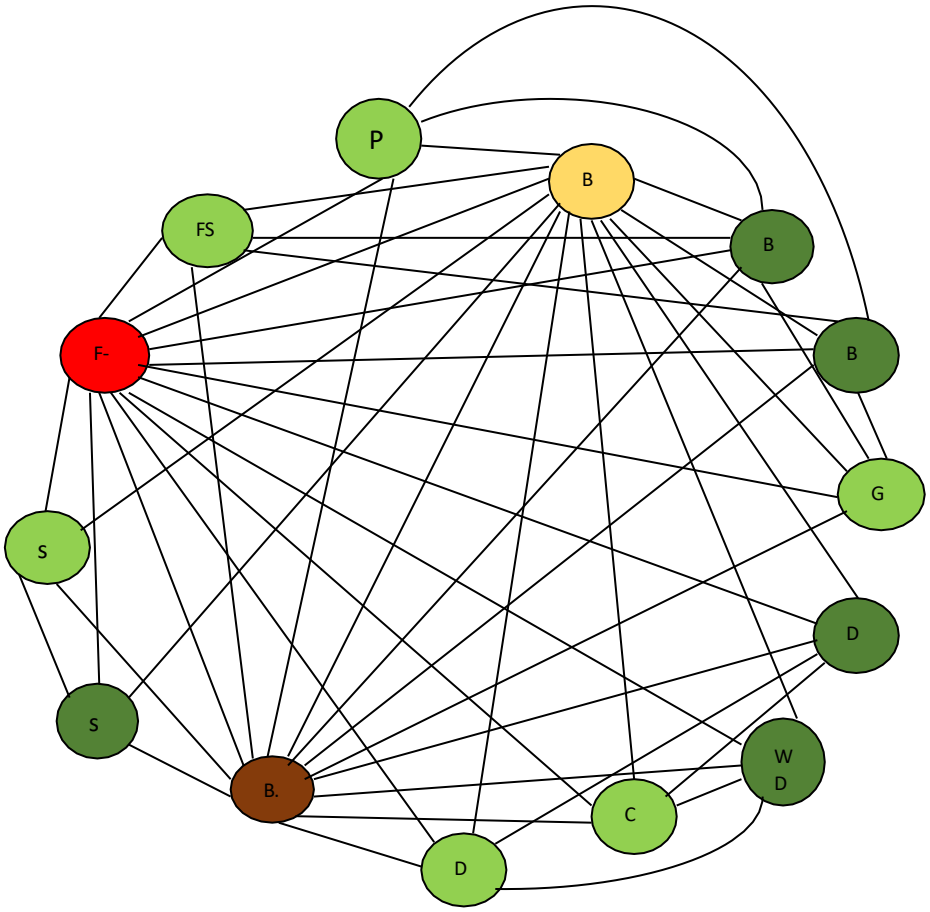
Optional courses need to be scheduled in non-overlapping time-slots.

8. SOLUTION

Considering each vertex as a course, an edge between two vertices is drawn only if there is a common student Graph of above courses given below.



After applying proposed coloring algorithm, the resultant graph given bellow is properly colored with 5 colors, hence the chromatic number is 5. This is the minimum numbers of none conflicting Lectures scheduling all the given courses



SOLUTION

In the above solution, the hard constraints are satisfied properly. The resultant minimum number of time slots needed is 4 which do not exceed the total available 8 periods. In the above solution, the hard constraints are completely satisfied. The resulting minimum number of lectures required is 5 which do not exceed the total of 7 periods available. BRF, B.O. and F-A/C are related to the all subjects, hence separate lectures will have to be provided them Lectures can be provided at the same time to vertex that have the same color. Course time table for given problem shows in following table.

Lectures	**Subjects**					
1.	F-a/c					
2.	BRF					
3.	B.O.					
4.	SS	BAI	BE	DP	WD	
5.	SP	FSI	PD	GST	CF	DPS

9. CONCLUSION

This paper describes a new graph (vertex coloring) coloring algorithm which is developed using well known Welch Powel and saturation degree ordering and the main concern of this algorithm is to find an optimal and unique solution for the same chromatic number, these algorithms provide more optimal solution, and thus it may be unique. in this problem two step method is being used, in the first step problem convert into graphically than in second step algorithm is apply to color the vertex of graph. This paper

focuses on solving the Scheduling problem more precisely and attempts to show that it provides a more accurate solution than the older algorithm for similar chromatic numbers. This paper focuses on solving the Scheduling problem more precisely and attempts to show that it provides a more accurate solution than the older algorithm for similar chromatic numbers.

