

# Evaluation of Reliability and Mean time to Failure of Sunflower oil Extraction Plant

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## Abstract:

This study presents a comprehensive reliability analysis and mean time to failure (MTTF) evaluation of complex sunflower oil extraction plant, integrating mechanical expelling, solvent extraction, and refining subsystems. The Boolean function technique models system behaviour under exponential and Weibull failure distributions, enabling precise probabilistic reliability trends and MTTF value sensitive to failure rates, highlighting critical components.

**Keywords:** Reliability, Mean time to Failure, Sunflower oil extraction, Time, Boolean function technique.

## 1. INTRODUCTION

The Boolean function technique is a systematic and mathematical approach used to evaluate the reliability and mean time to failure (MTTF) of complex engineering systems. It is especially useful for systems composed of multiple components arranged in series, parallel, or mixed configurations.

### Evaluation of system Reliability:

Using probability theory, the system reliability  $R(t)$  is obtained by replacing Boolean variables with component reliabilities and evaluating the Boolean expression.

### Evaluation of Mean Time to Failure (MTTF):

The MTTF is a key reliability index that represents the expected time until system failure. Once the system reliability function  $R(t)$  is derived using the Boolean function technique, MTTF is calculated as:

$$MTTF = \int_0^{\infty} R(t) dt$$

### Advantages of Boolean Function Technique:

The following are the advantages of Boolean function technique:

1. Provides clear logical structure of system operation.
2. Suitable for complex systems with dependent components
3. Easily integrates with probabilistic reliability models.
4. Helps identify critical components affecting system performance.

**About sunflower oil extraction plant:**

The process of extraction of sunflower oil from sunflower seeds starts with the cleaning of seeds and the separation of impurities and then diverges into two main extraction methods:

**Mechanical Expelling:**

Clean sunflower seeds undergo dehulling to separate the kernel from the hull. The kernels are then pressed (expelled) to produce oil and a resulting cake. The crude oil is filtered to become “Expelled Sunflower Oil”.

**Solvent Extraction:**

The Cake from the expelling process, or sometimes the deoiled cake from another process, undergoes solvent extraction to recover remaining oil. This results in “solvent Extracted Oil” and “Deoiled cake”.

**Refining Process:**

Both the expelled and solvent extracted oils undergo a series of refining step to achieve a high-quality “Expelled Refined Sunflower oil”:

**Degumming/ Neutralization/Washing:**

This step removes gums and free fatty acids.

**Bleaching:**

The oil is treated to remove colour pigments and impurities.

**Dewaxing:**

Waxes are removed to prevent the oil from becoming cloudy at lower temperatures.

**Deodorization:**

The final step uses steam distillation to remove volatile compounds responsible for unwanted odours and flavours.

## 2. LITERATURE REVIEW

In 1983, Fratta, Schneeweiss & Fuji [1] model system success and failure state using Boolean variable and derive system reliability expressions by algebraic manipulation. The approach simplifies reliability evaluation of complex systems compared to conventional probability-based methods.

In 1986, Gupta et al.,[2] analyses a four-generator power plant using the Boolean function technique. Reliability and MTTF are derived by representing system success through Boolean functions. The study demonstrates that Boolean modelling provides exact reliability expressions without using minimal cut or path sets.

In 1986, Aggarwal et al.,[3] evaluated the reliability behaviour of power plant under arbitrary failure time distributions using this technique. Unlike exponential assumptions, the model allows flexible life time distributions, showing that Boolean methods are adaptable and robust for realistic reliability analysis.

In 1991, Gupta & Singhal [4] apply this technique to evaluate the reliability and mean time to failure (MTTF) of a power plant. The system structure is expressed as a Boolean function of its component, and reliability expressions are obtained analytically. The study highlights the effectiveness of Boolean expansion in handling large systems.

In 1994, Somani & Trivedi [5] submitted NASA technical report develops Boolean algebraic methods for analysing phased-mission systems. The study establishes a strong theoretical framework for reliability evaluation using Boolean logic influencing later research in system reliability modelling.

In 1999, Costas [6] used an algorithm for reliability analysis of phased-mission systems using Boolean algebraic methods. The approach efficiently handles system configuration changes across mission phases, demonstrating the flexibility of Boolean modelling in dynamic systems.

