

# Stochastic Order Level Inventory Model with Exponentially Increasing Demand under Two Levels of Storage

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## Abstract

The present paper deals with a single period inventory model with exponentially increasing demand for both deteriorating and non-deteriorating items which are stored in two different warehouses. The demand is assumed to occur in a uniform pattern during a planning period. Both non-deteriorating items and deteriorating items are considered for the discussion and the results obtained are shown to agree with classical models. Inventory models for deteriorating items have been considered by many researchers in the recent times. A known fact is that it is very difficult to develop a model that reflects the reality as close as possible simultaneously simple for analysis.

**Keywords:** Inventory model, Exponentially increasing demand, Deterioration, Own warehouse, Rented warehouse.

## Introduction

Inventory may be regarded as those goods or items that are procured, stored and used for day-to-day functioning of the organization. The classical inventory models assume that the demand during an inventory cycle occurs at a constant rate so that the on-hand inventory at any time 't' can be represented as a linear function of time. This assumption is may not be true in all practical situations and on-hand inventory may vary at a constant rate and may vary with time. When the items are kept as an inventory for filling the demand, there may be deterioration of items in the inventory system, which may occur due to one or many reasons viz., storage condition, weather condition etc.

An extensive research work has been carried out by many researchers in the field of deterioration including Shah and Jaiswal (1990) have studied a periodic review inventory model for items that deteriorate continuously in time. Dye (2002) developed an EOQ model for deteriorating items allowing shortages and backlogging. An EOQ model of deteriorating items with time varying demand and shortages have been suggested by Chung, K.J and Hwang, Y.F (2003). Skouri and Papachritos (2003) have discussed about EOQ of deteriorating items under delay in payments. Ghare and Schrader (1963) categorised the inventory deterioration into three types: direct spoilage, physical depletion and deterioration. Direct spoilage refers to the unusable state of inventory items caused by breakage during transaction or by sudden accidental events. For example, quality and effectiveness of some medicines might be reduced in the event of non-functioning of refrigerator caused by sudden load shedding or absence of power supply for hours together. Deterioration on the other hand, refers to the slow but gradual loss of qualitative properties of an item with the passage of time. In fact no inventory item can avoid this kind of deterioration. This is inevitable. Wee

(1995) considered an inventory problem for deteriorating items with shortages. Teng and Chang (2005) discussed an EOQ model for deteriorating items where demand depends on stock and price. Inventory model with power pattern demand for deteriorating items is developed by Rao and Sarma (1991).

We now develop a single period inventory model with exponentially increasing demand for non-deteriorating items first and then followed by deteriorating items under  $L_2$ -system like the one discussed by Hartley (1976), Sarma & Sastry (1988). Whenever initial on-hand inventory exceeds the storage capacity of the OW, the excess stock will be kept in a RW. The goods kept in RW are prone to a lower rate of deterioration than the goods kept in OW. This is one of the reasons for costly holding charges in RW.

### Model Assumptions and Notations

We adopt the following notations in carrying out the mathematical models.

- T : Constant length of the period  
H, F : Unit cost of holding / unit time in OW & RW respectively  
such that  $F > H$   
 $\pi$  : Unit cost of shortage per unit time  
x : Random variable representing the demand during the  
period, with known probability density function  $f(x)$   
W : Storage capacity of OW  
S : Order level  
 $\theta_1, \theta_2$  : Rates of deterioration per unit time in OW & RW  
respectively, such that  $\theta_2 < \theta_1$ .

We now assume that

1. The replenishment rate is infinite and replenishment size is constant.
2. Lead time is zero and all shortages are backlogged.
3. There is no extra fixed cost associated with the use of RW.
4. The stock kept in OW is used only after exhausting the stock kept in RW.
5. The exponential demand pattern is uniform throughout the period.
6. A constant fraction of the on-hand inventory gets deteriorated per unit time.

The problem is to determine the optimal value of S which minimises the sum of holding, shortage and deterioration costs. The case of non-deteriorating items is studied first and followed by the case of deteriorating items.

### Model with Non- Deteriorating Items

In this case the demand during the period T is a random variable X. Depending on the values of S and X, the following two cases arise:

CASE (i): When  $X \leq S$ .

In this case shortages do not occur and the system ends with  $(S-x)$  units. Again  $(S-x) \leq W$  and  $(S-x) > W$  are the two possible contexts to be considered.

Let  $Q(t)$  denote the on-hand inventory and  $t_w$  be the time by which inventory level drops to zero in the RW. Clearly,  $Q_R(0) = Z$  and  $Q_R(t_w) = 0$ , so that shortages arise only during  $(t_1, T)$ .

