# Multifractal Analysis of Pre-Seismic VLF Signal Anomalies

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**Abstract:**
This study investigates multifractal characteristics of sub-ionospheric Very Low Frequency (VLF) signals recorded from the Bafa transmitter (Turkey) for five earthquakes (2011–2013). The Wavelet Transform Modulus Maxima (WTMM) method is used to analyze nonlinear variability before and after seismic events. Significant deviations in scaling behavior and widening of the singularity spectrum were observed prior to earthquakes, indicating ionospheric perturbations associated with seismic activity.

**Keywords:** VLF signals, Earthquake precursors, Multifractal analysis, WTMM, Ionospheric perturbations.

## 1. INTRODUCTION

Electromagnetic anomalies in the ionosphere have been widely studied as potential earthquake precursors. VLF signals are particularly sensitive to ionospheric disturbances and can provide useful information about lithosphere–ionosphere coupling processes. Multifractal analysis is an effective technique to characterize nonlinear and complex signals across multiple scales.

## 2. MULTIFRACTAL ANALYSIS

Multifractal analysis provides quantitative interpretation of various characteristics of VLF signal during earthquake. In this work WTMM based multifractal analysis method is used in order to obtain the $\tau(q) vs\ q$ curve and singularity spectrum for the VLF signal. The irregular behavior of VLF signal is often represented by the singularities of the series. The strength of singularities is often characterized by Hölder exponent. The Hölder exponent $\alpha(t_0)$ of VLF signal $f(t)$ at $t_0$ is defined as the largest exponent such that there exits a polynomial $P_n(t)$ of order $n$ that satisfies

$$|f(t) - P_n(t - t_0)| \leq C|t - t_0|^{\alpha} \qquad \dots\dots\dots\dots(1)$$

For $t$ in the neighborhood of $t_0$. The Hölder exponent is a function defined for each point of $f(t)$, and it describes the local regularity of the function (or distribution). This leads naturally to a generalization of the $D(H)$ spectrum (Muzy et al., 1998). Hereafter we will denote $D(H)$ the Housdroff dimension of the set where the Hőlder exponent is equal to $H$

$$D(H) = dim_h\{t|\ H(t) = H\} \qquad \dots\dots\dots\dots(2)$$

Where $H$ is no longer restricted to [0, 1], but a priori can take on positive as well as negative values (Bacry et al., 1993).

If the used wavelet function satisfies the admissibility condition, the wavelet transform can mirrored the local characteristics of $f(t)$ at a point $t_0$. More precisely, we have the following power law relation (Enescu et al., 2006):

$$Wf(t_0, s) \approx |s|^{\alpha(t_0)} \qquad \dots\dots\dots\dots(3)$$

Where $\alpha(t_0)$ is the Hölder exponent (or singularity strength). Equation (3) tell that, when investigating the local scaling behavior of the computed wavelet coefficients with an analyzing wavelet, whose first *N* moments vanish, one can generally detect all the Hölder exponents of $f(t)$ that are smaller than *N*. The scaling parameter (the so-called *Hurst exponent*) estimated when analyzing time series by using "monofractal" techniques is a global measure of self-similarity in a time series, while the singularity strength $H$ can be considered a local version (i.e. it describes "local similarities") of the *Hurst exponent*.

To characterize the singular behavior of functions, it is sufficient to consider the values and position of the WTMM. The WTMM representation is used to define the multifractal formulism (Muzy et al., 1993; Arnedo et al., 1995) at fixed scales. If the position of all local maxima at a fixed scale $s$ is denoted by $\{t_n(s)\}$, where $n$ is an integer. By summing up the $q$ power of all the WTMM, we obtained the partition function $Z(s,q)$:

$$Z(s,q) = \sum_n |Wf(t_n(s),s)|^q \approx s^{\tau(q)} \quad \dots\dots\dots\dots\dots\dots\dots..(4)$$

By varying $q$ in Equation (4), it is possible to characterize selectively the fluctuations of time series: positive $q$'s accentuate the "strong" in homogeneities of the signal, while negative $q$'s accentuate the "smoothest" ones. In this work, we have employed a slightly different formula to compute the partition function $Z$ by using the "supremum method", which prevents divergences from appearing in the calculation of $Z(s,q)$, for $q < 0$ (e.g. Arneodo et al., 1995).

Often scaling behavior is observed for $Z(s,q)$ and the spectrum $\tau(q)$, which describes how $Z$ scales with $s$ can be defined:

$$Z(s,q) = \approx s^{\tau(q)} \quad \dots\dots\dots\dots\dots\dots\dots..(5)$$

If the exponents of $\tau(q)$ define a straight line, the analyzed signal has a monofractal structure. However, if the signal under study is multifractal, then different parts of the signal are characterized by different values of $\alpha$ (Oswiecimka et al., 2006). The negative of the Legendre transformation $\tau(q)$ can be interpreted as singularity spectrum, so that $f(\alpha) = -(\tau(q) - q\alpha)$ (Vicsek, 1993). Thus, from the renormalization theory in statistical mechanics, interpreting the use of $\alpha$ as an energy cascade, the legendry transform $f(\alpha)$ is thus the analogue of the informational entropy (Stanley and Meakin, 1988).

Figure 1: An Example of f(α) − α curve for any VLF signal observed

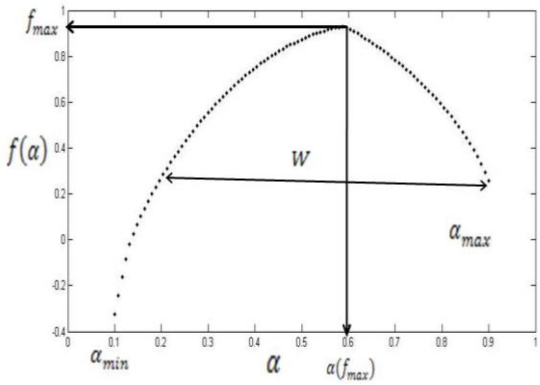



Figure 1 is an example of $f(\alpha) - \alpha$ curve obtained for any VLF signal observed. The width of $f(\alpha)$ taken at $f = 0$ characterizes the strength of the multifractality. In order to characterize and to compare the strength of the multifractality for VLF signal, we use many parameters of $f(\alpha)$ spectrum like $\alpha_{min}$ ,

$\alpha_{max}$ ,($\boldsymbol{w} \equiv \alpha_{max} - \alpha_{min}$ ), $f_{max}$ and $\alpha(f_{max})$. It is interpreted as follows: $f(\alpha)$ spectrum looks like an upside-dwon parabola (Gaussian shape) in which peaks occur at $\alpha(f_{max})$ and having the spread form $\alpha_{min}$ to $\alpha_{max}$ and $\boldsymbol{w}(\alpha_{min} - \alpha_{max})$ quantifies the non-uniformaity of the fractal. Which clearly confirms the non-uniqueness of the Hölder exponent $\alpha$ and thus the multifractality of the process.