In 2006, Rahmat, Jovanonic & Lo, [7] applied Boolean truth table method to estimate the reliability of UPS systems. System states are represented using Boolean variables, and reliability is computed by enumerating successful operational states.

In 2014, Gupta and Shrma [8] evaluates reliability parameters of a solar panel system using Boolean algebra techniques. The system configuration is modelled using Boolean logic, and reliability expression re derived to assess performance under different operational conditions.

In 2016, Iqbal & Uduman [9] analyses a paper manufacturing plant using Boolean function techniques integrated with fuzzy logic. The system reliability is expressed through Boolean structure functions, while fuzziness accounts for uncertainty in component failure data.

In 2018, Navyata, Sharma & Gupta [10] used this approach to analyse the reliability of dual-channel logic communication system. The Boolean model effectively captures redundancy and switching behaviour, enabling accurate reliability evaluation.

In 2017, Garg [11] applies the Boolean function technique to evaluate reliability factors in a refinery unit. The Boolean representation helps identify critical subsystem affecting overall reliability and supports decision-making for maintenance planning.

In 2023, Ismail [12] explores the mathematical relationship between reliability functions and Boolean rings. The study provides theoretical foundation for Boolean reliability modelling and shows how Boolean algebraic structures can simplify reliability computations.

In 2024, Kumar [13] used this technique to assess the reliability and MTTF of power plant operations. The author demonstrates how Boolean algebra reduces computational complexity and provides closed-form reliability expressions, making it suitable for modern industrial reliability analysis.

In 2025, Kumar, Kumar & Srivastava [14] integrate this technique with pentagonal fuzzy numbers to model uncertainty in component reliability. The approach is applied to a sugar plant system, showing that Boolean methods can be extended to fuzzy environments for more realistic reliability assessment.

In 2025, Malik, Balram & Yadav [15] evaluates the reliability of a fast-food manufacturing system using this method combined with the path tracing method. Reliability and mean time to system failure are derived, proving the applicability of a Boolean models beyond power systems.

### 3. SYSTEM TRANSITION DIAGRAM

The system under consideration is shown in the following figure.