If  $Q_R(t)$  denotes the inventory level at time 't' ( $0 \leq t \leq t_w$ ), then the differential equation governing the inventory system is given by

$$\frac{dQ_R(t)}{dt} = -xe^{\alpha t}, \quad 0 \leq t \leq t_w \quad \dots\dots (1)$$

The solution of the equation (1), using the boundary conditions (at  $t = 0, Q_R(0) = Z$  and at  $t = t_w, Q_R(t_w) = 0$ ), is given by

$$Q_R(t) = Z - \frac{x}{\alpha} (e^{\alpha t_w} - 1) \quad \dots\dots (2)$$

so that

$$t_w = \frac{1}{\alpha} \log\left(1 + \frac{Z\alpha}{x}\right) \quad \dots\dots (3)$$

Suppose  $(S-x) \leq W$ . In this case we have  $(S-W) \leq x \leq S$ . The inventory carried out in RW is given by

$$Q_R = \int_0^{t_w} Q_R(t) dt$$

$$= \int_0^{t_w} \left\{ Z - \frac{x}{\alpha} (e^{\alpha t_w} - 1) \right\} dt \quad \dots\dots (4)$$

$$\therefore Q_R = \frac{(x + Z\alpha)}{\alpha^2} \log\left(1 + \frac{Z\alpha}{x}\right) - \frac{Z}{\alpha} \quad \dots\dots (5)$$

where  $Z = (S-W)$  is the capacity of the RW.

The inventory carried in both the warehouses is given by

$$Q = \int_0^T Q(t) dt \quad \dots\dots (6)$$

where  $Q(t)$  is the inventory level at time  $t$ , which is the solution of the differential equation (using the boundary conditions at  $t = 0, Q(0) = S$  and at  $t = T, Q(T) = 0$ ) given by

$$\frac{dQ(t)}{dt} = -xe^{\alpha t}, \quad 0 \leq t \leq T \quad \dots\dots (7)$$

$$\therefore Q = \int_0^T \left\{ S - \frac{x}{\alpha} (e^{\alpha t} - 1) \right\} dt$$

$$\therefore Q = \frac{(x + S\alpha)T}{\alpha} - \frac{x}{\alpha^2} (e^{\alpha T} - 1) \quad \dots\dots (8)$$

The inventory carried out in the OW is then given by

$$Q_0 = Q - Q_R$$

$$= \frac{(x + S\alpha)T}{\alpha} - \frac{x}{\alpha^2} (e^{\alpha T} - 1) - \frac{(x + Z\alpha)}{\alpha^2} \log\left(1 + \frac{Z\alpha}{x}\right) + \frac{Z}{\alpha}$$

..... (9)

Hence when  $(S-x) \leq W$ , we have the expected inventory cost per unit time and is given by

$$C_1(S) = \frac{F}{T} \int_{S-W}^S \left[ \frac{(x + Z\alpha)}{\alpha^2} \log\left(1 + \frac{Z\alpha}{x}\right) - \frac{Z}{\alpha} \right] f(x) dx +$$

$$\frac{H}{T} \int_{S-W}^S \left[ \frac{(x + S\alpha)T}{\alpha} - \frac{x}{\alpha^2} (e^{\alpha T} - 1) - \frac{(x + Z\alpha)}{\alpha^2} \log\left(1 + \frac{Z\alpha}{x}\right) + \frac{Z}{\alpha} \right] f(x) dx$$

..... (10)

When  $(S-x) > W$ , the ending stock would be in RW and proceeding as above, we get the expected inventory holding cost per unit time and is given by

$$C_2(S) = \frac{F}{T} \int_0^{S-W} \left[ \frac{(x + Z\alpha)}{\alpha^2} \log\left(1 + \frac{Z\alpha}{x}\right) - \frac{Z}{\alpha} \right] f(x) dx + \frac{HW}{T} \int_0^{S-W} f(x) dx$$

..... (11)

The average inventory in RW is given by

$$A_1 = \frac{1}{T} \int_0^{t_w} Q_R(t) dt$$

$$= \frac{1}{T} \left[ \frac{(x + Z\alpha)}{\alpha^2} \log\left(1 + \frac{Z\alpha}{x}\right) - \frac{Z}{\alpha} \right] \dots\dots (12)$$