## 3. EARTHQUAKE SELECTION AND DATA

In this work VLF signal transmitted form famous Bafa transmitter ($37.240\,N, 27.190\,E$) located at Turkey have been analyzed, which was recorded at Sudden Ionospheric Monitoring (SID) Station and available online at website: http://sidstation.loudet.org. In order to identify the effect of geomagnetic storm on the transmitted VLF signals we may consider the $D_{st}$ index of corresponding earthquakes which are obtained form World Data Center, Kyoto, Japan through website: http:// swdcdb.kugi.koyotee.ac.jp.

## 4. RESULTS AND DISCUSSION

To study the multifractal variability of VLF signal during Earthquake, we have taken five earthquake having magnitude greater than 5. In this analysis VLF signal before 10 days and 4 days after the main shock have been analyzed to study the pre-seismic and post effect. During the period of analysis geo-magnetic condition are very quiet ($D_{st} < 30$). The detail of Earthquake taken under consideration is given in Table 1

Table 1- Details of Earthquake taken under study

| ***SN.*** | ***Date (yyyy/mm/dd)*** | ***Time (UTC)*** | ***Mag.*** | ***Depth (km)*** | ***Epicenter*** | ***Location*** |
|---|---|---|---|---|---|---|
| 1. | 2011/11/09 | 19:23 | 5.7 | 4.8 | Edremit South of Van City (39.35° N, 27.01° E) | Van city, eastern Turkey (38.49° N, 43.38° E) |
| 2. | 2012/04/16 | 11:23 | 5.6 | 10.5 | At sea +50 km form Kalamata (37.03° N, 22.11° E) | Southern Greece (36.77° N, 21.69° E) |
| 3. | 2012/06/10 | 12:44 | 6 | 30 | 33 km South West Fethiye (36.65° N, 29.12° E) | Dodecanese Islands, Greece (36.37° N, 28.95° E) |
| 4. | 2012/09/22 | 03:52 | 5.6 | 9.9 | 4 km from Kiato (38.07° N, 22.44° E) | *Corinthia*, Greece (37.93° N, 22.93° E) |
| 5. | 2013/01/08 | 14:16 | 5.7 | 9.8 | 195 km North West Izmir (38.26°N, 27.09°E) | Aegean Sea, Turkey (39.66° N, 24.54° E) |

Case I: Van City, Turkey Earthquake (November 9, 2011)

Significant deviation in scaling curves was observed one day before the earthquake, indicating increased nonlinearity. The singularity spectrum showed a wider Gaussian shape two days before the event, confirming enhanced multifractality.

