Vertex Antimagic Edge Slither Labeling of Sierpinski Gasket Graph

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Abstract:
In this modern era, mathematics and technology go hand-in-hand paving way for better lifestyle. To comprehend many situations, it requires special tool and applications by which those situations can become manageable. One such mathematical tool is the concept of graphs whose applications spread far and wide. Graphs are not complex structures that are hard to fathom. Neither they are just dots and lines nor intricate edifices. but they represent multifarious situations into simple structures of dots and lines. Of all topics in graph theory, the one that stood out and provided the much needed impetus is the concept of vertex antimagic edge labeling. The concept of vertex antimagic edge slither labeling is a special case of vertex antimagic edge labeling. In this paper, the admittance of vertex antimagic edge slither labeling to the Sierpinski gasket graph is established.

Keywords: Antimagic Labeling, Sierpinski Gasket Graph, Slither Labeling

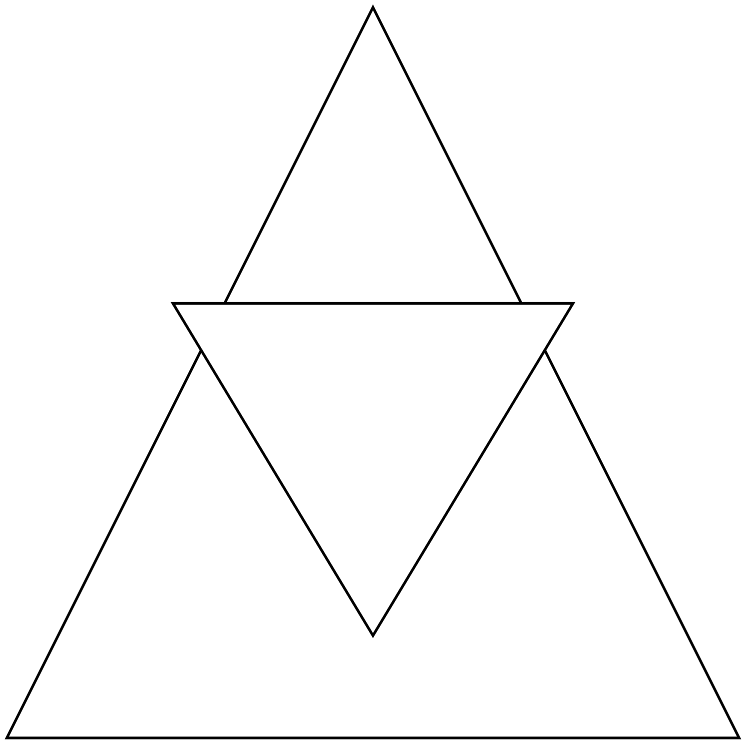
Introduction:
Graphs are not just the combination of dots and lines, but they are the depiction of many real life situations that are beyond human intellect. The concept of graph originated from a real life problem, the Konigsberg bridge problem that was solved by the renowned Mathematician Leonhard Euler in the year 1736 who have the solution to the problem in negative. From its origin till this day, graph theory has been used in various applications at different levels for easy understanding. From medieval time to modern era, graph theory has made a greater impact in helping to understand the near imaginative things. With much more in the offing, the concept of graphs has helped the technology driven humanity to reach greater heights.

Antimagic Labeling:
Labeling in graph theory is defined as the assignment of integers to the vertices and edges. There are different types of labeling defined in graph theory, but the one that inspired the most is the concept of vertex antimagic edge labeling. The concept of magic labeling in graphs was first introduced by J. Sedlacek [8]. J.A. Macdougall, M. Miller, Slamin and Wallis [5] prolonged this concept of magic labeling to vertex magic labeling. Through this graph researcher discovered the notion of antimagic labeling. This was first done by the famous graph theorists duo of N. Hartsfield and J. Ringel [3]. They were the first researchers to prove the admittance of antimagic labeling to Cycles C_n, Paths P_n, Wheels W_n and the complete graphs K_n. The perception of (a, d) antimagic labeling was defined the famous researchers R. Bodendiek and G. Walther [1]. The classical definition of (a, d) antimagic labeling given by Martin Baca and Mirka Miller [4] is as follows; a graph G is said to be an antimagic edge labelled graph if there exists a positive integer a and a non-negative integer d and a bijection $f_1 : E \to \{1, 2, \ldots, |E(G)|\}$ such that the induced

mapping given by g: $V(G) \rightarrow W$, where $W = \{a, a+d, a+2d, \ldots, a+|V-1|d\}$ is also a bijection. These results were effectively composed and accumulated by J.A. Gallian [2].

Sierpinski Gasket Graph:

The Sierpinski gasket graph S_n is a special type of tessellation formed from an equilateral triangle. This graph is formed by recursively joining the midpoints of an equilateral triangle. The number of vertices in the graph Sn is given by V(G) = $3^{n+1} + 3$ and the number of edges is E(G) = $\frac{3^{n+1}+3}{2}$. One of the most important applications of the Sierpinski gasket graph is in their recursive structure that is well used in communication networks.