Let A denote the average inventory carried in both warehouses then

$$A = \frac{1}{T} \int_0^{t_1} Q(t) dt$$

$$A = \frac{1}{T} \left[ \frac{(x + S\alpha)t_1}{\alpha} - \frac{x}{\alpha^2} (e^{\alpha t_1} - 1) \right] \dots\dots (13)$$

Since  $Q_0(t_1) = 0$  it follows that  $t_1 = \frac{1}{\alpha} \log\left(1 + \frac{S\alpha}{x}\right)$ , which gives

$$A = \frac{1}{T} \left[ \frac{(x + S\alpha)}{\alpha^2} \log\left(1 + \frac{S\alpha}{x}\right) - \frac{S}{\alpha} \right] \dots\dots (14)$$

Hence the average inventory time units in OW is given by

$$\begin{aligned}
 A_2 &= A - A_1 \\
 &= \frac{1}{\alpha T} \left[ \frac{(x + S\alpha)}{\alpha} \log\left(1 + \frac{S\alpha}{x}\right) - S - \frac{(x + Z\alpha)}{\alpha} \log\left(1 + \frac{Z\alpha}{x}\right) + Z \right] \\
 &\dots\dots (15)
 \end{aligned}$$

To compute the back order position, we first note that the inventory position during  $(t_1, T)$  is given by the differential equation

$$\frac{dQ_B(t)}{dt} = -xe^{\alpha t}, \quad t_1 \leq t \leq T$$

so that

$$Q_B(t) = \frac{-x}{\alpha} (e^{\alpha t} - e^{\alpha t_1}), \quad t_1 \leq t \leq T$$

Then the average back order position is given by

$$\bar{B} = \frac{1}{T} \int_{t_1}^T Q_B(t) dt$$

$$\therefore \bar{B} = \frac{-xe^{\alpha T}}{\alpha^2 T} + \frac{(x + S\alpha)}{\alpha^2 T} + \frac{(x + S\alpha)}{\alpha T} \left\{ T - \frac{1}{\alpha} \log\left(1 + \frac{S\alpha}{x}\right) \right\} \dots\dots (16)$$

CASE (ii): When  $X > S$ .

In this case shortages arise and the system contains both positive and negative inventories and the expected cost per unit time is given by

$$C_3(S) = FA_1 + HA_2 - \pi \bar{B}$$

$$\begin{aligned}
 &= \frac{F}{\alpha T} \int_S^\infty \left[ \frac{(x + Z\alpha)}{\alpha} \log\left(1 + \frac{Z\alpha}{x}\right) - Z \right] f(x) dx + \\
 &\frac{H}{\alpha T} \int_S^\infty \left[ \left\{ \frac{(x + S\alpha)}{\alpha} \log\left(1 + \frac{S\alpha}{x}\right) - S \right\} - \left\{ \frac{(x + Z\alpha)}{\alpha} \log\left(1 + \frac{Z\alpha}{x}\right) - Z \right\} \right] f(x) dx \\
 &+ \frac{\pi}{\alpha T} \int_S^\infty \left[ \frac{x e^{\alpha T}}{\alpha} - \frac{(x + S\alpha)}{\alpha} - (x + S\alpha) \left\{ T - \frac{1}{\alpha} \log\left(1 + \frac{S\alpha}{x}\right) \right\} \right] f(x) dx \\
 &\dots\dots (17)
 \end{aligned}$$

Hence the total expected cost of the entire inventory system is given by

$$C(S) = C_1(S) + C_2(S) + C_3(S)$$

$$\begin{aligned}
 C(S) &= \frac{(F - H)}{\alpha T} \int_{S-W}^\infty \left[ \frac{(x + Z\alpha)}{\alpha} \log\left(1 + \frac{Z\alpha}{x}\right) - Z \right] f(x) dx \\
 &+ \frac{H}{\alpha} \int_{S-W}^S (x + S\alpha) f(x) dx - \frac{H}{\alpha^2 T} \int_{S-W}^S x (e^{\alpha T} - 1) f(x) dx \\
 &+ \frac{F}{\alpha T} \int_0^{S-W} \left[ \frac{(x + Z\alpha)}{\alpha} \log\left(1 + \frac{Z\alpha}{x}\right) - Z \right] f(x) dx + \frac{HW}{T} \int_0^{S-W} f(x) dx \\
 &- \frac{HS}{\alpha T} \int_S^\infty f(x) dx + \frac{(H + \pi)}{\alpha T} \int_S^\infty \left[ \frac{(x + S\alpha)}{\alpha} \log\left(1 + \frac{S\alpha}{x}\right) \right] f(x) dx \\
 &+ \frac{\pi}{\alpha^2 T} \int_S^\infty x e^{\alpha T} f(x) dx - \frac{\pi}{\alpha^2 T} \int_S^\infty (x + S\alpha) f(x) dx \\
 &- \frac{\pi}{\alpha} \int_S^\infty (x + S\alpha) f(x) dx \\
 &\dots\dots (18)
 \end{aligned}$$

