A Unified Risk-Neutral Stochastic Simulation Framework for Securities-Backed (Lombard) Lending Business Context, Structural Wrong-Way Risk, and Endogenous Loss Modeling

Arghya Ghosh[1], Dr. Ramesh Bhavisetti[2]

[1]Vice President, JP Morgan Chase, Cama Industrial Estate, Goregaon, Mumbai, Maharashtra, India - 400063.
[2]Assistant Professor, R.V.R.R College of Education, Guntur, Andhra Pradesh, India – 522006.

Abstract:
Securities-Backed Lending (SBL), also known as Lombard lending, is widely used in private banking and wealth management to provide liquidity against pledged equities or liquid portfolios. While such loans are over-collateralised and frequently margined, they exhibit strong market-credit interaction: equity drawdowns increase loan-to-value ratios, trigger margin calls and liquidation, and materially elevate default risk. This paper develops a unified risk-neutral stochastic simulation framework for SBL that jointly models interest rates, equity collateral dynamics, and equity-driven default through an intensity (Cox) construction where default intensity depends on the loan-to-value ratio. Recovery and loss-given-default are endogenously determined by collateral value and liquidation haircuts. A fully reproducible Monte Carlo study on simulated market-consistent data is provided, including figures and tables suitable for publication.

Index Terms: Securities-Backed Lending, Lombard loans, collateralised lending, wrong-way risk, Monte Carlo simulation, Cox process, endogenous LGD.

I. BUSINESS CONTEXT AND MOTIVATION

A. What is an SBL (Lombard) loan?

Securities-Backed Lending (SBL) refers to loans granted against pledged marketable securities, typically equities or liquid diversified portfolios. The loan is secured by collateral that is marked-to-market, with contractual margin thresholds that can trigger margin calls or forced liquidation.

B. How banks make business from SBL

SBL is a core product in private banking and wealth management due to its attractive risk-adjusted profitability and client demand for liquidity without selling securities. Banks generate revenue through:

- Net interest margin: loan pricing as a spread over funding (e.g., $r_t + s$), sometimes with floors.
- Cross-selling and client retention: retaining assets under management and enhancing relationship value.
- Fee income: structuring fees, custody, and related services (jurisdiction dependent).

C. Why unified market-credit modelling is essential

Despite over-collateralisation, SBL loans exhibit *structural wrong-way risk*: declines in equity collateral value simultaneously (i) reduce collateral coverage and (ii) increase the borrower's distress likelihood. Hence, default risk cannot be treated as independent from market risk. Additionally, recoveries are driven by collateral liquidation and haircuts, leading to *endogenous loss-given-default (LGD)*. These features make SBL a natural application for a unified stochastic simulation framework.

II. RISK-NEUTRAL VALUATION SETUP

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$ be a filtered probability space satisfying the usual conditions. Let the short rate be r_t and define the money-market account (numeraire)′

$$B_t = \exp\left(\int_0^t r_s\,ds\right). \quad (1)$$

For any integrable payoff X_T measurable at time T, the arbitrage-free time-t value is

$$V_t = \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\int_t^T r_s\,ds\right) X_T \mid \mathcal{F}_t\right]. \quad (2)$$

We use Q to ensure consistency between discounting and asset drifts in simulation and to facilitate valuation adjustments and expected loss computations.

III. UNIFIED STOCHASTIC DYNAMICS FOR RATES, EQUITY COLLATERAL, AND THE LOAN

A. Correlated risk drivers

Let $\mathbf{W}_t = (W_t^{(r)}, W_t^{(S)}, W_t^{(\lambda)})^\top$ be a 3-dimensional Brownian motion with correlation matrix Σ and Cholesky factor L such that $\Sigma = LL^\top$. Correlated increments are generated via

$$\Delta\mathbf{W} = L\Delta\mathbf{Z}, \quad \Delta\mathbf{Z} \sim \mathrm{N}(0, I). \quad (3)$$

This permits explicit modelling of rate-equity correlation and market-credit co-movement.

B. Interest rates (Hull–White short rate)

We model r_t via the Hull–White model [1]:

$$dr_t = (\theta(t) - ar_t)\,dt + \sigma_r\,dW_t^{(r)}, \quad (4)$$

where $a > 0$ is the mean reversion and $\sigma_r > 0$ the rate volatility. In a production setting, $\theta(t)$ is calibrated to the initial curve; the numerical study adopts a constant θ for clarity.

C. Equity collateral under the risk-neutral measure

Let S_t denote the equity collateral price. Under Q, the risk neutral equity dynamics are [2]

$$\frac{dS_t}{S_t} = r_t\,dt + \sigma_S\,dW_t^{(S)}. \quad (5)$$

Collateral value is defined as

$$C_t = \alpha S_t, \quad (6)$$

where α is the number of pledged shares (or an effective exposure scaling for a portfolio proxy).

