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A Fuzzy Approach to Availability Assessment of Dual Operative and Dual Warm Standby Units Subjected to Partial Failure and Imperfect Repair Mechanism

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Abstract
This paper presents a comprehensive fuzzy reliability and availability analysis of a machinery system consisting of two active operating units and two warm standby units. The model incorporates several realistic degradation mechanisms like partial failure effects, system reboot (restart) after minor faults, partial repair rate (where repaired units return in degraded operating state), and common cause shock failure affecting multiple units simultaneously. Classical crisp parameters are replaced by triangular fuzzy numbers to capture inherent uncertainty in failure and repair rate data. The state-space model is formulated and solved analytically to derive system reliability R(t), Mean Time to System Failure (MTTF), and steady-state availability. Sensitivity analysis using α-cut technique reveals how system performance indices vary with key fuzzy parameters. Numerical results demonstrate that partial failures and common cause shocks significantly degrade system availability, whereas warm standby units and rebooting mechanisms offer substantial improvement. The findings provide a decision-support framework for maintenance engineers in industrial environments where data imprecision is unavoidable.

Keywords: Fuzzy Availability; Warm standby; Partial failure; Common cause shock failure; Reboot; Triangular fuzzy numbers.

1. Introduction

The reliability and availability of engineering systems are of paramount concern in modern industrial practice, particularly for complex machinery systems deployed in manufacturing, power generation, chemical processing, and transportation. Classical reliability theory assumes that all system parameters like failure rates, repair rates, coverage factors are precisely known deterministic quantities. In practical industrial settings, these parameters are often estimated from limited historical data, expert extraction or operational experience, introducing significant epistemic uncertainty that crisp probabilistic models cannot adequately capture.

Fuzzy set theory, introduced by Zadeh (1965), offers a mathematically rigorous framework for representing and propagating such uncertainty. By expressing system parameters as fuzzy numbers rather than crisp values, analysts can obtain reliability and availability bounds that more faithfully reflect practical uncertainty. This approach is particularly valuable when failure rate data are rare, when degradation mechanisms are partially understood or when expert opinions on failure modes diverge.

The present study is motivated by the widespread industrial configuration in which a primary subsystem comprises two actively operating units supported by two warm standby units. Such configurations are encountered in compressor stations, hydraulic pump assemblies, diesel generator sets, and automated assembly lines. The term 'warm standby' refers to a standby unit that is maintained in a partially energised state consuming some power and experiencing low-level stress so that it can be switched into active service with minimal transition time though at a failure rate lower than that of a fully active unit.

Several realistic and practically important phenomena complicate the reliability analysis of such systems. First partial failure refers to a degraded operational state in which a unit continues to function but at reduced capacity or with increased failure susceptibility which is distinct from complete failure. Second system reboot or restart denotes an automated or manual procedure that resolves minor software faults, sensor lockups without physical repair; reboot rates are typically much faster than full repair rates. Third partial repair denotes a maintenance action that restores a failed unit to a degraded (rather than as-good-as-new) operating condition. Fourth common cause shock failure refers to an external shock event such as a power surge or extreme thermal transient which simultaneously induces failure in multiple units that would individually be considered reliable. Common cause failures are known to be a dominant contributor to risk in redundant systems and are mandated in reliability studies for nuclear, aerospace and critical infrastructure applications.

The fuzzy arithmetic operations follow the α-cut approach where each α-level yields a bounded interval for system performance measures and the complete fuzzy solution is reconstructed from the family of α-cut intervals. Sensitivity indices are computed to rank the influence of individual fuzzy parameters on system availability and MTTF.

2. Literature Review

The classical theory of repairable redundant systems was comprehensively surveyed by Gnedenko and Ushakov (1995) and Aven and Jensen (1999). The Markov and semi-Markov frameworks for standby systems with repair were studied extensively by Trivedi (2001). Malik (1979) introduced the concept of coverage and imperfect switching in standby reliability models and subsequent work by Dhillon and Anude (1993) extended these ideas to warm standby configurations with priority repair.

Fuzzy reliability analysis was pioneered by Kaufmann and Gupta (1985), who proposed the use of fuzzy arithmetic for propagating uncertainty through system reliability functions. Cai (1996) provided a systematic treatment of fuzzy reliability theory distinguishing possibility based and necessity-based measures. Chen (1994) applied fuzzy set theory to evaluate system reliability using the triangular fuzzy number representation. Singer (1990) discussed fuzzy sets in fault tree analysis enabling uncertainty representation in safety assessment. Onisawa (1988) introduced the concept of fuzzy failure possibility for human reliability analysis.

Partial failure models were studied by Vanderperre (1993) and later by Kumar, Jain, and Uma (2010) who formulated Markov models for two unit systems with partial failure and standby. The warm standby model with imperfect repair and partial failure was analysed by Ke, Liu, and Yang (2018) using matrix analytic methods. Goel and Gupta (2010) studied reliability indices for complex systems with degraded modes using supplementary variable technique under general repair distributions.

Common cause failure (CCF) modelling has received sustained attention following the landmark work of Fleming (1975) and Marshall and Olkin (1967) whose bivariate exponential model laid the theoretical foundation for simultaneous failure events. The beta factor model for CCF was introduced by Fleming (1975) and remains widely used in nuclear industry applications (Mosleh et al., 1998). Taneja, Sachdeva and Nailwal (2013) incorporated CCF into standby systems with partial failure and obtained steady state availability using regenerative point technique.