The optimal value of S denoted by  $S^0$  is the solution of  $\frac{dC(S)}{dS} = 0$ , which is given by the integral equation

$$\begin{aligned}
 & (F - H) \int_{S-w}^{\infty} \log\left\{1 + \frac{Z\alpha}{x}\right\} f(x) dx + H\alpha T \int_{S-w}^S f(x) dx \\
 & + F \int_0^{S-w} \log\left\{1 + \frac{Z\alpha}{x}\right\} f(x) dx + (H + \pi) \int_S^{\infty} \log\left\{1 + \frac{S\alpha}{x}\right\} f(x) dx \\
 & = \pi\alpha T \int_S^{\infty} f(x) dx \\
 & \dots\dots (19)
 \end{aligned}$$

When  $F = H$ , the equation (19) reduces to

$$\int_0^S f(x) dx = \frac{\pi}{(H + \pi)} \dots\dots (20)$$

This agrees with that of Naddor (1966).

The integrals involved in (18) and (19) can be evaluated explicitly for some specific function  $f(x)$  only. For such functions we can obtain optimum order level say  $S^0$  by solving (19) for  $S$  and the minimum total expected cost of the system can be obtained by substituting  $S = S^0$  in (18). This is illustrated in the following section.

**Sensitivity Analysis of the Model**

Let the demand density be

$$\begin{aligned}
 f(x) &= \frac{1}{2} x^2 e^{-x}, x \geq 0 \\
 &= 0, \text{ otherwise}
 \end{aligned}$$

For this demand density, (18) becomes

$$\begin{aligned}
 C(S) = & \frac{(F - H)Z^2(Z + 1)e^{-Z}}{4T} - \frac{(F - H)Z^3\alpha e^{-Z}}{4T} + \frac{H}{2\alpha} \left\{ 1 - \frac{(e^{\alpha T} - 1)}{\alpha T} \right\} \\
 & \left[ (Z^3e^{-Z} - S^3e^{-S}) + 3(Z^2e^{-Z} - S^2e^{-S}) + 6\{(Z + 1)e^{-Z} - (S + 1)e^{-S}\} \right] \\
 & + \frac{HS}{2} \left[ Z^2e^{-Z} - S^2e^{-S} + 2\{(Z + 1)e^{-Z} - (S + 1)e^{-S}\} \right] + \frac{FZ^3\alpha}{4T} (1 - e^{-Z}) \\
 & + HW \left\{ 2 - 2(1 + Z)e^{-Z} - Z^2e^{-Z} \right\} - \frac{HS(S^2 + 2S + 2)e^{-S}}{2\alpha T} + \frac{FZ^2}{4T} (1 - (1 + Z)e^{-Z}) \\
 & + \frac{(H + \pi)S(S^2 + 2S + 2)e^{-S}}{2\alpha T} + \frac{(H + \pi)S^2(S + 1)e^{-S}}{4T} - \frac{(H + \pi)S^3\alpha e^{-S}}{4T} \\
 & + \frac{\pi(S^3 + 3S^2 + 6S + 6)e^{-S}(e^{\alpha T} - 1 - \alpha T)}{2\alpha^2 T} - \frac{\pi S(S^2 + 2S + 2)e^{-S}(1 + \alpha T)}{2\alpha T}
 \end{aligned}$$

..... (21)

and the optimum order level  $S = S^0$  is the solution of

$$\begin{aligned}
 & \frac{(F - H)Z\alpha(1 + Z)e^{-Z}}{2} - \frac{(F - H)Z^2\alpha^2 e^{-Z}}{4} + \frac{H\alpha T(Z^2 + 2Z + 2)e^{-Z}}{2} \\
 & - \frac{(H + \pi)\alpha T(S^2 + 2S + 2)e^{-S}}{2} + \frac{FZ\alpha\{1 - (1 + Z)e^{-Z}\}}{2} + \frac{FZ^2\alpha^2(e^{-Z} - 1)}{4} \\
 & + \frac{(H + \pi)S\alpha(S + 1)e^{-S}}{2} - \frac{(H + \pi)S^2\alpha^2 e^{-S}}{4} = 0
 \end{aligned}$$

Reconsider the hypothetical values of the parameters of section (2.4) then substituting these values in (21) and by applying Newton Rhapsion method we find that the optimal order quantity is  $S^0 = 340$  units and the associated minimum cost which is obtained from (20) is Rs. 2200.