Figure 2 τ(q): curves for VLF signal during November 9, 2011 earthquake

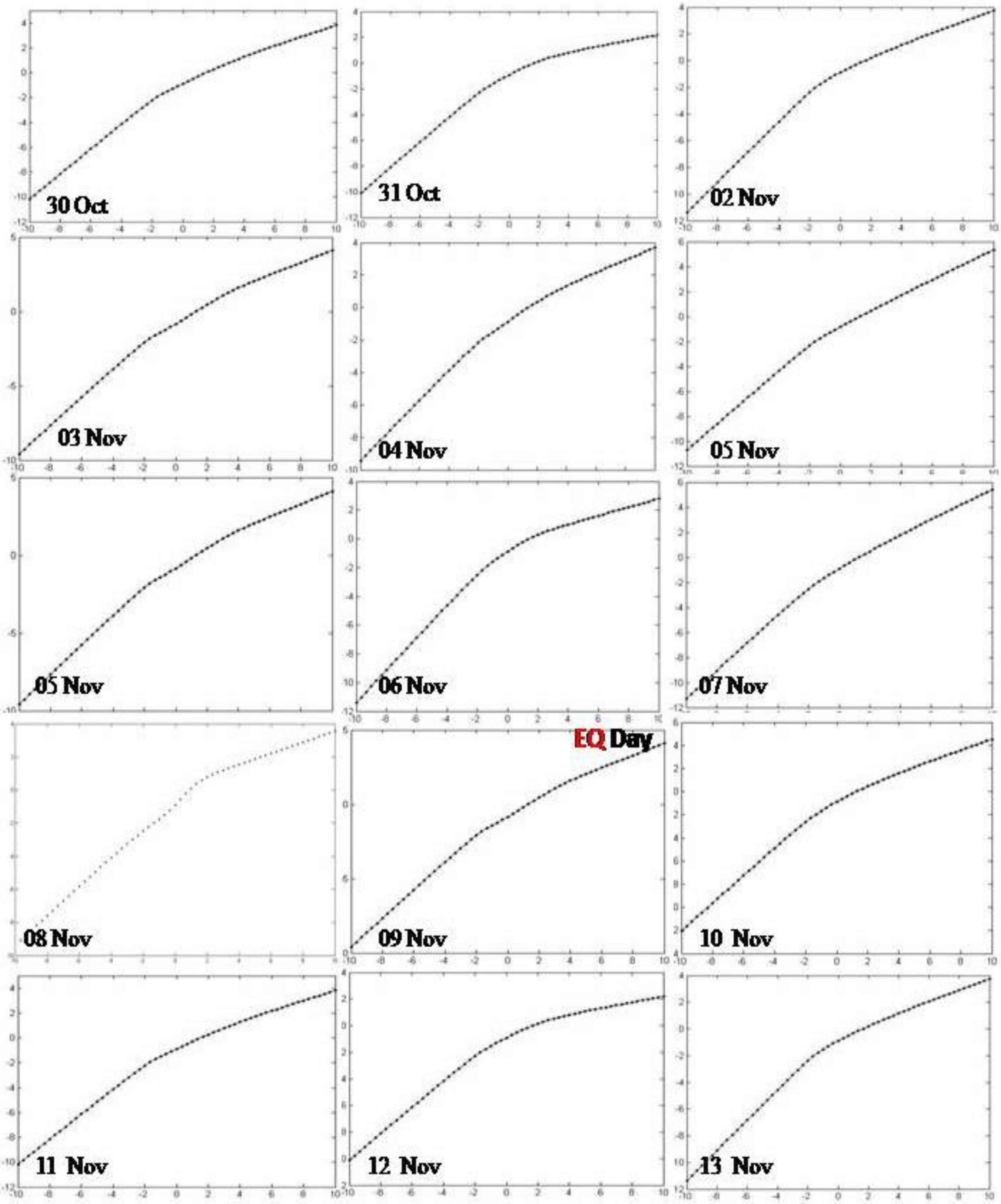
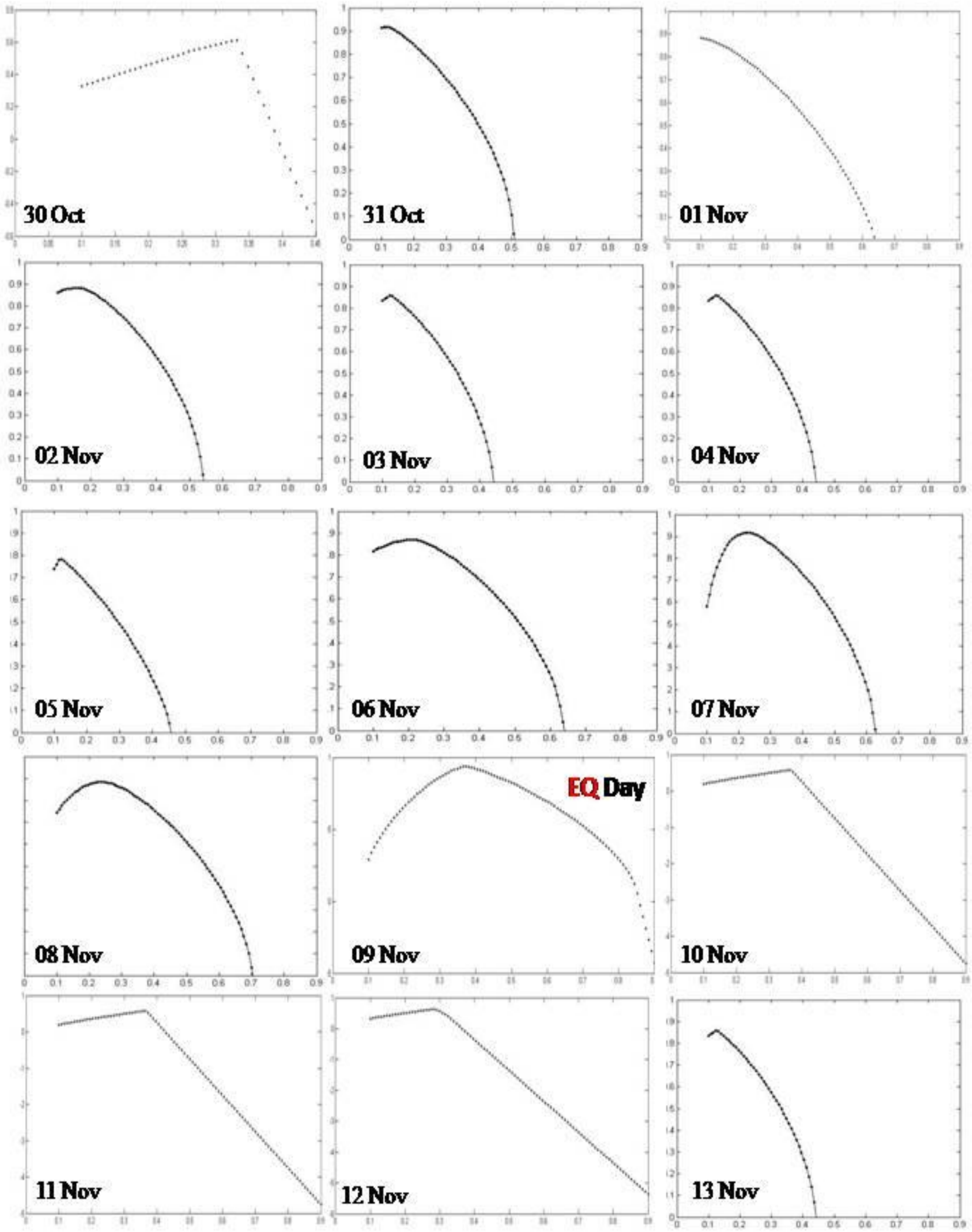


Figure 3 τ(q): Singularity spectrum for VLF signal during November 9, 2011 earthquake




Case II: Southern Greece Earthquake (April 16, 2012)
The multifractal parameters showed strong deviation two days prior to the earthquake. The singularity spectrum widened and flattening regions appeared near the main shock, suggesting increased intermittency.