REFERENCES

1. Welch, D. J., & Powel, M. B. (1967). "An upper bound for the chromatic number of a graph and its application to timetabling problems". The Computer Journal, vol.10(1),pp. 85-86.
2. Mansuri A., Gupta V. and Chandel R.S.(2010) "Coloring Programs in Graph Theory" International Journal of Math. Analysis, Vol. 4,pp. 2473 - 2479
3. Dahlan Abdullah (2019)"Lecture Scheduling System Using Welch Powell Graph Coloring Algorithm in Informatics Engineering Departement of Universitas Malikussale" Journal of Physics: Conference Series 1363 012074
4. Peck, J. E. L., Williams, M. R.(1966)."Examination Scheduling," Communications of the ACM, 9(6), pp.433-434.
5. Wood, D. C.(1969). "A Technique for Coloring a Graph Applicable to Large Scale Timetabling Problems," The Computer Journal, 12, pp. 317-319.
6. Johnson, D. S., Aragon, C. R., McGeoch, L. A., and Schevon, C. (1991) "Optimization by Simulated Annealing: An Experimental Evaluation; Part II, Graph Coloring and Number Partitioning," Operations Research, 39(3), pp. 378- 406.,The IBM Research Symposia Series, New York, NY: Plenum Press, 1972, pp. 85-103
7. Kiaer, L., Yellen, J. (1992). "Weighted Graphs and University Timetabling"Computers and Operations Research, 19(1), pp. 59-67.
8. Méndez Díaz, I., Zabala, P. (2008). A cutting plane algorithm for graph coloring. Discrete Applied Mathematics, 156(2): 159-179.
9. Lucet, C., Mendes, F., Moukrim, A. (2006). An exact method for graph coloring. Computers & Operations Research, 33(8): 2189-207.
10. Malviya A., Agrawal B., Kumar S.and Mansuri A., (2022) "Study of algorithm for coloring invariousgraph".International Journal of Statistics and Applied Mathematics,Vol,7(2),pp.88-91.DOI:
11. https://doi.org/10.22271/maths.2022.v7.i2b.798
12. Segundo, P.S. (2012). A new DSATUR-based algorithm for exact vertex coloring. Computer & Operations Research, 39(7): 1724-1733.
13. Shukla A.N. ,Bharti V. and Garag M.L. (2019), "A Link List based Exact Algorithm for Graph Coloring Problem" Revue d 'Intelligence Artificiel,33(3),pp.189-195.
14. Biswas S.,Nusrat S.A.,Sharmin N.and Rahman M (2023) " Graph Coloring in University Timetable Scheduling" I.J. Intelligent Systems and Applications, Vol .3, .pp16-32
15. Sunanto U.A.M., Napitupulu H., Carnia E., Supriatna A.K.(2022) " Course Scheduling of Undergraduate Study Program of Mathematics Universitas Padjadjaran Case Using Gra Coloring with Modified Algorithm", International Journal of Mathematics Trends and Technology, Vol. 68 pp.61-69.
16. Klotz, W. (2002). Graph coloring algorithms Verlag nicht ermittelbar. mathematik.tu-clausthal.de pp.1-9 Gebremedhin, A. H. (1999). Parallel graph coloring. Candidatus scientarum, University of Bergen.

17. Cipta H., Widyasari R.,Batubara F.H.(2023) “Graph Coloring Implementation Using Welch Powel Algorithm In Lecture Scheduling Design For mathematics Department ” M ATHLINE Journal Matematika Dan Pendidikan Matematika,Vol 8,pp1383-1398.
18. Gangrade A., Agrawal B., Kumar S. (2023). Subject Allocation Bipartite Graph coloring Samdarshi, UGCCare, Vol.16(4), pp.3128-3132.
19. Gangrade A., Agrawal B., Kumar S. and Mansuri A. (2022) **"** A study of applications of graph coloring in various fields**".** International Journal of Statistics and Applied Mathematics; Vol **7(2)**, pp. 51-53. https://doi.org/10.22271/maths.2022.v7.i2a.795.
20. Malviya A., Agrawal B., Kumar S. (2023). “A new approach on Star chromatic number of splitting complete bipartite graph” Samdarshi, UGC Care, Vol.16(5), pp.2581-3986.