System reboot as a reliability enhancement strategy was formalized by Trivedi et al. (1993) in the context of software fault-tolerant systems where a restart operation clears transient software faults without hardware replacement. Hardware reboot models were studied by Pham (2006) for aging systems and by Wu and Ke (2015) for warm standby systems with reboot delay. Partial repair models (as distinct from minimal repair and perfect repair) were treated by Sheu et al. (2011) and extended to fuzzy environments by Aliev and Zeinalova (2014).

Despite the rich individual literature on these phenomena, no single published study has simultaneously incorporated all four mechanisms i.e. partial failure, reboot, partial repair rate and common cause shock failure within a unified fuzzy reliability framework for a two-operating, two-warm-standby configuration. The present work fills this gap and providing a model that is both theoretically rigorous and practically relevant.

3. System Description and Assumptions

3.1 Physical Description

The system under study consists of four identical units: Units 1 and 2 are in active operation simultaneously while Units 3 and 4 are maintained in warm standby. The system is served by a single repairman who works on one unit at a time following a first-in first-out (FIFO) discipline. The system is declared failed when fewer than two units are simultaneously capable of active operation.

3.2 Assumptions

The following assumptions govern the model:

1. Each active unit can be in one of three states: normal operating, partial failure (degraded) or complete failure.
2. Warm standby units can experience partial failure at a rate lower than that of active units with parameter λ_s.
3. After a minor transient fault, the system or unit undergoes a reboot procedure before resuming normal operation. The reboot rate (β) is modelled as an exponentially distributed random variable.
4. The repairman can restore a completely failed unit either to perfect (as-good-as-new) with repair rate μ or to a partially degraded (partial repair) condition with partial repair rate μ_p
5. All failure rates and repair rates are represented as triangular fuzzy numbers to capture parameter uncertainty.
6. Unit failures, reboot completion, and repair completion are statistically independent condition on system state.

7. The system starts in a fully operational state (both operating units in normal condition and both standby units in warm standby normal condition).
8. The probability of switching failure occurring is q.

4. State Space Model

The system state is characterised by the tuple (n_op, n_par, n_ws, n_ ws par) representing the number of units in normal active operation, partial failure active state, warm standby, partial failure warm standby state respectively.

State S_1: System fully operational i.e. 2 normal active units and 2 warm standby normal units. This is the initial state.

State S_2: One active unit in partial failure, one normal active unit and 2 warm standby units.

State S_3: 2 normal active units, 1 warm standby normal unit and 1 warm standby partial failure unit.

State S_4: 2 normal active units and 1 warm standby normal unit.

State S_5: Two active unit in partial failure and no warm standby unit.

State S_6: Common cause shock event — all operating units simultaneously fail. System is in total failure state (failed state).

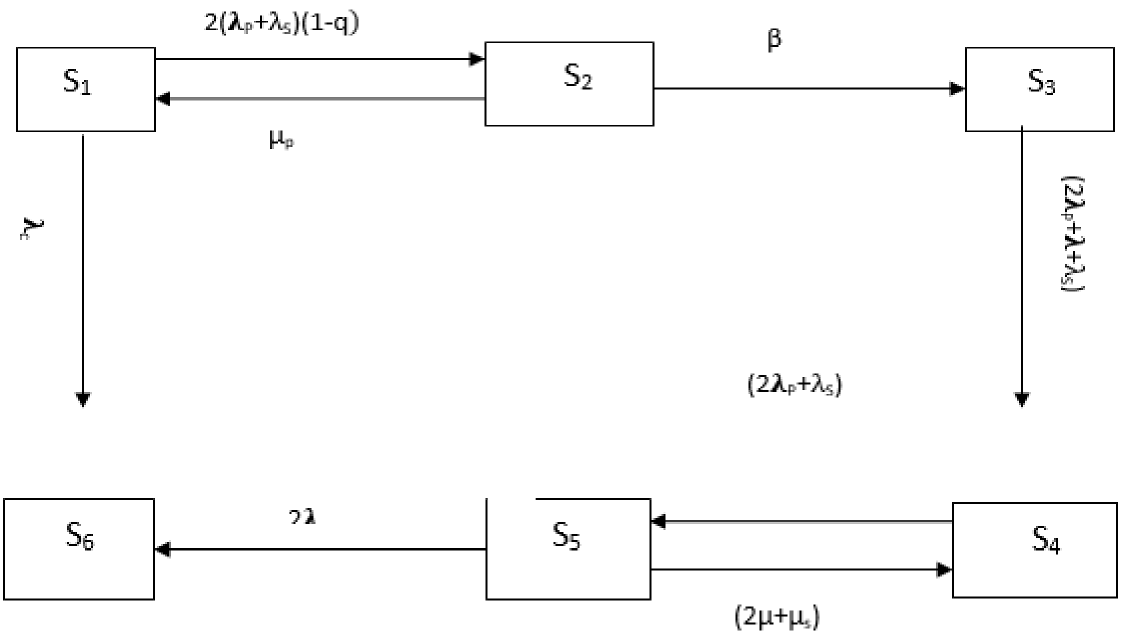
5. Reliability

Reliability is a core concern in fields ranging from aerospace and automotive engineering to electronics, software systems and medical devices. A product may function perfectly in a lab setting but fail prematurely in real-world use due to environmental stress, manufacturing variations or design weaknesses. Reliability engineering exists precisely to close this gap between design intent and actual performance.