Figure 4 τ(q): curves for VLF signal during April 16, 2012 earthquake

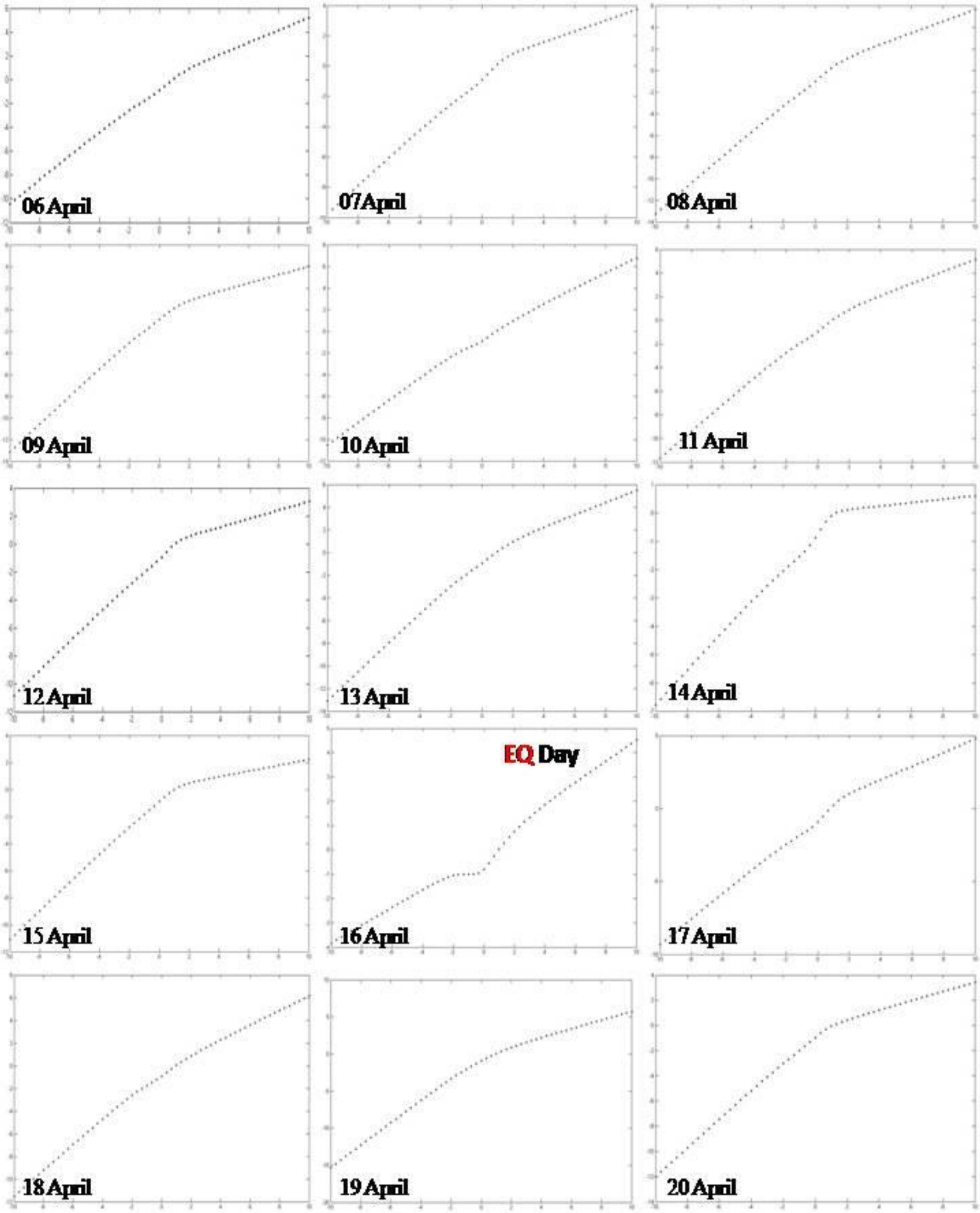
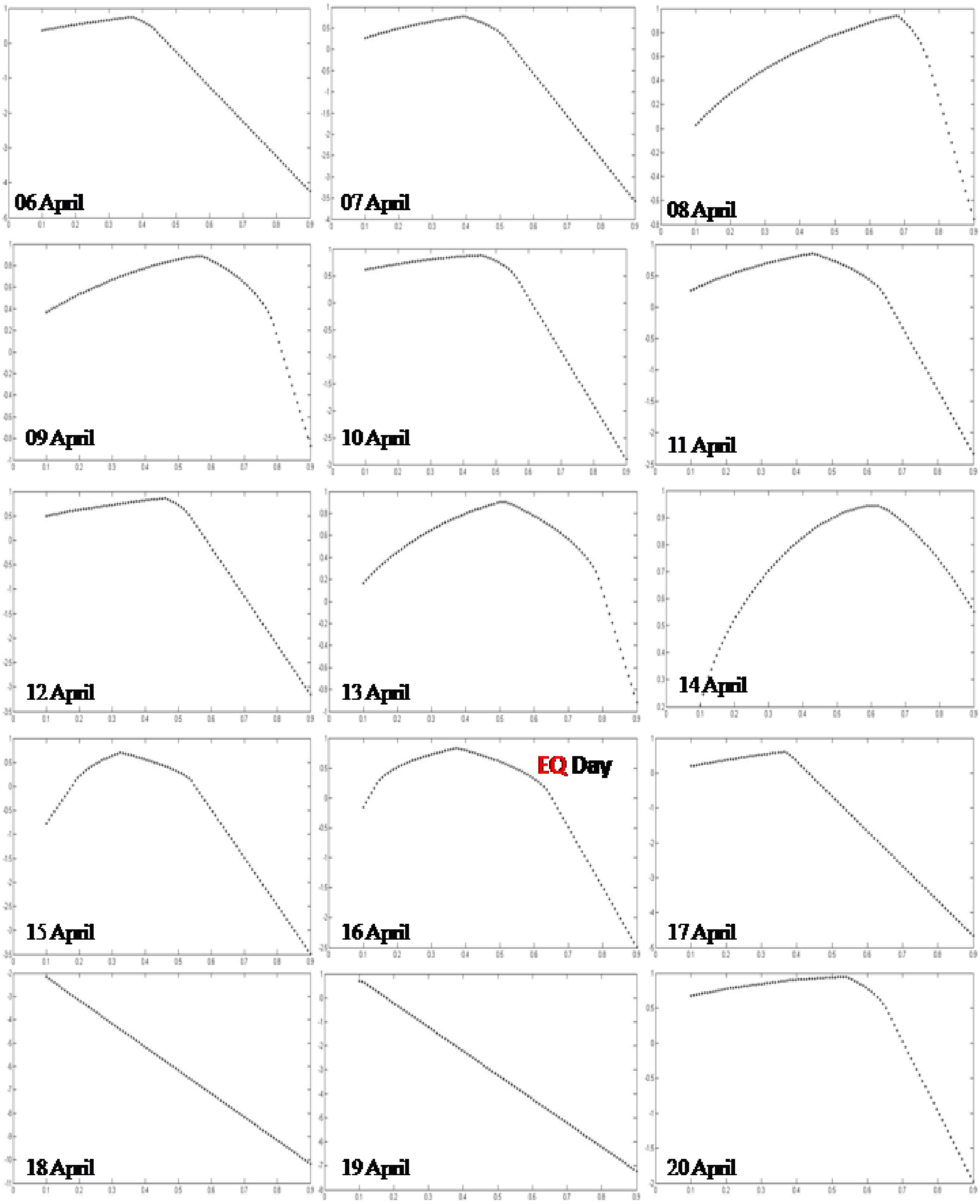


Figure 5: Singularity spectrum for VLF signal during April 16, 2012 earthquake




Case III: Dodecanese Islands, Greece Earthquake (June 10, 2012)

Scaling curves became highly nonlinear approaching the earthquake day. The singularity spectrum shifted toward lower Hölder exponents and attained greater width, indicating strong multifractal behavior.