### Model with Deteriorating Items

In this case the demand will be treated as a random variable X, during the period T. Depending on the values of S and x of X, the following cases arise:

CASE (i): When  $x \leq S$ .

In this case the shortages do not arise. The period begins with S units and ends with (S - x) units. If  $(S - x) \leq W$ , the ending stock will be in OW only. Otherwise, if  $(S - x) > W$ , the RW contains (S - x - W) units by the end of the period. The above argument holds good when there is no deterioration. Since a constant rate of deterioration is acting on the system, the consumption of holding cost shall consider the effect of deterioration. Consider the case when  $0 \leq x \leq (S - W)$ . The inventory situation in this case is shown in figure (1), which is drawn for uniform demand.

The inventory area in figure (1) in RW is divided into two parts namely  $B_1$  and  $B_2$ , where  $B_1$  indicates the inventory time units when x units have been consumed from RW. It follows that the time required to consume exactly x units is given by

$$T_\beta = \frac{1}{(\alpha + \theta_2)} \log\{1 + (\alpha + \theta_2)\} \dots\dots (22)$$

Which can be obtained by putting  $Z = x$  in  $t_w = \frac{1}{(\alpha + \theta_2)} \log\left\{1 + \frac{Z(\alpha + \theta_2)}{a}\right\}$ .

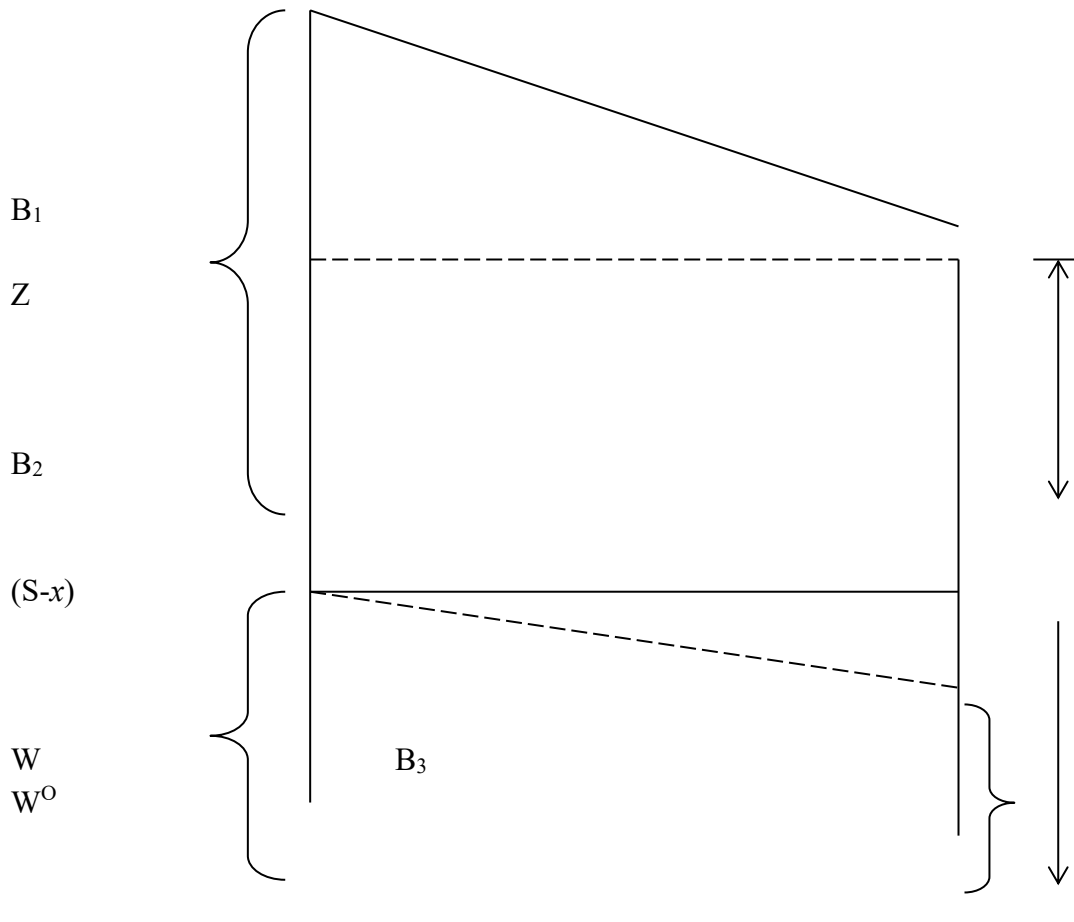


Figure 1  
Note that  $T_\beta = T$ , when  $\beta = 0$  as expected.