D. Loan balance and loan-to-value ratio

Assuming the loan accrues at the short rate (stylised, but consistent with the simulation numeraire),′

$$L_t = L_0 \exp\left(\int_0^t r_s\,ds\right). \quad (7)$$

The loan-to-value ratio (LTV) is

$$LTV_t = \frac{L_t}{C_t} \quad (8)$$

The LTV is the central state variable connecting market dynamics to credit outcomes through margining and default intensity.

IV. EQUITY-DRIVEN DEFAULT, MARGINING, AND ENDOGENOUS RECOVERY

A. Default time as a Cox process

We define default time τ using a Cox process [3]:

$$\tau = \inf\left\{t \geq 0 : \int_0^t \lambda_s\, ds \geq E\right\}, \quad E \sim \mathrm{Exp}(1), \quad (9)$$

independent of F_t. This yields conditional survival

$$\mathbb{Q}(\tau > t \mid \mathcal{F}_t) = \exp\left(-\int_0^t \lambda_s\, ds\right). \quad (10)$$

B. Structural wrong-way risk via LTV-dependent intensity

To represent the empirical reality that distress risk rises sharply when collateral coverage deteriorates, we set intensity as an increasing function of LTV:

$$\lambda_t = \lambda_0 \exp(\beta(LTV_t - LTV^*)_+). \quad (11)$$

where $(x)_+ = \max(x,0)$, λ_0 is a base intensity, LTV^* a reference (maintenance-like) level, and $\beta > 0$ controls sensitivity. This specification produces *structural wrong-way risk* even if $\rho_{S\lambda} = 0$ because the default intensity directly linked to loan-to-value.

C. Margin calls and liquidation triggers

Define stopping times for margin call and liquidation:

$$\tau_m = \inf\{t : \mathrm{LTV}_t > LTV^*\}, \quad (12)$$

$$\tau_\ell = \inf\{t : \mathrm{LTV}_t > LTV_{liq}\}. \quad (13)$$

These are path-dependent and naturally handled in Monte Carlo simulation. In the numerical study, liquidation is treated as a close-out event that realises loss (if any) based on collateral liquidation value.

Breaching the maintenance LTV triggers a margin call, requiring the borrower to restore collateral coverage by posting additional securities or reducing the loan balance. If the LTV continues to deteriorate and exceeds the liquidation threshold, the lender initiates immediate collateral liquidation to prevent further losses. The separation of these thresholds provides an operational buffer that balances client relationship management against timely risk mitigation.

D. Endogenous recovery and LGD

Let $h \in [0,1)$ denote a liquidation haircut capturing slippage, bid-ask, and market impact. In simple term haircut is the reduction applied to the market value of collateral to reflect how much money the bank can actually raise if it has to sell the collateral quickly. It accounts for lower prices in forced sales due to bid-ask spreads, limited liquidity, and market impact. As a result, the bank typically recovers less than the quoted market value of the collateral.

At close-out (default or liquidation) time $\acute{\tau} = \min(\tau, \tau_l)$, recovery is limited by collateral liquidation proceeds:

$$Rec_{\acute{\tau}} = \min(L_{\acute{\tau}}, (1-h)C_{\acute{\tau}}). \quad (14)$$

Thus, the loss is

$$LGD_{\acute{\tau}} = L_{\acute{\tau}} - Rec_{\acute{\tau}} = \max(L_{\acute{\tau}} - (1-h)\, C_{\acute{\tau}}, 0). \quad (15)$$

Unlike standard unsecured credit, LGD is stochastic and jointly driven by equity and rates.

V. EXPECTED LOSS AND MONTE CARLO ESTIMATION

A. Discounted expected loss

A risk-neutral discounted expected loss (EL) over horizon T can be expressed as

$$EL = E^Q\left[\exp\left(-\int_0^{\acute{\tau}} r_s ds\right) LGD_{\acute{\tau}} 1_{\acute{\tau}<T}\right] \quad (16)$$

B. Discrete-time estimator

On a grid $0 = t_0 < \cdots < t_N = T$, the simulation uses the Cox construction by comparing cumulative hazard $H_{t_j} = \int_0^{t_j} \lambda_s ds$ against $E \sim \mathrm{Exp}(1)$. With path wise discount factor $D(0,t_j) = \exp(-\int_0^{t_j} r_s ds)$, the estimator is

$$\widehat{EL} = \frac{1}{M}\sum_{i=1}^{M} D^{(i)}(0,\acute{\tau}^{i})LGD_{\acute{\tau}^{i}}^{i}\, 1_{\{\acute{\tau}^{i}\leq T\}}(17)$$

By standard Monte Carlo arguments, $\widehat{EL} \rightarrow$ EL almost surely as $M \rightarrow \infty$.

VI. NUMERICAL STUDY ON SIMULATED MARKET-CONSISTENT DATA

This section reports a reproducible simulation study.

A. Inputs and calibration rationale

The chosen parameters reflect stylised, market-consistent magnitudes for an equity-collateralised loan: moderate rate levels, equity volatility typical of single-name risk, conservative margin thresholds, and a non-trivial liquidation haircut. The base intensity λ_0 and sensitivity β are selected such that default risk is low under normal LTV levels but rises sharply under collateral drawdowns.