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Figure 6: $\tau(q)$ curves for VLF signal during June 10, 2012 earthquake

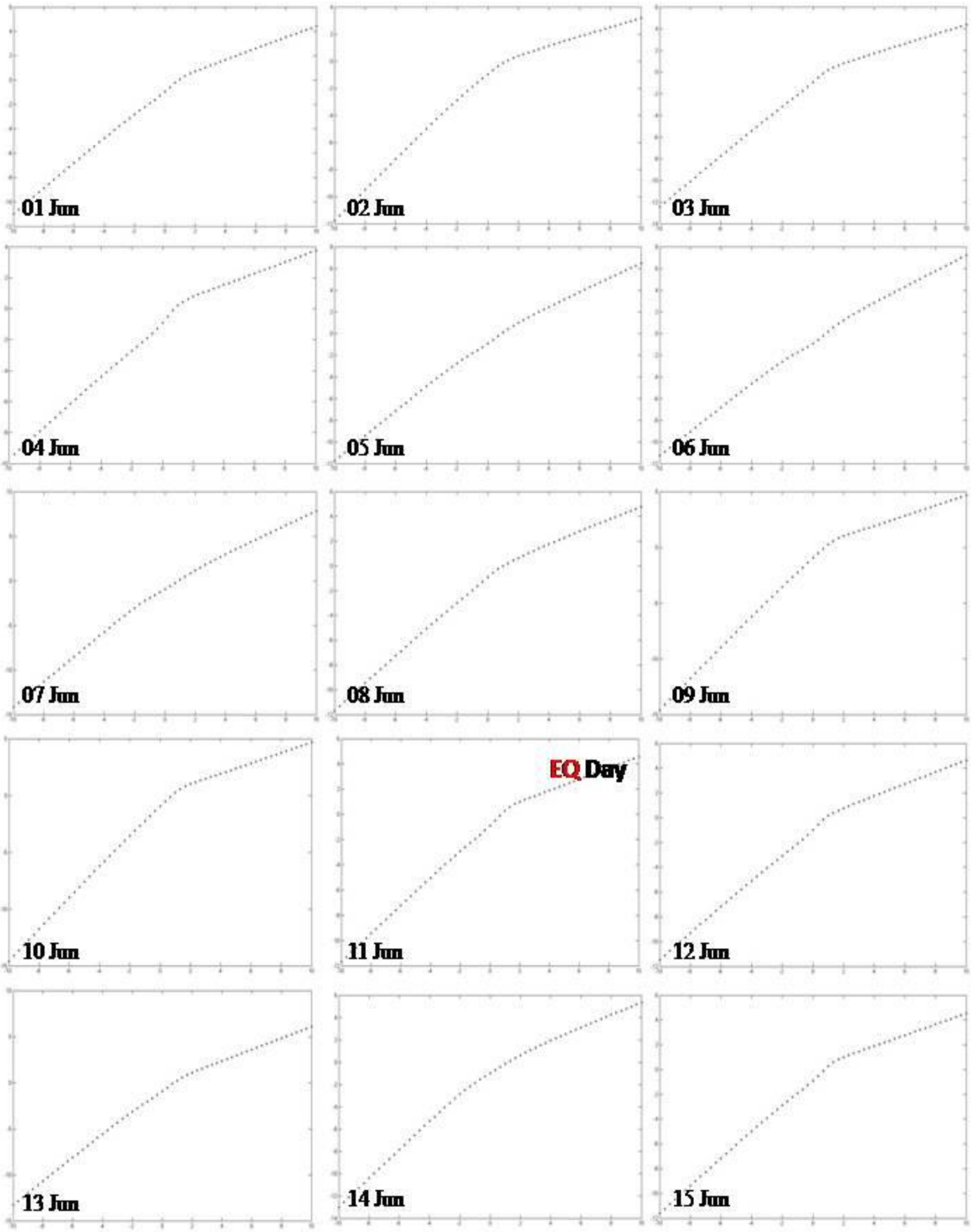
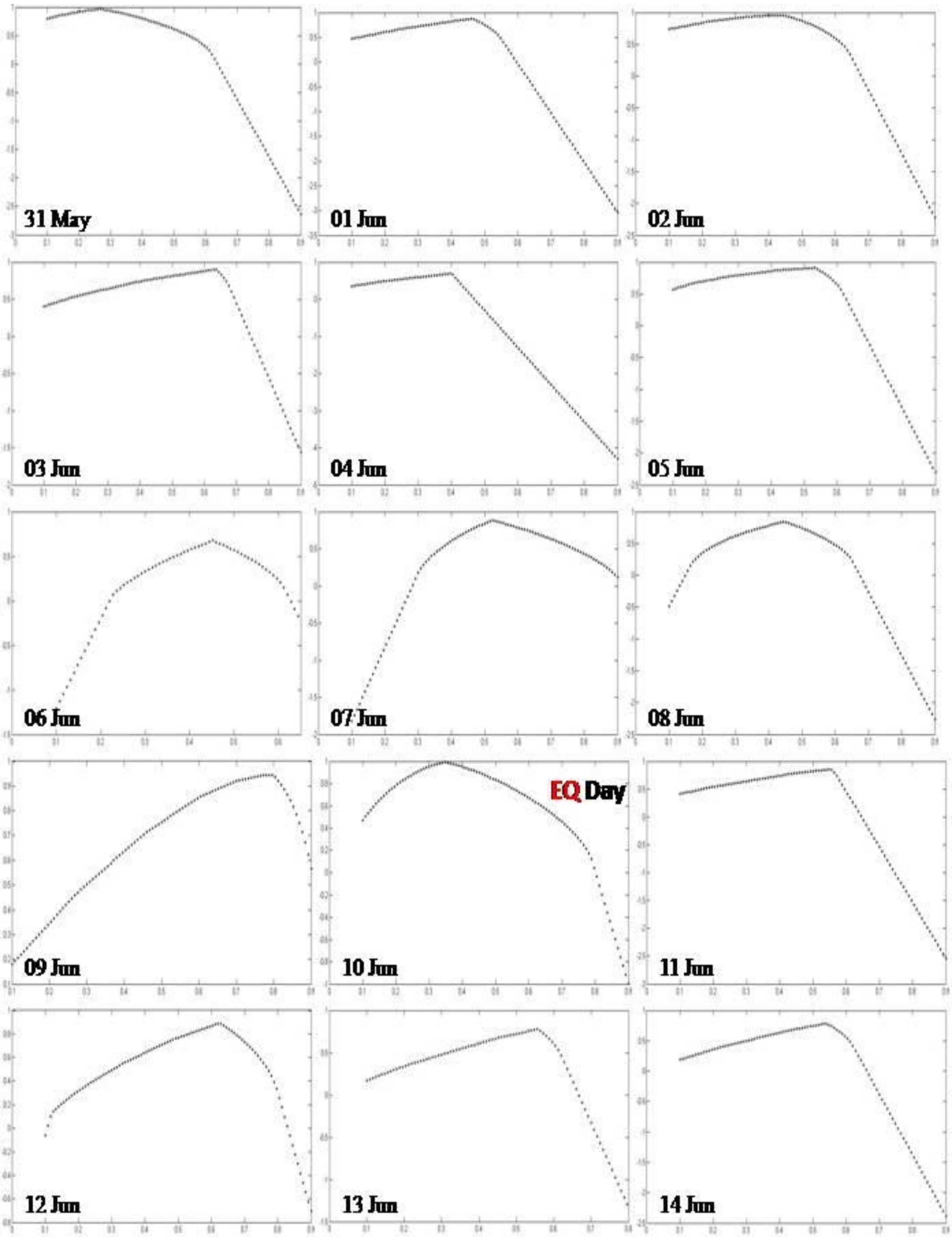