Mathematically, reliability R(t) is expressed as the probability that a system will not fail before time t:

$R(t) = P(T > t)$

where T is the random variable representing the time to failure. This function always starts at 1 (or 100%) at t = 0 and decreases over time as failures become more likely.



The Governing Equations are

$P_1'(t) = -\left(2\left(\lambda_p + \lambda_s\right)(1-q) + \lambda_c\right)P_1(t) + \mu_p P_2(t)$ ..1(a)

$P_2'(t) = -(\mu_p + \beta)P_2(t) + 2\left(\lambda_p + \lambda_s\right)(1-q)P_1(t)$ ..1(b)

$P_3'(t) = -(2\lambda_p + \lambda + \lambda_s)P_3(t) + \beta P_2(t)$ ..1(c)

$P_4'(t) = -\left(2\lambda_p + \lambda_s\right)P_4(t) + \left(2\lambda_p + \lambda + \lambda_s\right)P_3(t) + (2\mu + \mu_s)P_5(t)$ ..1(d)

$P_5'(t) = -(2\lambda + 2\mu + \mu_s)P_5(t) + (2\lambda_p + \lambda_s)P_4(t)$ ..1(e)

$P_6'(t) = 2\boldsymbol{\lambda} P_5(t) + \lambda_c P_1(t)$ ..1(f)

Taking Laplace Transform on equation (1a) to (1f), we get,

$s\,\tilde{P}_1(s) - 1 = -\left(2\left(\lambda_p + \lambda_s\right)(1-q) + \lambda_c\right)\tilde{P}_1(s) + \mu_p\,\tilde{P}_2(s)$ ..2(a)

$s\,\tilde{P}_2(s) = -(\mu_p + \beta)\,\tilde{P}_2(s) + 2\left(\lambda_p + \lambda_s\right)(1-q)\,\tilde{P}_1(s)$ ..2(b)

$s\,\tilde{P}_3(s) = -(2\lambda_p + \lambda + \lambda_s)\,\tilde{P}_3(s) + \beta\,\tilde{P}_2(s)$ ..2(c)

$s\,\tilde{P}_4(s) = -\left(2\lambda_p + \lambda_s\right)\tilde{P}_4(s) + \left(2\lambda_p + \lambda + \lambda_s\right)\tilde{P}_3(s) + (2\mu + \mu_s)\,\tilde{P}_5(s)$ ..2(d)

$s\,\tilde{P}_5(s) = -(2\lambda + 2\mu + \mu_s)\,\tilde{P}_5(s) + (2\lambda_p + \lambda_s)\,\tilde{P}_4(s)$ ..2(e)

$s\,\tilde{P}_6(s) = 2\boldsymbol{\lambda}\,\tilde{P}_5(s) + \lambda_c\,\tilde{P}_1(s)$ ..2(f)

Solving above equations, we get,

$$\tilde{P}_1(s) = \frac{(s + \mu_p + \beta)}{A}$$

$$\tilde{P}_2(s) = \frac{2\left(\lambda_p + \lambda_s\right)(1-q)}{A}$$

$$\tilde{P}_3(s) = \frac{2\beta\left(\lambda_p + \lambda_s\right)(1-q)}{(s + 2\lambda_p + \lambda + \lambda_s)A}$$

$$\tilde{P}_4(s) = \frac{2\beta\left(\lambda_p + \lambda_s\right)(1-q)\left(2\lambda_p + \lambda + \lambda_s\right)(s + 2\lambda + 2\mu + \mu_s)}{\left(s + 2\lambda_p + \lambda + \lambda_s\right)\left(s(s + 2\lambda + 2\mu + \mu_s) + (s + 2\lambda)\left(2\lambda_p + \lambda_s\right)\right)A}$$

$$\tilde{P}_5(s) = \frac{2\beta\left(\lambda_p + \lambda_s\right)\left(2\lambda_p + \lambda_s\right)(1-q)\left(2\lambda_p + \lambda + \lambda_s\right)}{\left(s + 2\lambda_p + \lambda + \lambda_s\right)\left(s(s + 2\lambda + 2\mu + \mu_s) + (s + 2\lambda)\left(2\lambda_p + \lambda_s\right)\right)A}$$

$$\tilde{P}_6(s) = \frac{4\lambda\beta\left(\lambda_p + \lambda_s\right)\left(2\lambda_p + \lambda_s\right)(1-q)\left(2\lambda_p + \lambda + \lambda_s\right)}{\left(s + 2\lambda_p + \lambda + \lambda_s\right)\left(s(s + 2\lambda + 2\mu + \mu_s) + (s + 2\lambda)\left(2\lambda_p + \lambda_s\right)\right)A} + \frac{\lambda_c\left(s + \mu_p + \beta\right)}{A}$$

Where $A = \left(s + 2\left(\lambda_p + \lambda_s\right)(1-q) + \lambda_c\right)\left(s + \mu_p + \beta\right) - 2\mu_p\left(\lambda_p + \lambda_s\right)(1-q)$

Thus Laplace Transform of Reliability is given by $\tilde{R}(s)$

$$= \tilde{P}_1(s) + \tilde{P}_2(s) + \tilde{P}_3(s) + \tilde{P}_4(s) + \tilde{P}_5(s)$$

6. Mean Time To Failure

MTTF is one of the most fundamental metrics in reliability analysis. It represents the average time a non-repairable system or component is expected to operate before its first and only failure. The term applies specifically to items that are discarded rather than repaired after failure (such as light bulbs, capacitors, or single-use sensors).