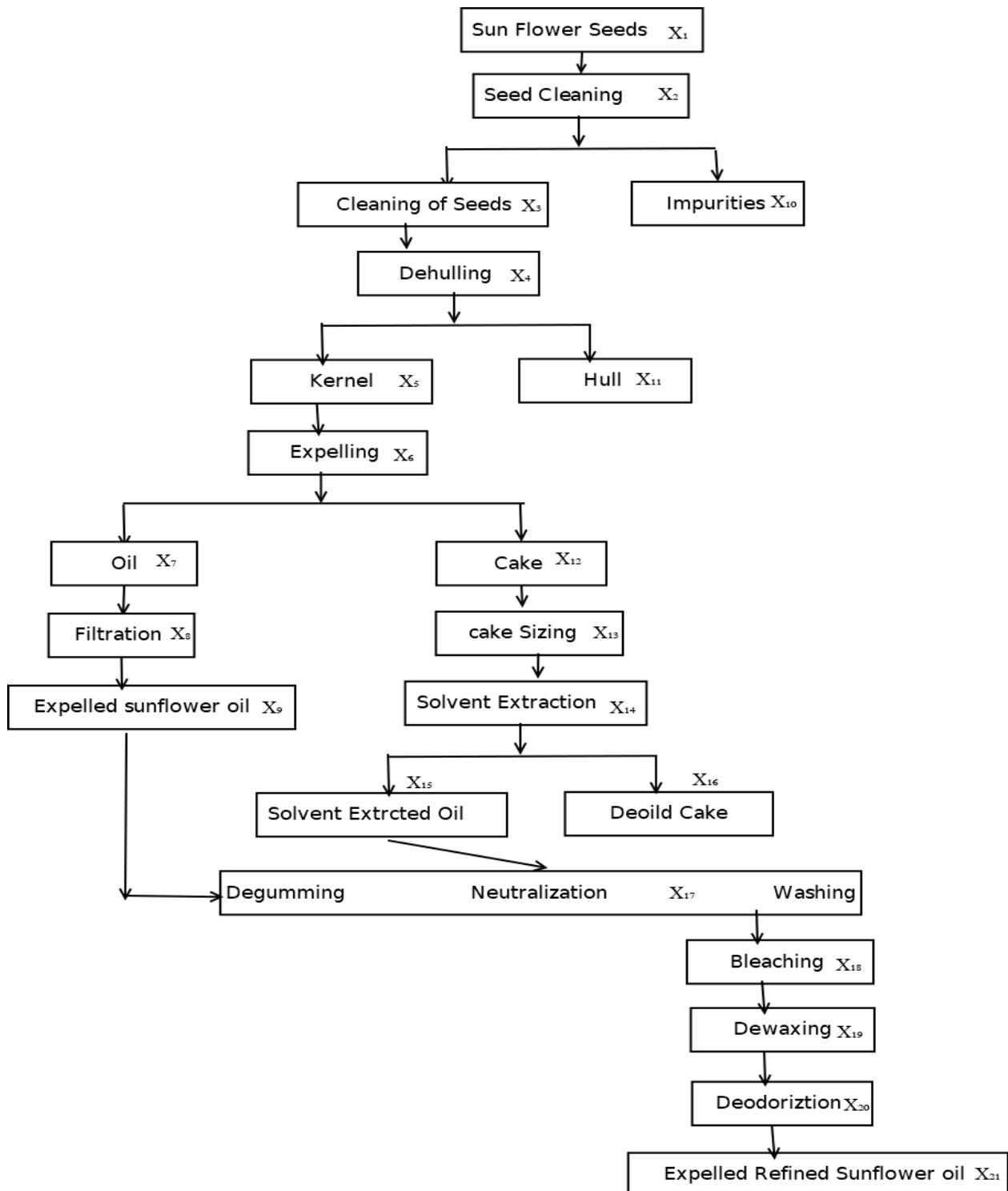


Fig 1. System Configuration

### 4. ASSUMPTIONS

1. The reliabilities of all constituent component of the system are known.
2. The States of all components are independent.

3. The state of each component and of the whole system is either good or bad.
4. The failure time of all components are arbitrary.
5. All components are always operating (no stand by or switched redundancy)
6. There is no repair facility.

## 5. NOTATIONS:

$X_1, X_2, X_3, \dots, X_{20}, X_{21}$  States of each subsystem of the System Configuration

$X_k^1$  Negation of  $X_k$  ( $k = 1 - 21$ )

$\wedge$  Conjunction

$\vee$  Disjunction

$x_k = \begin{cases} 0, & \text{in bad state,} \\ 1, & \text{in good state} \end{cases} \quad (k = 1 - 21)$

$\Pr(f = 1)$  The probability of the successful operation of the function  $f$ .

## 6. FORMATION OF THE MATHEMATICAL MODEL

By using Boolean function technique, the logical matrix is expressed as

$$f(x_1, x_2, x_3, \dots, x_{20}, x_{21}) = \begin{vmatrix} x_1 & x_2 & x_{10} \\ x_1 & x_2 & x_3 & x_4 & x_{11} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{16} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \end{vmatrix} \quad (1)$$

## 7. SOLUTION OF THE MODEL:

By the application of algebra of logic, equation (1) may be written as

$$f(x_1, x_2, x_3, \dots, x_{20}, x_{21}) = |x_1 x_2 n(x_3, x_4, \dots, x_{20}, x_{21})| \quad (2)$$

Where

$$n(x_3, x_4, \dots, x_{20}, x_{21}) = \begin{vmatrix} x_{10} \\ x_3 & x_4 & x_{11} \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{16} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \end{vmatrix} \quad (3)$$

Substituting,  $k_1 = x_{10}$ ,  $k_2 = x_3 x_4 x_{11}$ ,  $k_3 = x_3 x_4 x_5 x_6 x_{12} x_{13} x_{14} x_{16}$ ,

$k_4 = x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{17} x_{18} x_{19} x_{20} x_{21}$ ,

$k_5 = x_3 x_4 x_5 x_6 x_{12} x_{13} x_{14} x_{15} x_{17} x_{18} x_{19} x_{20} x_{21}$

From equation (3) we obtain

$$n(x_3, x_4, \dots, x_{20}, x_{21}) = \begin{vmatrix} k_1 \\ k_1^1 & k_2 \\ k_1^1 & k_2^1 & k_3 \\ k_1^1 & k_2^1 & k_3^1 & k_4 \\ k_1^1 & k_2^1 & k_3^1 & k_4^1 & k_5 \end{vmatrix} \quad (4)$$