TABLE I- SIMULATION AND MODEL PARAMETERS.

Quantity	Value	Notes
Horizon T	1.0	years
Paths M	200,000	Monte Carlo
Steps/year	252	daily grid
$r0$	3.0%	initial short rate
a	0.20	mean reversion
σr	1.0%	rate vol
$S0$	100	equity price
σS	30%	equity vol
Loan L_0	60	notional
LTV^*	0.70	reference level
LTV_{maint}	0.75	maintenance
LTV_{liq}	0.85	liquidation
Haircut h	10%	liquidation discount
$\lambda 0$	2.0%	base intensity
β	25	LTV sensitivity
$\rho S\lambda$	-0.4	WWR correlation

TABLE II- KEY OUTPUTS (WWR VS NO-WWR).

Quantity	Value	Notes
Expected Loss (EL)	0.0000	discounted, per loan
Default Probability (PD)	0.0301	over horizon
Avg. λ_T	40963767749.2401	end intensity
Avg. LTV_T	0.6566	end LTV
EL (no WWR)	0.0000	$\rho_{S\lambda} = 0$
PD (no WWR)	0.0301	$\rho_{S\lambda} = 0$

B. Key outputs and wrong-way risk comparison

We report discounted expected loss (EL) and default probability (PD) over the horizon. To highlight the impact of marketcredit co-movement, we compare a wrong-way risk (WWR) setting with negative equity-credit correlation to a no-WWR setting with $\rho_{S\lambda} = 0$. Note that even the no-WWR case retains structural coupling via (11).

C. Figures

Fig. 1 shows representative equity collateral paths. Fig. 2 shows the induced LTV dynamics and the relation to maintenance and liquidation thresholds.

VII. DISCUSSION: PRACTICAL RISK MANAGEMENT IMPLICATIONS

The model links bank operational mechanics (margining and liquidation) to a rigorous stochastic structure. Key implications include:

- Margin policy and haircuts: tighter LTV_{maint} and larger haircuts reduce tail losses but may increase forced liquidation frequency and client friction.
- Concentration risk: single-name collateral increases volatility and jump-to-liquidity risk; portfolio collateral reduces LTV variance.

Sample equity collateral paths (risk-neutral)

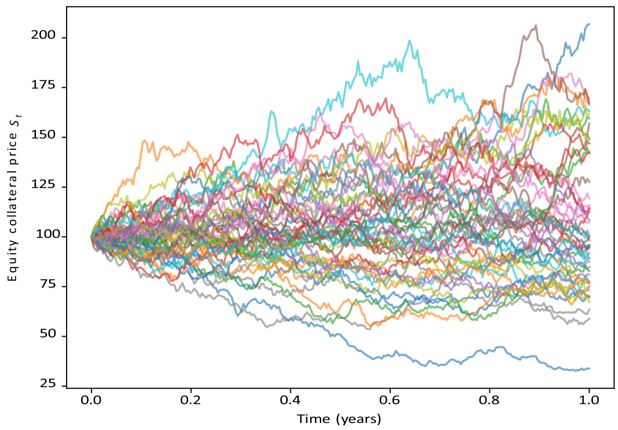

Fig. 1. Sample risk-neutral equity collateral paths S_t under the unified simulation.

Sample LTV paths with margin thresholds

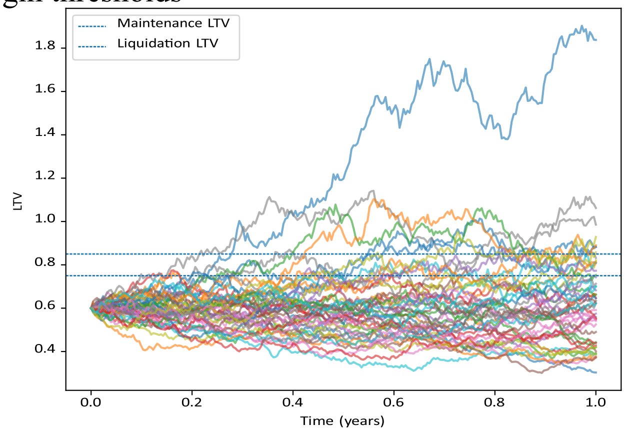

Fig. 2. Sample LTV paths with maintenance and liquidation thresholds.

- Stress testing: structural wrong-way risk implies that equity stress scenarios amplify both PD and LGD, requiring joint simulation rather than independent shocks.

VIII. CONCLUSION

We developed a unified risk-neutral stochastic simulation framework for equity-collateralised lending that captures business-relevant mechanics of SBL: collateral marking, margin thresholds, liquidation, structural wrong-way risk, and endogenous recovery. The accompanying numerical study demonstrates how LTV-driven intensity and equity-credit dependence materially influence expected loss and trigger frequencies. The framework provides a consistent foundation for pricing, expected loss, and stress testing of Lombard lending portfolios.

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