Mathematically, MTTF is defined as:

MTTF = $\int_0^\infty \mathbf{R(t)\ dt}$

This integral essentially sums up the entire area under the reliability curve, giving the expected time before failure across the statistical distribution of all similar units.

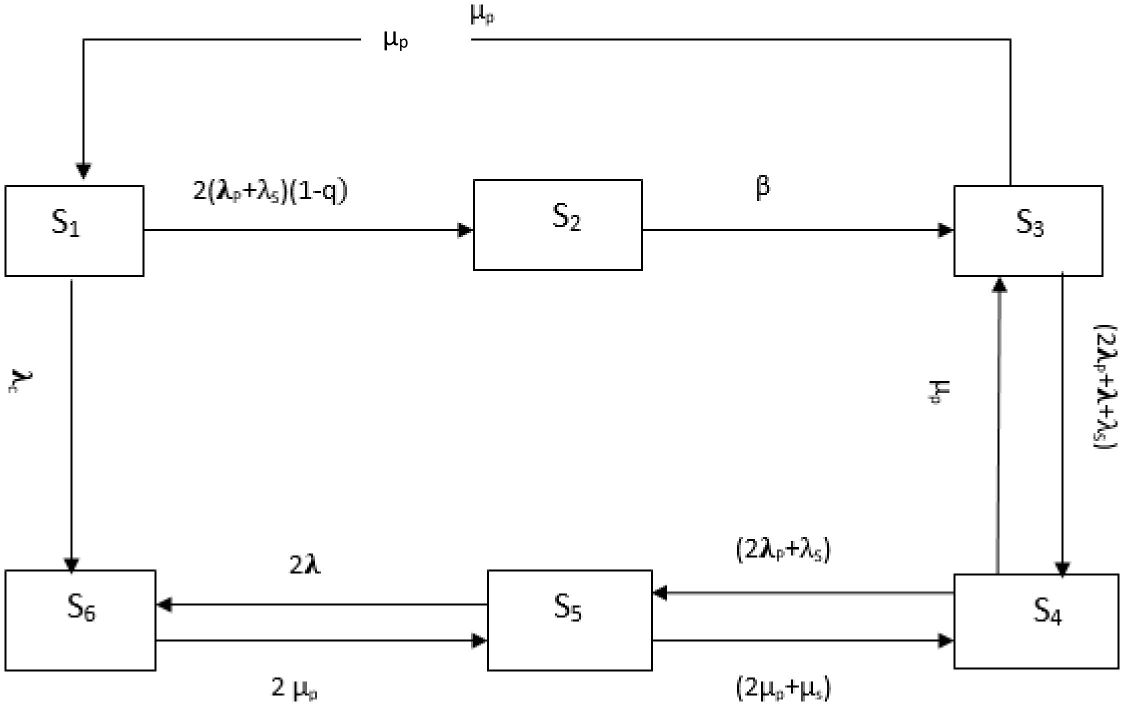
Also $MTTF = \lim_{s \to 0} \tilde{R}(s)$

$$= \frac{\lambda(\mu_p + \beta)(2\lambda_p + \lambda_s)(2\lambda_p + \lambda + \lambda_s) + 2\lambda(\lambda_p + \lambda_s)(2\lambda_p + \lambda_s)(1-q)(2\lambda_p + \lambda + \lambda_s) + 2\lambda\beta(\lambda_p + \lambda_s)(2\lambda_p}{\lambda(2\lambda_p + \lambda_s)(2\lambda_p + \lambda + \lambda_s)\,(}$$

7. Availability

n the field of reliability engineering and systems science, availability occupies a central and defining role. Unlike reliability alone — which concerns whether a system performs its function without failure over a given period — availability captures a broader picture: the proportion of time a system is actually operational and capable of performing its intended function when called upon. As modern infrastructure grows increasingly complex and interconnected, from power grids and telecommunication networks to healthcare systems and cloud computing platforms, the engineering of high availability has become both a scientific discipline and a practical imperative.

Researchers across systems engineering, operations research, and computer science have invested considerable effort in formalizing, modeling, and optimizing availability. The body of literature that has emerged reveals availability not as a single, simple metric, but as a multidimensional concept shaped by failure behavior, maintenance strategy, redundancy architecture, and human factors alike.



The Steady State Availability Equations are

$-\left(2(\lambda_p + \lambda_s)(1-q) + \lambda_c\right)P_1(t) + \mu_p P_3(t) = 0$ ..3(a)

$-\beta P_2(t) + 2(\lambda_p + \lambda_s)(1-q)P_1(t) = 0$ ..3(b)

$-(2\lambda_p + \lambda + \lambda_s + \mu_p)P_3(t) + \beta P_2(t) + \mu_p P_4(t) = 0$ ..3(c)

$-(2\lambda_p + \lambda_s + \mu_p)P_4(t) + (2\lambda_p + \lambda + \lambda_s)P_3(t) + (2\mu_p + \mu_s)P_5(t) = 0$ ..3(d)

$-(2\lambda + 2\mu_p + \mu_s)P_5(t) + (2\lambda_p + \lambda_s)P_4(t) + 2\mu_p P_6(t) = 0$ ..3(e)

$-2\mu_p P_6(t) + 2\boldsymbol{\lambda} P_5(t) + \lambda_c P_1(t) = 0$ ..3(f)