Where

$$k_1^1 = |x_{10}^1| \quad (5)$$

$$k_2^1 = \begin{vmatrix} x_3^1 \\ x_3 & x_4^1 \\ x_3 & x_4 & x_{11}^1 \end{vmatrix} \quad (6)$$

$$k_3^1 = \begin{vmatrix} x_3^1 \\ x_3 & x_4^1 \\ x_3 & x_4 & x_5^1 \\ x_3 & x_4 & x_5 & x_6^1 \\ x_3 & x_4 & x_5 & x_6 & x_{12}^1 \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13}^1 \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14}^1 \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{16}^1 \end{vmatrix} \quad (7)$$

$$k_4^1 = \begin{vmatrix} x_3^1 & & & & & & & & & & & & \\ x_3 & x_4^1 & & & & & & & & & & & \\ x_3 & x_4 & x_5^1 & & & & & & & & & & \\ x_3 & x_4 & x_5 & x_6^1 & & & & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_7^1 & & & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8^1 & & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9^1 & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17}^1 & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17} & x_{18}^1 & & & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17} & x_{18} & x_{19}^1 & & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17} & x_{18} & x_{19} & x_{20}^1 & & \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21}^1 \end{vmatrix} \quad (8)$$

$$k_5^1 = \begin{vmatrix} x_3^1 & & & & & & & & & & & & \\ x_3 & x_4^1 & & & & & & & & & & & \\ x_3 & x_4 & x_5^1 & & & & & & & & & & \\ x_3 & x_4 & x_5 & x_6^1 & & & & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12}^1 & & & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13}^1 & & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14}^1 & & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15}^1 & & & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17}^1 & & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17} & x_{18}^1 & & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17} & x_{18} & x_{19}^1 & & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17} & x_{18} & x_{19} & x_{20}^1 & \\ x_3 & x_4 & x_5 & x_6 & x_{12} & x_{13} & x_{14} & x_{15} & x_{17} & x_{18} & x_{19} & x_{20} & x_{21}^1 \end{vmatrix} \quad (9)$$

$$k_1^1 k_2 = \begin{vmatrix} x_3 & x_4 & x_{10}^1 & x_{11} \end{vmatrix} \quad (10)$$

$$k_1^1 k_2^1 k_3 = \begin{vmatrix} x_3 & x_4 & x_5 & x_6 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{16} \end{vmatrix} \quad (11)$$

$$k_1^1 k_2^1 k_3^1 k_4^1 = \begin{vmatrix} x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \end{vmatrix} \quad (12)$$

$$k_1^1 k_2^1 k_3^1 k_4^1 k_5^1 = \begin{vmatrix} x_3 & x_4 & x_5 & x_6 & x_7^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \end{vmatrix} \quad (13)$$

Making use of (10) to (13) in equation (4), one can obtain

$$n(x_3, x_4, \dots, x_{20}, x_{21}) = \begin{vmatrix} x_{10} \\ x_3 & x_4 & x_{10}^1 & x_{11}^1 \\ x_3 & x_4 & x_5 & x_6 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{16} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{220} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \end{vmatrix} \quad (14)$$

$$f(x_1, x_2, \dots, x_{20}, x_{21}) = \begin{vmatrix} x_1 & x_2 & x_{10} \\ x_1 & x_2 & x_3 & x_4 & x_{10}^1 & x_{11} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{16} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12}^1 & x_{13}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12}^1 & x_{13}^1 & x_{14}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10}^1 & x_{11}^1 & x_{12}^1 & x_{13}^1 & x_{14}^1 & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9^1 & x_{10}^1 & x_{11}^1 & x_{12} & x_{13} & x_{14} & x_{15} & x_{16}^1 & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} \end{vmatrix}$$

(15)

Finally, the probability of the successful operation (i.e. Reliability) of the complex system is given by

$$\begin{aligned} R_s &= P_r \{ f(x_1, x_2, \dots, x_{20}, x_{21}) = 1 \} \\ &= R_1 R_2 R_{10} + R_1 R_2 R_3 R_4 Q_{10} R_{11} + R_1 R_2 R_3 R_4 R_5 R_6 Q_{10} Q_{11} R_{12} R_{13} R_{14} R_{16} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} Q_{12} R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} R_{12} Q_{13} R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} R_{12} R_{13} Q_{14} R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 Q_{10} Q_{11} R_{12} R_{13} R_{14} Q_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 Q_7 Q_{10} Q_{11} R_{12} R_{13} R_{14} R_{15} Q_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 Q_8 Q_{10} Q_{11} R_{12} R_{13} R_{14} R_{15} Q_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 Q_9 Q_{10} Q_{11} R_{12} R_{13} R_{14} R_{15} Q_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \end{aligned} \quad (16)$$