The inventory time units in RW are given by

$$B_1 = \frac{\left\{x - x\left(\frac{T_\beta}{T}\right)\right\}}{\theta_2} \dots\dots (23)$$

Since  $Q(T_\beta) = \left\{x - x\left(\frac{T_\beta}{T}\right)\right\}$  is the inventory level at  $T_\beta$ , it should be zero, had there been no deterioration.

The unconsumed stock in RW is  $(S - x - W)$  units at time  $t = 0$  which becomes  $(S - x - W^0)$  by time  $T_\beta$ .

Hence we get

$$B_2 = \frac{[(Z - x)\{1 - e^{-\theta_2 T_\beta}\}]}{\theta_2} \dots\dots (24) \text{ and } B_3 = \frac{[W - We^{-\theta_1 T_\beta}]}{\theta_1} \dots\dots (25)$$

Therefore, the expected holding cost for the case  $0 \leq x \leq (S - W)$  is given by

$$C_1(S) = \int_0^{S-W} [F(B_1 + B_2) + HB_3] f(x) dx \dots\dots (26)$$

Now, consider the case  $(S - x) \leq W$ . Then we have  $(S - W) \leq x \leq S$ . The inventory situation for this case is depicted in figure (1) and the figure is drawn for uniform demand. In this case we denote by  $C_1, C_2, C_3$  and  $C_4$  as the areas shown in figure (2).