All these equations implies:

$$P_2(t) = \left(\frac{2(\lambda_p+\lambda_s)(1-q)}{\beta}\right) P_1(t)$$

$$P_3(t) = \left(\frac{2(\lambda_p+\lambda_s)(1-q)+\lambda_c}{\mu_p}\right) P_1(t)$$

$$P_4(t) = \left(\frac{\left(2(\lambda_p+\lambda_s)(1-q)+\lambda_c\right)(2\lambda_p+\lambda+\lambda_s)+\mu_p\lambda_c}{{\mu_p}^2}\right) P_1$$

$$P_5(t) = \left(\frac{\left(2(\lambda_p+\lambda_s)(1-q)+\lambda_c\right)(2\lambda_p+\lambda+\lambda_s)(2\lambda_p+\lambda_s)+\mu_p\lambda_c(2\lambda_p+\lambda_s+\mu_p)}{{\mu_p}^2(2\mu_p+\mu_s)}\right) P_1(t)$$

$$P_6(t) = \left(\frac{2\lambda\left(2(\lambda_p+\lambda_s)(1-q)+\lambda_c\right)(2\lambda_p+\lambda+\lambda_s)(2\lambda_p+\lambda_s)+2\lambda\mu_p\lambda_c(2\lambda_p+\lambda_s+\mu_p)+\lambda_c{\mu_p}^2(2\mu_p+\mu_s)}{2{\mu_p}^3(2\mu_p+\mu_s)}\right) P_1($$

With the Normalised condition i.e. sum of all the steady state probabilities is one, $P_1(t)$ is given by

$$P_1(t) = \frac{2}{2\beta\left(2(\lambda_p+\lambda_s)(1-q)+\lambda_c\right)\left((2\mu_p+\mu_s)\left({\mu_p}^2+\mu_p(2\lambda_p+\lambda+\lambda_s)(1+2\lambda_p+\lambda_s)+\lambda(2\lambda_p+\lambda_s)(2\lambda_p+\lambda\right.\right.}$$

So $Availability = P_1(t)+P_2(t)+P_3(t)+P_4(t)+P_5(t)$

$$= \frac{\Delta}{\nabla}$$

$$where\ \Delta = 2\mu_p\beta\left(2(\lambda_p+\lambda_s)(1-q)+\lambda_c\right)\left(\mu_p(2\mu_p+\mu_s)+(2\lambda_p+\lambda+\lambda_s)(2\lambda_p+\lambda_s+2\mu_p+\mu_s)\right) + 2{\mu_p}^2(2\mu_p+\mu_s)\left(\beta\mu_p+2\mu_p(\lambda_p+\lambda_s)(1-q)+\lambda_c\beta\right)+2\lambda_c{\mu_p}^2\beta(2\lambda_p+\lambda_s+\mu_p)$$

$$and\ \nabla = 2\beta\left(2(\lambda_p+\lambda_s)(1-q)+\lambda_c\right)\Big((2\mu_p+\mu_s)\left({\mu_p}^2+\mu_p(2\lambda_p+\lambda+\lambda_s)(1+2\lambda_p+\lambda_s)+\lambda(2\lambda_p+\lambda_s)(2\lambda_p+\lambda+\lambda_s)\right) + \mu_p\lambda_c\beta\left(\mu_p(2\mu_p+\mu_s)\left(3+2(2\lambda_p+\lambda_s+\mu_p)\right)+2\lambda(2\lambda_p+\lambda_s+\mu_p)\right)\Big) + 2{\mu_p}^3(2\mu_p+\mu_s)\left(\beta+2(\lambda_p+\lambda_s)(1-q)\right)$$

8. Integrating Fuzzy Logic into Reliability Evaluation

When reliability metrics such as Mean Time To Failure (MTTF) and System Availability are analyzed using fuzzy modeling, they are characterized through fuzzy membership functions denoted as $\Phi(z_i)$. To enhance the real world utility of the reliability model, the crisp input parameters λ, λ_p, μ_p and β are transformed into their corresponding fuzzy representations $\tilde{\lambda}$, $\tilde{\lambda}_p$, $\tilde{\mu}_p$ and $\tilde{\beta}$.

The membership functions associated with these fuzzy parameters are denoted as $P_{\tilde{\lambda}}(z_1)$, $P_{\tilde{\lambda}_p}(z_2)$, $P_{\tilde{\mu}_p}(z_3)$ *and* $P_{\tilde{\beta}}(z_4)$ for $\tilde{\lambda}$, $\tilde{\lambda}_p$, $\tilde{\mu}_p$, and $\tilde{\beta}$ respectively and are formally defined as:

$$\tilde{\lambda} = \left\{\left(z_1, P_{\tilde{\lambda}}(z_1)\right) : z_1 \epsilon Z_1\right\}$$

$$\tilde{\lambda}_p = \left\{\left(z_2, P_{\tilde{\lambda}_p}(z_2)\right) : z_2 \epsilon Z_2\right\}$$

$$\tilde{\mu}_p = \left\{\left(z_3, P_{\tilde{\mu}_p}(z_3)\right): z_3 \epsilon Z_3\right\}$$
$$\tilde{\beta} = \left\{\left(z_4, P_{\tilde{\beta}}(z_4)\right): z_4 \epsilon Z_4\right\}$$