Figure 7: Singularity spectrum for VLF signal during June 10, 2012 earthquake




Case IV: Kiato Corinthia, Greece Earthquake (September 22, 2012)
Large deviations in scaling behavior were observed before the main shock. The singularity spectrum achieved maximum width one day prior, confirming enhanced multifractality and ionospheric disturbance.

Figure 8 τ(q): curves for VLF signal during September 22, 2012 earthquake

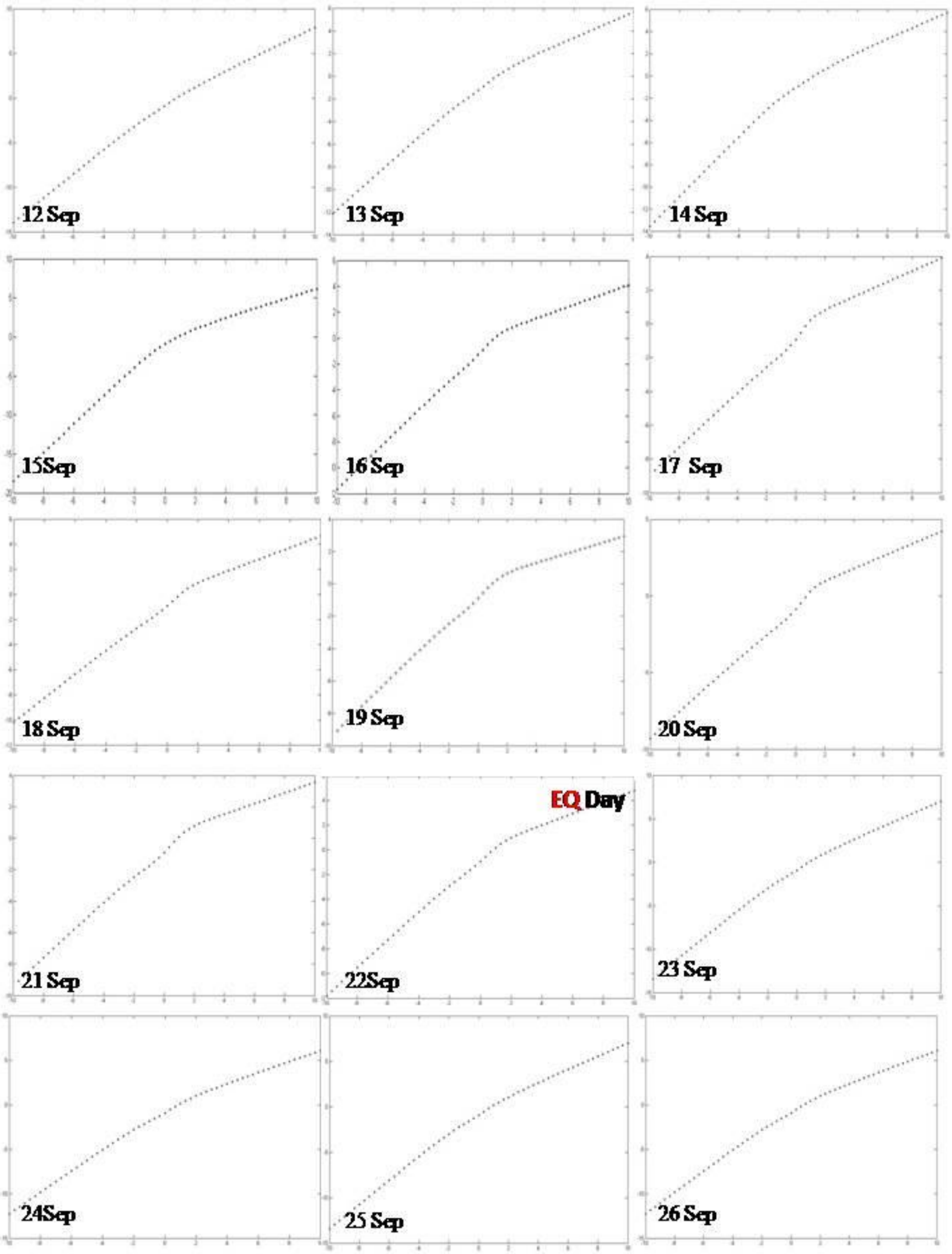
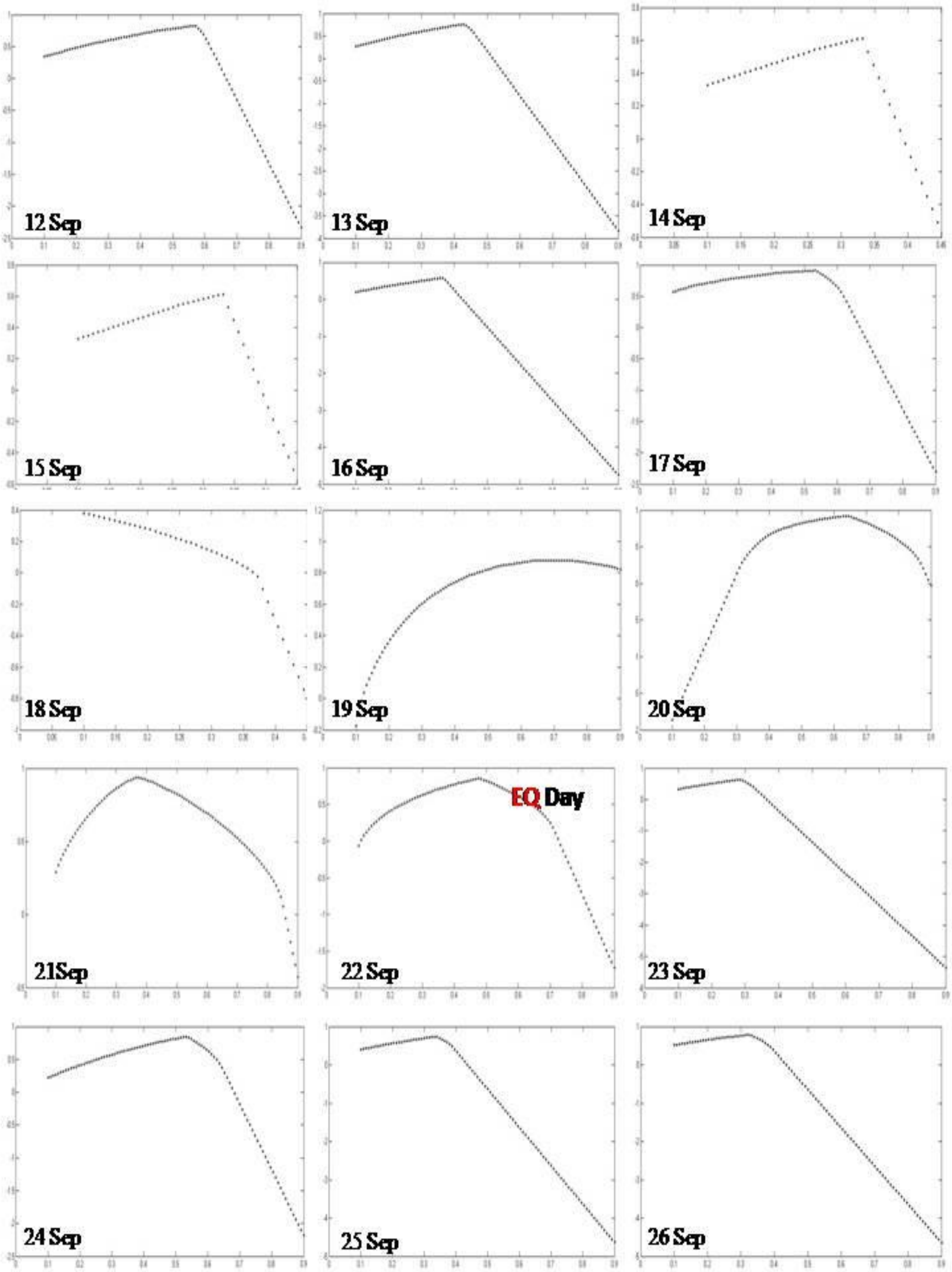


Figure 9: Singularity spectrum for VLF signal during September 22, 2012 earthquake




Case V: Aegean Sea, Turkey Earthquake (January 8, 2013)
Significant nonlinear deviations were observed one day before the earthquake. The singularity spectrum broadened and shifted toward lower values, indicating strong ionospheric perturbations.