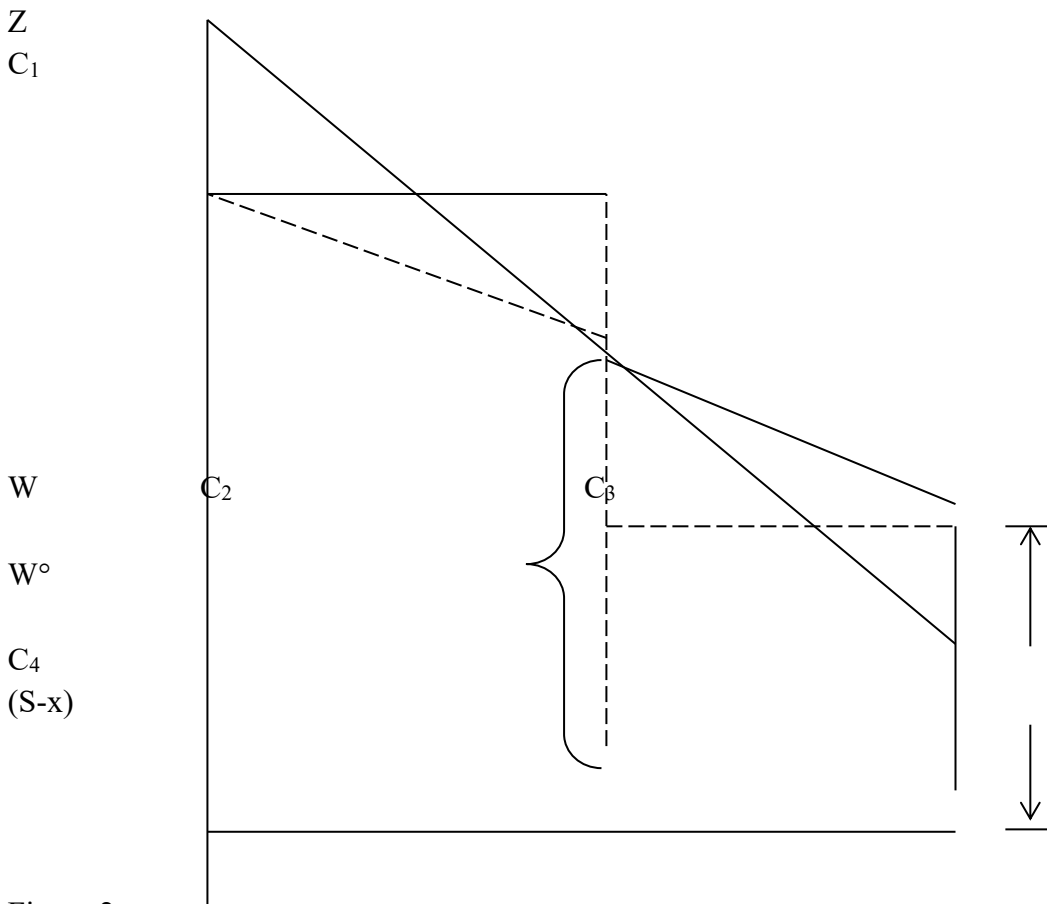


Figure 2

$$C_1 = \frac{[Z - xt_w]}{\theta_2} \dots\dots (27)$$

where

$$t_w = \frac{1}{(\alpha + \theta_2)} \log \left\{ 1 + \frac{Z(\alpha + \theta_2)}{a} \right\} \dots\dots (28)$$

$$C_2 = \frac{[W - W^0]}{\theta_1} \dots\dots (29)$$

and

$$W^0 = We^{-\theta_1 t_w} \dots\dots (30)$$

To compute  $C_3$ , we first note that at time  $t_w$  the OW contains only  $W^0$  units, of which  $[W^0 - (S - x)]$  units will be consumed in presence of deterioration. It takes  $\hat{t}$  units of time, for this to happen.

$$\therefore \hat{t} = \frac{1}{(\alpha + \theta_1)} \log \left\{ 1 + \frac{\hat{W}(\alpha + \theta_1)}{a} \right\} \dots\dots (31)$$

where

$$\hat{W} = [W^0 - (S - x)] \dots\dots (32)$$

Now

$$C_3 = \frac{[\hat{W} - x \hat{t}]}{\theta_1} \dots\dots (33)$$

The  $(S - x)$  units in the OW are subject to deterioration only. (It is easy to see that  $\hat{t} = (t - t_w)$  when there is no deterioration)

As such we have

$$C_4 = (S - x)[1 - e^{-\theta_1 \hat{t}}] \dots\dots (34)$$

Therefore, the total expected holding cost in this case is given by

$$C_2(S) = \int_{S-W}^S [FC_1 + H(C_2 + C_3 + C_4)]f(x)dx \dots\dots (35)$$

CASE (ii) When  $x > S$ .

In this case shortages arise and the demand is to be treated as a random variable. For a given value  $x$  of  $X$ , the areas of  $A_1, A_2, A_3$  and  $A_4$  can be used to yield the expected cost function given by

$$C_3(S) = \int_S^{\infty} [FA_1 + H(A_2 + A_3) + \pi A_4] f(x) dx \quad \dots\dots (36)$$

where  $A_1$  is the average inventory carried in RW and is given by

$$A_1 = \frac{[Z - xe^{\alpha t_w}]}{\theta_2},$$

$A_2$  denotes inventory units carried during  $(0, t_w)$  in OW and is given by

$$A_2 = \frac{[W - W^0]}{\theta_1}, A_3 = \frac{[W^0 - xe^{\alpha t^*}]}{\theta_1},$$

where  $t^* = \frac{1}{(\alpha + \theta_1)} \log \left\{ 1 + \frac{W^0(\alpha + \theta_1)}{a} \right\}$

and  $A_4$  is the amount of shortage during  $(t_1, T)$  and is given by

$$A_4 = \frac{xe^{\alpha t} (T - t_1)^2}{2}$$

The cost of deteriorated items during the entire period is given by

$$D(S) = D_1(S) + D_2(S) + D_3(S) \quad \dots\dots (37)$$

where  $D_1(S) = C \int_0^{S-W} [Z - xe^{\alpha t} ] f(x) dx$

$$D_2(S) = C \int_{S-W}^S [1 - xe^{\alpha(\hat{t} + t_w)}] f(x) dx$$

$$D_3(S) = C \int_S^{\infty} [S - xe^{\alpha t_1}] f(x) dx$$

Hence the expected cost of the entire inventory system is given by

$$C(S, \theta_1, \theta_2) = C_1(S) + C_2(S) + C_3(S) + D(S) \quad \dots\dots (38)$$

where  $C_1(S), C_2(S), C_3(S)$  and  $D(S)$  are given in the equations (34), (35), (36) and (37) respectively.