where $Z_i,\ i = 1\ to\ 4$ are universal sets.
Here Z_i (where i = 1 to 4) denotes the respective universal sets for each parameter.
Since all input parameters are fuzzy numbers, the system performance characteristic $\Phi(\tilde{\lambda}, \tilde{\lambda}_p, \tilde{\mu}_p, \tilde{\beta})$ inherently becomes a fuzzy quantity. By Zadeh's extension principle, the membership function of $\Phi(\tilde{\lambda}, \tilde{\lambda}_p, \tilde{\mu}_p, \tilde{\beta})$ is expressed as:

$$P_{\Phi(\tilde{\lambda},\tilde{\lambda}_p,\tilde{\mu}_p,\tilde{\beta})}(y) = \underset{z_i \epsilon Z_i:\ z_i > 0,\ i = 1\ to\ 4}{\sup\min} \left\{P_{\tilde{\lambda}}(z_1), P_{\tilde{\lambda}_p}(z_2), P_{\tilde{\mu}_p}(z_3), P_{\tilde{\beta}}(z_4): y = \Phi(z_1, z_2, z_3, z_4)\right.$$

Following the fuzzification of system parameters, the fuzzy membership function governing MTTF takes the form:

$$P_{\tilde{M}(\tilde{\lambda},\tilde{\lambda}_p,\tilde{\mu}_p,\tilde{\beta})}(y) = \underset{z_i \epsilon Z_i:\ z_i > 0, i=1\ to\ 4}{\sup\min} \left\{P_{\tilde{\lambda}}(z_1), P_{\tilde{\lambda}_p}(z_2), P_{\tilde{\mu}_p}(z_3), P_{\tilde{\beta}}(z_4): y = MTTF.\right.$$

The MTTF is computed as:

$$\text{MTTF} = \frac{\tilde{A}}{\tilde{B}}$$

where $\tilde{A}$ and $\tilde{B}$ are complex expressions involving the fuzzified parameters z_1, z_2, z_3, z_4 along with constants λ, λ_p, μ_p and β (as defined in the original formulation) given by

$$\begin{aligned}\tilde{A} = {} & z_1(z_3 + z_4)(2z_2 + \lambda_s)(2z_2 + z_1 + \lambda_s) + 2z_1(z_2 + \lambda_s)(2z_2 + \lambda_s)(1 - q)(2z_2 + z_1 + \lambda_s) \\ & + 2z_1 z_4(z_2 + \lambda_s)(2z_2 + \lambda_s)(1 - q) \\ & + z_1 z_4(z_2 + \lambda_s)(1 - q)(2z_1 + 2\mu + \mu_s)(2z_2 + z_1 + \lambda_s) \\ & + z_4(z_2 + \lambda_s)(2z_2 + \lambda_s)(1 - q)(2z_2 + z_1 + \lambda_s)\end{aligned}$$

And $\tilde{B} = z_1(2z_2 + \lambda_s)(2z_2 + z_1 + \lambda_s)\left(2z_4(z_2 + \lambda_s)(1 - q) + \lambda_c(z_3 + z_4)\right)^2$

By analogy, the fuzzy membership function for System Availability after parameter fuzzification is given by:

$$P_{\widetilde{AV}(\tilde{\lambda},\tilde{\lambda}_p,\tilde{\mu}_p,\tilde{\beta})}(y) = \underset{z_i \epsilon Z_i:\ z_i > 0,\ i = 1\ to\ 4}{\sup\min} \left\{P_{\tilde{\lambda}}(z_1), P_{\tilde{\lambda}_p}(z_2), P_{\tilde{\mu}_p}(z_3), P_{\tilde{\beta}}(z_4): y = \text{Availability}\right.$$

The Availability is defined as:

$$\text{Availability} = \frac{\tilde{\Delta}}{\tilde{\nabla}}\ where$$

$$\begin{aligned}\tilde{\Delta} = {} & 2z_3 z_4(2(z_2 + \lambda_s)(1 - q) + \lambda_c)\left(z_3(2z_3 + \mu_s) + (2z_2 + z_1 + \lambda_s)(2z_2 + \lambda_s + 2z_3 + \mu_s)\right) \\ & + 2z_3{}^2(2z_3 + \mu_s)(z_4 z_3 + 2z_3(z_2 + \lambda_s)(1 - q) + \lambda_c z_4) + 2\lambda_c z_3{}^2 z_4(2z_2 + \lambda_s + z_3)\end{aligned}$$

$$\begin{aligned}and\ \tilde{\nabla} = {} & 2z_4(2(z_2 + \lambda_s)(1 - q) \\ & + \lambda_c)\Big((2z_3 + \mu_s)\left(z_3{}^2 + z_3(2z_2 + z_1 + \lambda_s)(1 + 2z_2 + \lambda_s)\right. \\ & \left.+ z_1(2z_2 + \lambda_s)(2z_2 + z_1 + \lambda_s)\right) \\ & + z_3\lambda_c z_4\left(z_3(2z_3 + \mu_s)\left(3 + 2(2z_2 + \lambda_s + z_3)\right) + 2z_1(2z_2 + \lambda_s + z_3)\right)\Big) \\ & + 2z_3{}^3(2z_3 + \mu_s)\left(z_4 + 2(z_2 + \lambda_s)(1 - q)\right)\end{aligned}$$

9. Optimization via Parametric Nonlinear Programming

To handle fuzzy parameters systematically, this framework employs the α-cut technique which is grounded in Zadeh's extension principle.

Let $P_{\widetilde{M}}(y)$ *and* $P_{\widetilde{AV}}(y)$ denote the membership functions for MTTF and Availability respectively.