Figure 10: τ(q) curves for VLF signal during January 8, 2013 earthquake

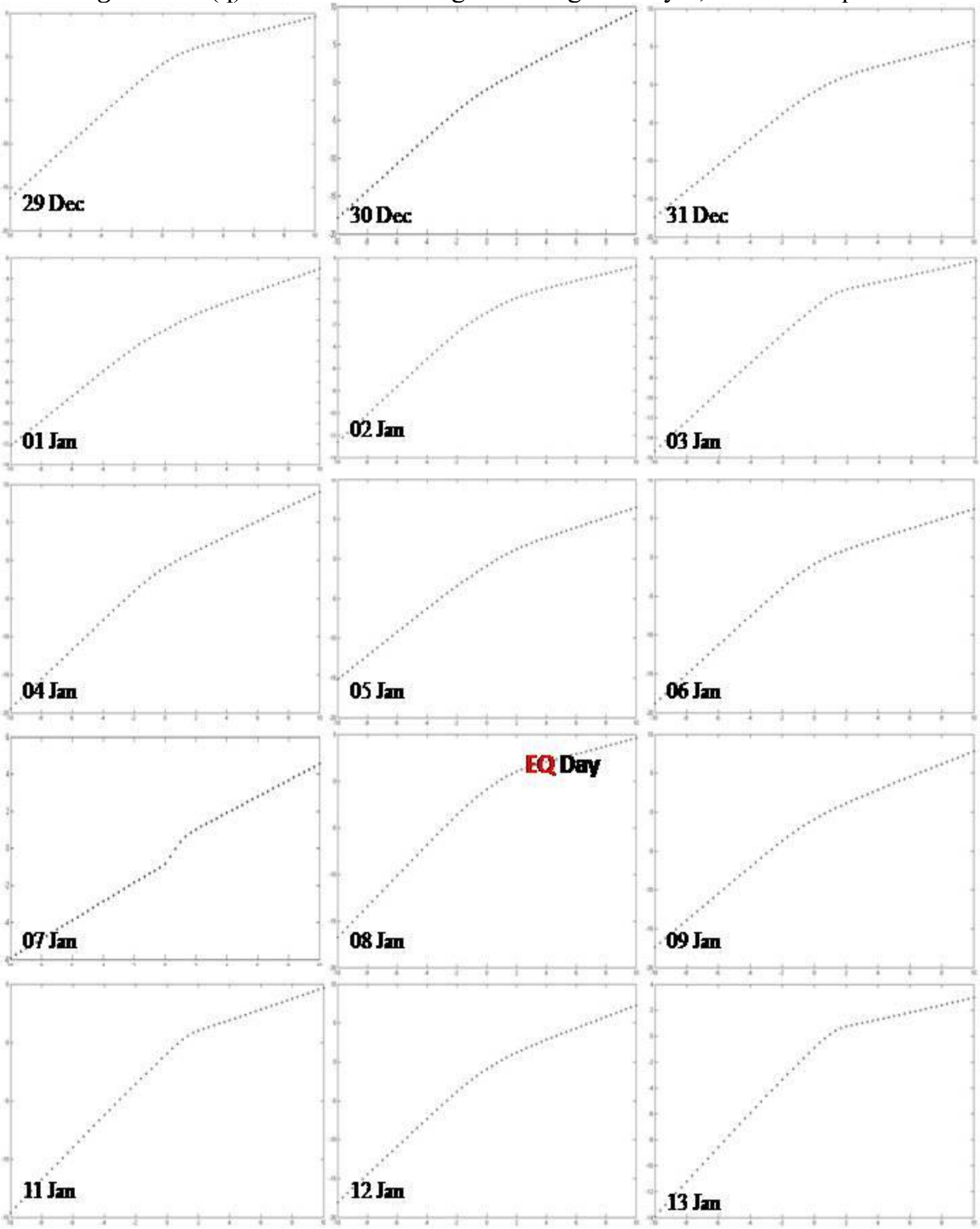
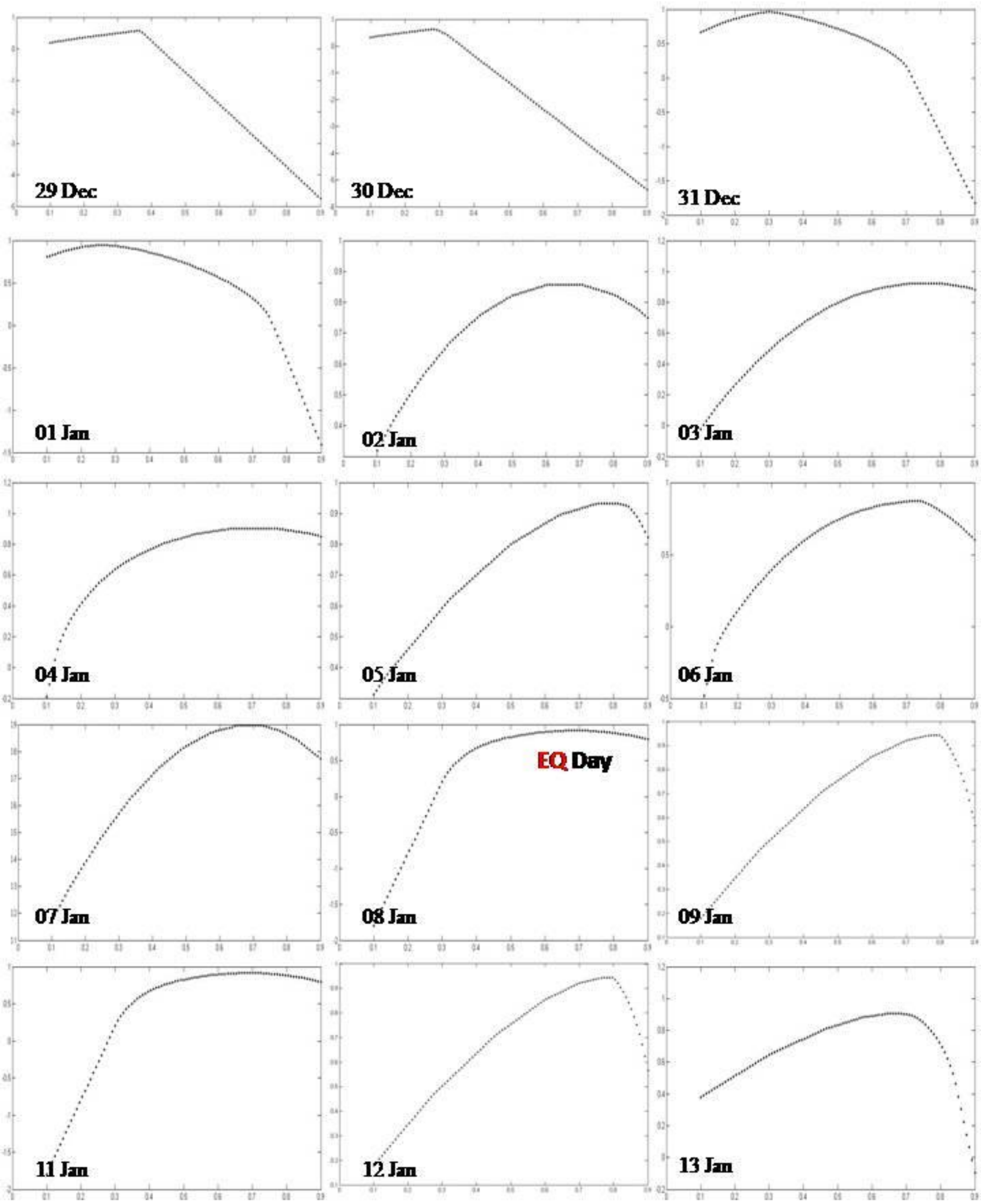


Figure 11: Singularity spectrum for VLF signal during January 8, 2013 earthquake




## 5. CONCLUSION

The multifractal analysis of VLF signals revealed significant anomalies prior to earthquakes, including deviation in scaling behavior and widening of the singularity spectrum. These results support the hypothesis that electromagnetic emissions generated during earthquake preparation processes influence ionospheric conditions and can be detected using multifractal parameters.

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