The optimal value of S is the solution of  $C'(S, \theta_1, \theta_2) = 0$ . From the quantities defined earlier, we have the following derivatives

$$C'_1 = \frac{F}{\theta_2} \int_0^{S-W} \left[ 1 - e^{\frac{-\theta_2}{\alpha+\theta_2} \log\{1+(\alpha+\theta_2)\}} \right] f(x) dx$$

$$C'_2 = \frac{F}{\theta_2} \int_{S-W}^S \left[ 1 - \frac{x}{a+Z(\alpha+\theta_2)} \right] f(x) dx + \frac{H}{\theta_1} \int_{S-W}^S \left[ \frac{x \{1 + \theta_1 W t'_w e^{-\theta_1 t_w}\}}{a + \hat{W}(\alpha + \theta_1)} - 1 \right] f(x) dx$$

$$+ H \int_{S-W}^S \left[ \left\{ 1 - e^{-\theta_1 \hat{t}} \right\} + \left\{ \frac{\theta_1 (S-x) (\hat{W})' e^{-\theta_1 \hat{t}}}{a + \hat{W}(\alpha + \theta_1)} \right\} \right] f(x) dx$$

where  $\hat{W}$  is defined as in equation (32)

$$t'_w = \frac{1}{a + Z(\alpha + \theta_2)}, \hat{t} \text{ is defined as in equation (31)}$$

$$(\hat{W})' = - \left[ 1 + \frac{\theta_1 W e^{-\theta_1 t_w}}{a + Z(\alpha + \theta_2)} \right]$$

$$C'_3 = \frac{F}{\theta_2} \int_S^\infty \left[ 1 - \frac{x e^{\alpha t_w}}{a + Z(\alpha + \theta_2)} \right] f(x) dx + H \int_S^\infty \left[ \frac{x \alpha e^{\alpha t^*}}{\{a + \hat{W}(\alpha + \theta_1)\} \{a + Z(\alpha + \theta_2)\}} \right] f(x) dx$$

$$- \Pi \int_S^\infty \left[ \frac{(T - t_1) x e^{\alpha t} (a + \alpha W^0)}{\{a + \hat{W}(\alpha + \theta_1)\} \{a + Z(\alpha + \theta_2)\}} \right] f(x) dx$$

where

$$t^* = \frac{1}{(\alpha + \theta_1)} \log \left\{ 1 + \frac{W^0(\alpha + \theta_1)}{a} \right\}$$

$$D'(S) = C \int_0^{S-W} f(x) dx + C \int_{S-W}^S \left[ \frac{\{\theta_1 W e^{-\theta_1 t_w} + a + Z(\alpha + \theta_2)\} \alpha x e^{\alpha t_1}}{\{a + \hat{W}(\alpha + \theta_1)\} \{a + Z(\alpha + \theta_2)\}} \right] f(x) dx$$

$$+ C \int_S^\infty \left[ 1 - \frac{\alpha x e^{\alpha t_1} (a + \alpha W^0)}{\{a + W^0(\alpha + \theta_1)\} \{a + Z(\alpha + \theta_2)\}} \right] f(x) dx$$

where

$$t_1 = t^* + t_w$$

$$\therefore C'(S, \theta_1, \theta_2) = C'_1(S) + C'_2(S) + C'_3(S) + D'(S) \quad \dots \dots (39)$$

The solution of the above equation gives the optimal value  $S^*$  of  $S$ .

However, the equation (37) is complex in nature and one can use numerical methods or search methods like Genetic Algorithm to obtain optimum value of  $S$  say  $S^*$ .

**Conclusion**

In this paper an attempt has been made to derive an analytical solution to the optimal order quantity when the demand increases exponentially with time. Whenever there is large demand and longer shortage period the use of RW is recommended. Both non-deteriorating and deteriorating items were considered. However, a closed form solution could not be obtained. Numerical illustration can be carried out by choosing appropriate hypothetical parameter values. The applicability of the model requires specific parametric quantification with different distributions, even with the simple density functions the cost function becomes complex in nature. In analysing such inventory systems, it is always challenging to estimate the demand distribution. Most of the researchers use parametric approaches for estimation of the demand distribution, usually a Gamma distribution, (C.F Moonen (1962), Burgin (1972), Das (1975), a lognormal distribution (Beckmann and Bobkoski (1958), Tadikamalla (1984)) and other distributions. Since with small data sets, it is difficult to reject a null hypothesis of any of the above standard distributions it is interesting to study a non-parametric approach. Strijbosch and Heuts (1992) have used parametric and non-parametric approximation for the lead time demand distributions and demonstrated that the Kernel density (Non-Parametric approach) estimation approach has considerable advantage over the parametric density estimation in an (Q, s) inventory model with a cost criterion. A separate simulation study on these lines will be carried out highlighting the computational convenient over parametric approach. The pertinent results and analysis will be communicated elsewhere. This suggestion will enhance the further scope of this research problem.

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