$$\begin{aligned} R_s(t) &= R_1 R_2 R_{10} + R_1 R_2 R_3 R_4 (1 - R_{10}) R_{11} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 (1 - R_{10})(1 - R_{11}) R_{12} R_{13} R_{14} R_{16} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 (1 - R_{10})(1 - R_{11})(1 - R_{12}) R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 (1 - R_{10})(1 - R_{11}) R_{12} (1 - R_{13}) R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 (1 - R_{10})(1 - R_{11}) R_{12} R_{13} (1 - R_{14}) R_{17} R_{18} R_{19} R_{20} R_{21} \\ &+ R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 (1 - R_{10})(1 - R_{11}) R_{12} R_{13} R_{14} (1 - R_{16}) R_{17} R_{18} R_{19} R_{20} R_{21} \end{aligned}$$

$$\begin{aligned}
 &+R_1 R_2 R_3 R_4 R_5 R_6 (1-R_7)(1-R_{10})(1-R_{11}) R_{12} R_{13} R_{14} R_{15} (1-R_{16}) R_{17} R_{18} R_{19} R_{20} R_{21} \\
 &+R_1 R_2 R_3 R_4 R_5 R_6 R_7 (1-R_8)(1-R_{10})(1-R_{11}) R_{12} R_{13} R_{14} R_{15} (1-R_{16}) R_{17} R_{18} R_{19} R_{20} R_{21} \\
 &+R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 (1-R_9)(1-R_{10})(1-R_{11}) R_{12} R_{13} R_{14} R_{15} (1-R_{16}) \\
 &\quad R_{17} R_{18} R_{19} R_{20} R_{21}
 \end{aligned}
 \tag{17}$$

$$\begin{aligned}
 R_S(t) = & R_1 R_2 R_{10} + R_1 R_2 R_3 R_4 R_{11} - R_1 R_2 R_3 R_4 R_{10} R_{11} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_{12} R_{13} R_{14} R_{15} - R_1 R_2 R_3 R_4 R_5 R_6 R_{10} R_{12} R_{13} R_{14} R_{15} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_{11} R_{12} R_{13} R_{14} R_{15} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_{10} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{10} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{17} R_{18} R_{19} R_{20} R_{21}
 \end{aligned}$$

$$\begin{aligned}
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} R_{13} R_{14} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} R_{13} R_{14} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} R_{13} R_{14} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} R_{13} R_{14} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_{10} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & \quad -R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{10} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & \quad -R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & -R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_9 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21} \\
 & +R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} R_{13} R_{14} R_{15} R_{16} R_{17} R_{18} R_{19} R_{20} R_{21}
 \end{aligned} \tag{18}$$

Where  $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, R_{11}, R_{12}, R_{13}, R_{14}, R_{15}, R_{16}, R_{17}, R_{18}, R_{19}, R_{20}, R_{21}$  are the reliabilities of the units and  $Q_k$  ( $k = 1 - 21$ ) are corresponding unreliability's.

## 8. PARTICULAR CASES

### CASE 1:

If reliabilities of each component of the complex system is  $R$ , equation (18) yields

$$R_5 = R^3 + R^5 - R^6 + R^{10} - 2R^{11} + R^{12} + R^{14} - R^{15} - 2R^{16} + 2R^{17} - 2R^{18} + 5R^{19} - 4R^{20} + R^{21} \tag{19}$$