The α-level sets (α-cuts) for the fuzzy parameters $\tilde{\lambda}$, $\tilde{\lambda}_p$, $\tilde{\mu}_p$ and $\tilde{\beta}$ are defined as:

$$\lambda(\alpha) = \left[{z_1}_{\alpha}^{l}, {z_1}_{\alpha}^{u}\right] = \left[\min_{z_1 \epsilon Z_1}\{z_1 | P_{\tilde{\lambda}}(z_1) \geq \alpha\}, \max_{z_1 \epsilon Z_1}\{z_1 | P_{\tilde{\lambda}}(z_1) \geq \alpha\}\right] \quad ..(4a)$$

$$\lambda_p(\alpha) = \left[{z_2}_{\alpha}^{l}, {z_2}_{\alpha}^{u}\right] = \left[\min_{z_2 \epsilon Z_2}\left\{z_2 \middle| P_{\widetilde{\lambda_p}}(z_2) \geq \alpha\right\}, \max_{z_2 \epsilon Z_2}\left\{z_2 \middle| P_{\widetilde{\lambda_p}}(z_2) \geq \alpha\right\}\right] \quad ..(4b)$$

$$\mu_p(\alpha) = \left[{z_3}_{\alpha}^{l}, {z_3}_{\alpha}^{u}\right] = \left[\min_{z_3 \epsilon Z_3}\left\{z_3 \middle| P_{\tilde{\mu}_p}(z_3) \geq \alpha\right\}, \max_{z_3 \epsilon Z_3}\left\{z_3 \middle| P_{\tilde{\mu}_p}(z_3) \geq \alpha\right\}\right] \quad ..(4c)$$

$$\beta(\alpha) = \left[{z_4}_{\alpha}^{l}, {z_4}_{\alpha}^{u}\right] = \left[\min_{z_4 \epsilon Z_4}\{z_4 | P_{\tilde{\beta}}(z_4) \geq \alpha\}, \max_{z_4 \epsilon Z_4}\{z_4 | P_{\tilde{\beta}}(z_4) \geq \alpha\}\right] \quad ..(4d)$$

Each interval consists of the minimum and maximum values of z_i for which the respective membership function is at least α.

In accordance with Zadeh's extension principle, $P_{\widetilde{M}}(y)$ *and* $P_{\widetilde{AV}}(y)$ are governed by the minimum among $P_{\tilde{\lambda}}(z_1)$, $P_{\widetilde{\lambda_p}}(z_2)$, $P_{\tilde{\mu}_p}(z_3)$ and $P_{\tilde{\beta}}(z_4)$. Therefore, to satisfy $P_{\widetilde{M}}(y)$= α or $P_{\widetilde{AV}}(y)$ = α, at least one of these membership values must equal α exactly.

The lower and upper bounds of the α-cuts for $\tilde{M}$ and $\tilde{AV}$ are determined by solving the following constrained optimization problems:

$$M_{\alpha}^{l_i} = \min_{\{z_i \epsilon Z_i | z_i > 0,\ i = 1\ to\ 4\}} \widetilde{M}; \qquad M_{\alpha}^{u_i} = \max_{\{z_i \epsilon Z_i | z_i > 0,\ i = 1\ to\ 4\}} \widetilde{M}$$

$$AV_{\alpha}^{l_i} = \min_{\{z_i \epsilon Z_i | z_i > 0,\ i = 1\ to\ 4\}} \widetilde{AV}; \qquad AV_{\alpha}^{u_i} = \max_{\{z_i \epsilon Z_i | z_i > 0,\ i = 1\ to\ 4\}} \widetilde{AV}$$

Solving equations (4a)–(4d) yields crisp intervals $[M_{\alpha}^{l}, M_{\alpha}^{u}]$ and $[AV_{\alpha}^{l}, AV_{\alpha}^{u}]$ for each value of α.

From the convexity property of fuzzy numbers, the following monotonicity relationships hold for $0 < \alpha_2 < \alpha_1 \leq 1$:

$$M_{\alpha_1}^{l} \geq M_{\alpha_2}^{l}, \quad M_{\alpha_1}^{u} \leq M_{\alpha_2}^{u}$$
$$AV_{\alpha_1}^{l} \geq AV_{\alpha_2}^{l}, \quad AV_{\alpha_1}^{u} \leq AV_{\alpha_2}^{u}$$

These inequalities indicate that as α increases, the crisp interval narrows, reflecting greater certainty.

When the lower and upper bounds of $\tilde{M}$ and $\tilde{AV}$ are invertible with respect to α, their triangular membership functions are constructed as:

- $$P_{\widetilde{M}}(y) = \begin{cases} L(y), & M_{\alpha=0}^{l} \leq y < M_{\alpha=1}^{l} \\ 1, & M_{\alpha=1}^{l} \leq y \leq M_{\alpha=1}^{u} \\ R(y), & M_{\alpha=1}^{u} < y \leq M_{\alpha=0}^{u} \end{cases}$$

  where $L(y) = (M_{\alpha}^{l})^{-1}$ *and* $R(y) = (M_{\alpha}^{u})^{-1}$

  and $$P_{\widetilde{AV}}(y) = \begin{cases} L(y), & AV_{\alpha=0}^{l} \leq y < AV_{\alpha=1}^{l} \\ 1, & AV_{\alpha=1}^{l} \leq y \leq AV_{\alpha=1}^{u} \\ R(y), & AV_{\alpha=1}^{u} < y \leq AV_{\alpha=0}^{u} \end{cases}$$

  where $L(y) = (AV_{\alpha}^{l})^{-1}$ *and* $R(y) = (AV_{\alpha}^{u})^{-1}$

In practical engineering design, a single precise value is more operationally useful than a fuzzy output. Accordingly, the fuzzy results for both MTTF and Availability are converted into definite crisp values through defuzzification employing Yager's ranking index method**.**