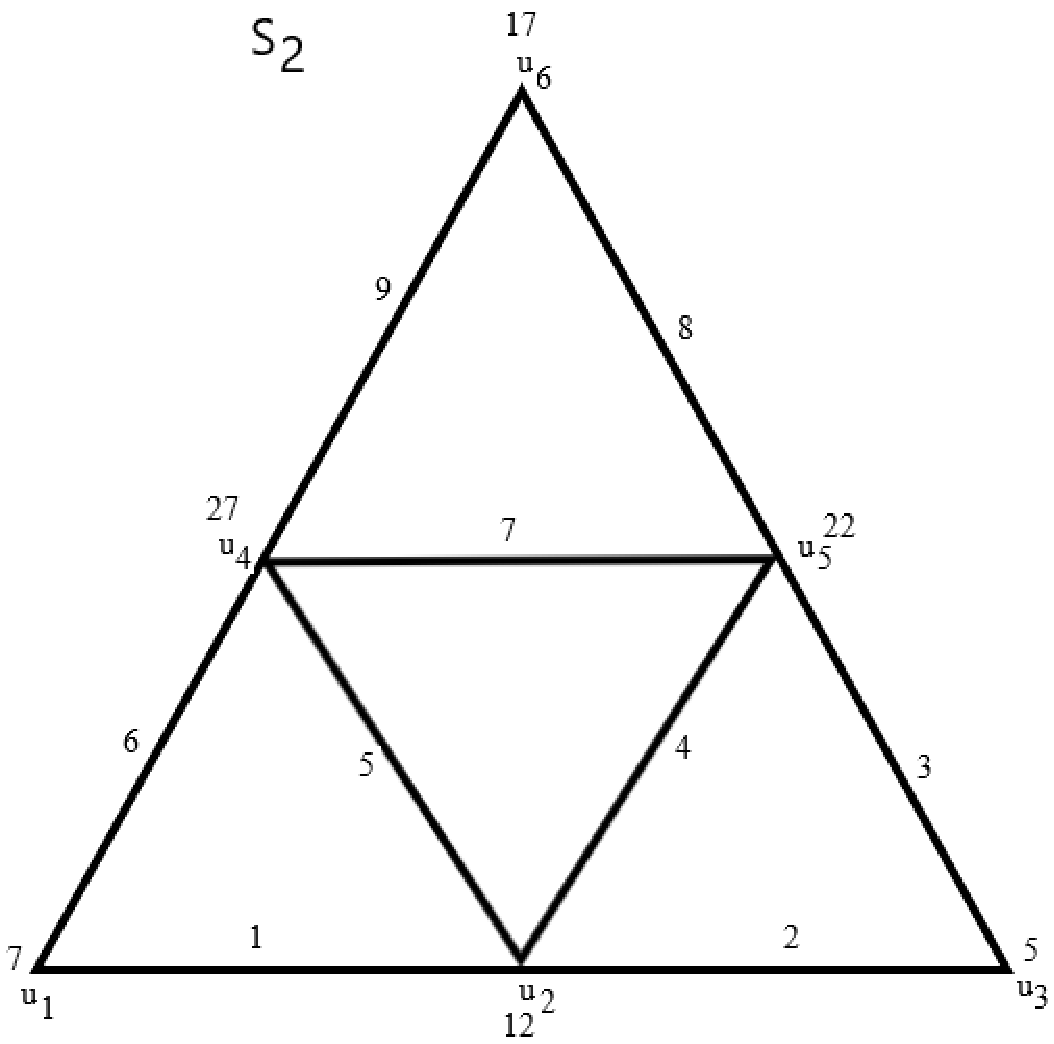
Sierpinski Gasket Graph

Slither Labeling Pattern: The slither labeling pattern as stated above is a special case of antimagic edge labeling. The method labeling the edges from right to left and left to right is referred as the slither labeling as the pattern of labeling resembles the slither movement of snakes. The edges are being labelled with positive integers such that none of the labels are recurrent. The edges of the Sierpinski graph is labelled using the slither pattern to prove that the graph admits vertex antimagic edge slither labeling.

Main Result:

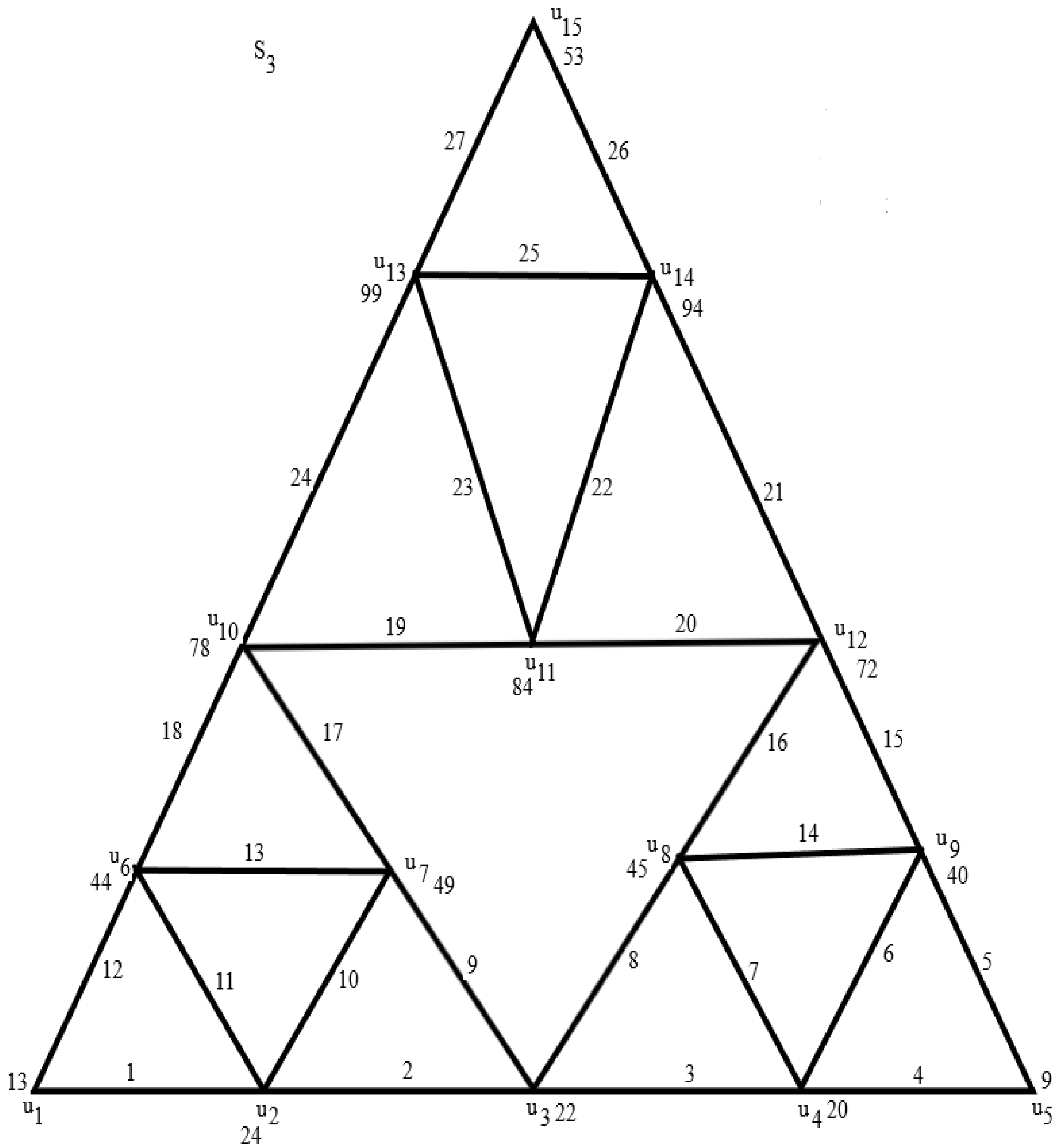
Theorem: The Sierpinski gasket graph S_n admits vertex antimagic edge slither labeling for $n \geq 2$.

Proof: Let S_n be the Sierpinski gasket formed on n vertices, $n \geq 2$. The graph S_1 is a 2-regular graph for which the theorem has already been proved.



Sierpinski Gasket graph S_2

Consider the Sierpinski gasket graph S_2. The edges of the graph were labelled with positive integers following the slither pattern. The labels of the edges are labelled in such a way that the labeling are all unique. The label of a vertex is the sum of labels of the edges that are incident with that vertex. The labels of the vertices of S_2 are as follows; $u_1 = (1 + 6) = 7$, $u_2 = (1 + 5 + 4 + 2) = 12$, $u_3 = 2 + 3 = 5$, $u_4 = (6 + 9 + 7 + 5) = 27$, $u_5 = (8 + 7 + 3 + 4) = 22$, $u_6 = (9 + 8) = 17$. From the above graph, it is evident that the labels of the vertices are all distinct and hence the graph admits vertex antimagic edge slither labeling. Hence the graph admits vertex antimagic edge slither labeling.



Sierpinski Gasket graph S_3

Consider the Sierpinski gasket graph S_3. The number of vertices of the graph is 15 and the number of edges is 27. As like the previous graph, the edges of the graph were labelled with positive integers following the slither pattern. From the above graph, it is evident that the labels of the vertices are all distinct and hence the graph S_3 admits vertex antimagic edge slither labeling. In a similar way, the graph S_n, $n \geq 4$ can be labelled using slither pattern the graphs admits vertex antimagic edge slither labeling.

Conclusion:

This paper deals with a special type of vertex antimagic edge labeling called the vertex antimagic edge slither labeling. The graph that is considered here is the Sierpinski Gasket graph. The edges of the graph are labelled with positive integers following the Slither pattern and the graph is proved to admit vertex antimagic edge slither labeling. In a similar way different graphs can be considered and the concept of vertex antimagic edge slither labeling can be applied.

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