## CASE 2: When failure rates follow Weibull Distribution:

Let failure rates of each unit is  $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9, \omega_{10}, \omega_{11}, \omega_{12}, \omega_{13}, \omega_{14}, \omega_{15}, \omega_{16}, \omega_{17}, \omega_{18}, \omega_{19}, \omega_{20}, \omega_{21}$  respectively, then from equation (18) reliability of the complex system at an instant  $t$  is given by

$$\begin{aligned}
 R_{SW}(t) = & \exp(-g_1 t^p) + \exp(-g_2 t^p) - \exp(-g_3 t^p) + \exp(-g_4 t^p) - \exp(-g_5 t^p) \\
 & - \exp(-g_6 t^p) + \exp(-g_7 t^p) + \exp(-g_8 t^p) - \exp(-g_9 t^p) - \exp(-g_{10} t^p) \\
 & + \exp(-g_{11} t^p) + \exp(-g_{12} t^p) + \exp(-g_{13} t^p) - \exp(-g_{14} t^p) - \exp(-g_{15} t^p) \\
 & + \exp(-g_{11} t^p) + \exp(-g_{12} t^p) + \exp(-g_{13} t^p) - \exp(-g_{14} t^p) - \exp(-g_{15} t^p) \\
 & - \exp(-g_{16} t^p) - \exp(-g_{17} t^p) + \exp(-g_{18} t^p) + \exp(-g_{19} t^p) - \exp(-g_{20} t^p) \\
 & - \exp(-g_{21} t^p) + \exp(-g_{22} t^p) + \exp(-g_{23} t^p) + \exp(-g_{24} t^p) + \exp(-g_{25} t^p) \\
 & - \exp(-g_{26} t^p) - \exp(-g_{27} t^p) - \exp(-g_{28} t^p) + \exp(-g_{29} t^p) + \exp(-g_{30} t^p) \\
 & - \exp(-g_{31} t^p) - \exp(-g_{32} t^p) - \exp(-g_{33} t^p) + \exp(-g_{34} t^p) + \exp(-g_{35} t^p) \\
 & + \exp(-g_{36} t^p) + \exp(-g_{37} t^p) - \exp(-g_{38} t^p) - \exp(-g_{39} t^p) + \exp(-g_{40} t^p) \\
 & + \exp(-g_{41} t^p) - \exp(-g_{42} t^p) - \exp(-g_{43} t^p) - \exp(-g_{44} t^p) - \exp(-g_{45} t^p) \\
 & + \exp(-g_{46} t^p) + \exp(-g_{47} t^p)
 \end{aligned} \tag{20}$$

Where  $p$  is a positive parameter and  $g_i$  are given by

$$g_1 = \omega_1 + \omega_2 + \omega_{10}$$

$$g_2 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_{11}$$

$$g_3 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_{10} + \omega_{11}$$

$$g_4 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15}$$

$$g_5 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{10} + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15}$$

$$g_6 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{11} + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15}$$

$$g_7 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{10} + \omega_{11} + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15}$$

$$g_8 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_9 = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{10} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{11} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{11} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{11} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{12} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{13} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{14} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{15} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{16} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{10} + \omega_{12} + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{17} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{18} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{19} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{20} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{11} + \omega_{12} + \omega_{13} \\ + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{21} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{22} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{23} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{24} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{10} + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{25} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{10} + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{26} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{10} + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{27} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{28} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{29} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{30} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{11} + \omega_{12} + \\ \omega_{13} + \omega_{14} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{31} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{32} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{12} + \omega_{13} \\ + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{33} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{12} + \omega_{13} \\ + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{34} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{12} + \omega_{13} \\ + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{35} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{36} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{37} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{38} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{39} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{40} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{41} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{42} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$g_{43} = \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{11} + \omega_{12} \\ + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}$$

$$\begin{aligned}
 g_{44} &= \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_{10} + \omega_{11} + \omega_{12} \\
 &\quad + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21} \\
 g_{45} &= \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_{10} + \omega_{11} + \omega_{12} \\
 &\quad + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21} \\
 g_{46} &= \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_9 + \omega_{10} + \omega_{11} + \omega_{12} \\
 &\quad + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21} \\
 g_{47} &= \omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5 + \omega_6 + \omega_7 + \omega_8 + \omega_9 + \omega_{10} + \omega_{11} + \omega_{12} \\
 &\quad + \omega_{13} + \omega_{14} + \omega_{15} + \omega_{16} + \omega_{17} + \omega_{18} + \omega_{19} + \omega_{20} + \omega_{21}
 \end{aligned}$$

### CASE 3: When failure rates follow exponential distribution

Exponential distribution is nothing but a particular case of Weibull distribution for  $p=1$  and is very useful in numerous practical problems, thus the reliability of the complex system in this case at an instant  $t$  is given by

$$\begin{aligned}
 R_{SE}(t) &= [R_{SW}(t)] \text{ at } p=1 \\
 R_{SE}(t) &= \exp(-g_1 t) + \exp(-g_2 t) - \exp(-g_3 t) + \exp(-g_4 t) - \exp(-g_5 t) \\
 &\quad - \exp(-g_6 t) + \exp(-g_7 t) + \exp(-g_8 t) - \exp(-g_9 t) - \exp(-g_{10} t) \\
 &\quad + \exp(-g_{11} t) + \exp(-g_{12} t) + \exp(-g_{13} t) - \exp(-g_{14} t) - \exp(-g_{15} t) \\
 &\quad - \exp(-g_{16} t) - \exp(-g_{17} t) + \exp(-g_{18} t) + \exp(-g_{19} t) - \exp(-g_{20} t) \\
 &\quad - \exp(-g_{21} t) + \exp(-g_{22} t) + \exp(-g_{23} t) + \exp(-g_{24} t) + \exp(-g_{25} t) \\
 &\quad - \exp(-g_{26} t) - \exp(-g_{27} t) - \exp(-g_{28} t) + \exp(-g_{29} t) + \exp(-g_{30} t) \\
 &\quad - \exp(-g_{31} t) - \exp(-g_{32} t) - \exp(-g_{33} t) + \exp(-g_{34} t) + \exp(-g_{35} t) \\
 &\quad + \exp(-g_{36} t) + \exp(-g_{37} t) - \exp(-g_{38} t) - \exp(-g_{39} t) + \exp(-g_{40} t) \\
 &\quad + \exp(-g_{41} t) - \exp(-g_{42} t) - \exp(-g_{43} t) - \exp(-g_{44} t) - \exp(-g_{45} t) \\
 &\quad + \exp(-g_{46} t) + \exp(-g_{47} t)
 \end{aligned} \tag{21}$$

### EVALUATION OF MTTF OF THE SYSTEM:

The expression for MTTF in this case is given by

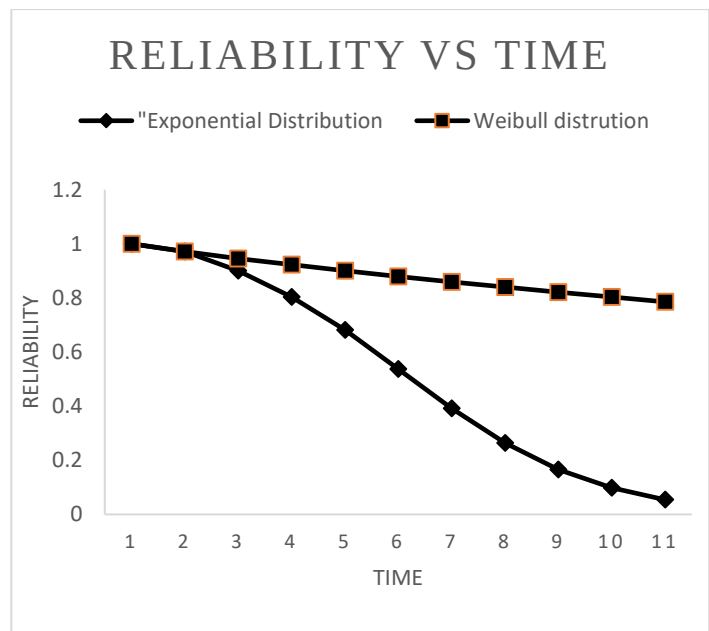
$$\begin{aligned}
 M.T.T.F &= \int_0^{\infty} R_{SE}(t) dt \\
 M.T.T.F &= \frac{1}{g_1} + \frac{1}{g_2} - \frac{1}{g_3} + \frac{1}{g_4} - \frac{1}{g_5} - \frac{1}{g_6} + \frac{1}{g_7} + \frac{1}{g_8} - \frac{1}{g_9} - \frac{1}{g_{10}}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{g_{11}} + \frac{1}{g_{12}} + \frac{1}{g_{13}} - \frac{1}{g_{14}} - \frac{1}{g_{15}} - \frac{1}{g_{16}} - \frac{1}{g_{17}} + \frac{1}{g_{18}} + \frac{1}{g_{19}} - \frac{1}{g_{20}} \\
 & - \frac{1}{g_{21}} + \frac{1}{g_{22}} + \frac{1}{g_{23}} + \frac{1}{g_{24}} + \frac{1}{g_{25}} - \frac{1}{g_{26}} - \frac{1}{g_{27}} - \frac{1}{g_{28}} + \frac{1}{g_{29}} + \frac{1}{g_{30}} \\
 & - \frac{1}{g_{31}} - \frac{1}{g_{32}} - \frac{1}{g_{33}} + \frac{1}{g_{34}} + \frac{1}{g_{35}} + \frac{1}{g_{36}} + \frac{1}{g_{37}} - \frac{1}{g_{38}} - \frac{1}{g_{39}} + \frac{1}{g_{40}} \\
 & + \frac{1}{g_{41}} - \frac{1}{g_{42}} - \frac{1}{g_{43}} - \frac{1}{g_{44}} - \frac{1}{g_{45}} + \frac{1}{g_{46}} + \frac{1}{g_{47}}
 \end{aligned} \tag{22}$$

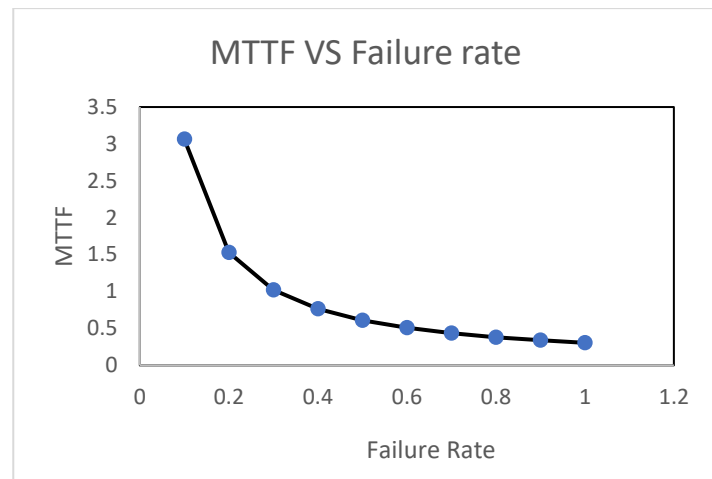
## 9. NUMERICAL COMPUTATION FOR RELIABILITY AND MTTF:

Setting  $\omega_i = 0.01$  for  $i=1-21$  and  $p=2$  in equation (20) and (21), one can compute Table 1 and setting  $\omega_i = \omega$  one can compute table 2 from equation (22).

| S.No. | Time(t) | System Reliability       |                      |
|-------|---------|--------------------------|----------------------|
|       |         | Exponential Distribution | Weibull Distribution |
| 1.    | 0       | 1                        | 1                    |
| 2.    | 1       | 0.9716                   | 0.9716               |
| 3.    | 2       | 0.9003                   | 0.9459               |
| 4.    | 3       | 0.8033                   | 0.9223               |
| 5.    | 4       | 0.6811                   | 0.9003               |
| 6.    | 5       | 0.5376                   | 0.8795               |
| 7.    | 6       | 0.3919                   | 0.8595               |
| 8.    | 7       | 0.2645                   | 0.8403               |
| 9.    | 8       | 0.1662                   | 0.8216               |
| 10.   | 9       | 0.0978                   | 0.8033               |
| 11.   | 10      | 0.0540                   | 0.7852               |



| SLNo | $\omega$ | MTTF     |
|------|----------|----------|
| 1    | 0.0      | $\infty$ |
| 2    | 0.1      | 3.0643   |
| 3    | 0.2      | 1.5321   |
| 4    | 0.3      | 1.0214   |
| 5    | 0.4      | 0.7660   |
| 6    | 0.5      | 0.6128   |
| 7    | 0.6      | 0.5107   |
| 8    | 0.7      | 0.4377   |
| 9    | 0.8      | 0.3830   |
| 10   | 0.9      | 0.3404   |
| 11   | 1.0      | 0.3064   |



## 10. INTERPRETATION OF THE RESULTS

Table 1 computes the variation of reliability with respect to time, when failure follows Exponential and Weibull distributions. A critical examination of Reliability vs time graph shows that the reliability of the complex system decreases approximately at a uniform rate in case of exponential distribution, where as it decreases very rapidly when failure follows Weibull distribution.

Further, Table 2 computes the mean time to failure i.e., MTTF at an instant  $t$ . An inspection of MTTF vs. failure rate graph reveals that in the beginning it decreases catastrophically, but later on, it decreases approximately at a uniform rate.

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