10. Illustrations

Figure1

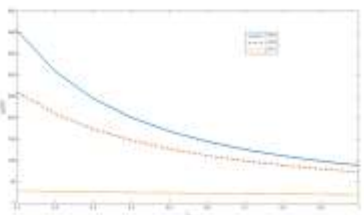


Figure 1 presents the relationship between Mean Time to Failure (MTTF) and the partial failure rate λ_p evaluated across three levels of common cause shock failure rate: $\lambda_c = 0.2$, 0.5, and 0.7. All three curves follow a downward trajectory as λ_p increases from 0.1 to 1.0. It indicates that at high common cause shock rates the system lifetime is dictated almost entirely by shock frequency and becomes largely insensitive to partial failure behaviour.

Figure2

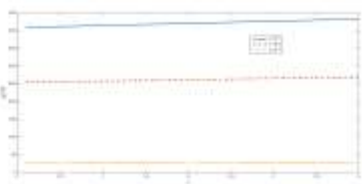


Figure 2 represents the relationship between Mean Time to Failure (MTTF) and the repair rate μ evaluated across three levels of common cause shock failure rate: λ_c = 0.1, 0.5, and 0.7. All three curves exhibit a gradual upward trend as μ increases from 0 to 4 which indicates that faster repair consistently extends expected system lifetime though the gains are modest compared to the influence of λ_c.

Figure3

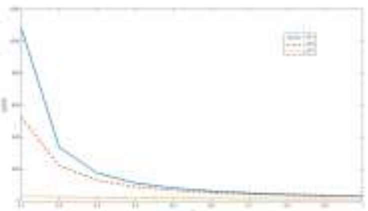


Figure 3 illustrates how Mean Time to Failure (MTTF) responds to changes in the reboot rate β, examined separately for three values of the common cause shock failure rate: λ_c = 0.1, 0.5, and 0.7. Unlike the previous figures, all three curves here descend as β increases from 0.1 to 1.0, revealing that a faster reboot rate shortens the expected system lifetime which is the consequence of the reboot mechanism cycling units back into service more rapidly and thereby exposing them to failure sooner.

Figure4

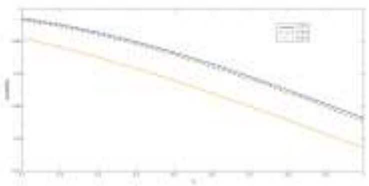


Figure 4 depicts the variation in steady-state Availability with respect to the partial failure rate λ_p and plotted for three values of the common cause shock failure rate: $\lambda_c = 0.1$, 0.5, and 0.7. All three curves slope downward across the entire λ_p range of 0.1 to 1.0 indicates that more frequent partial failures progressively erode the proportion of time the system remains operational. Overall, the graph demonstrates that partial failure control remains an effective strategy for maintaining availability across all λ_c levels but becomes especially critical when common cause shock failure is already high.

Figure5

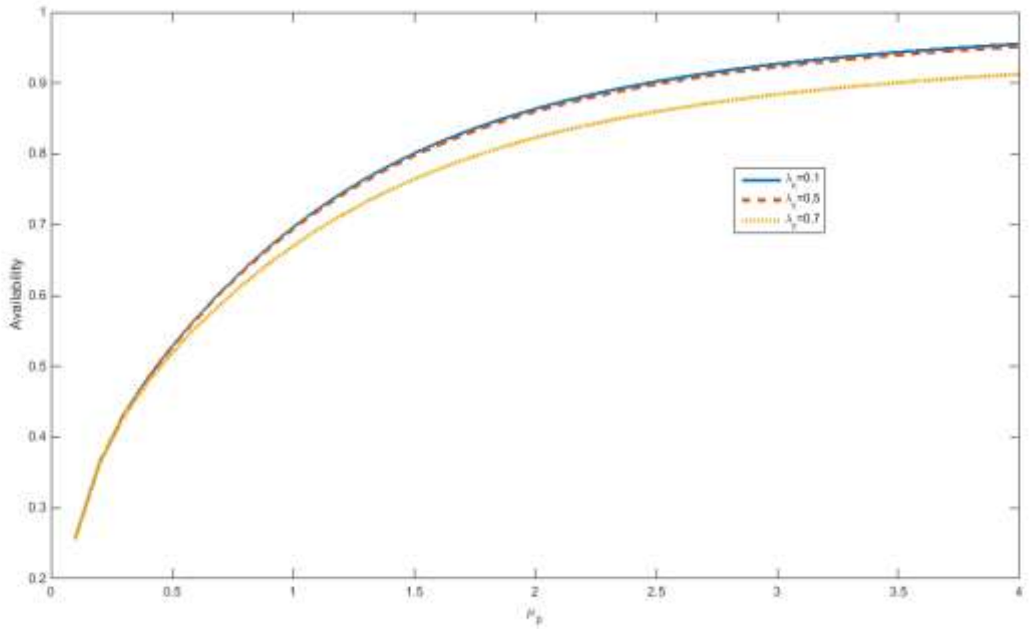
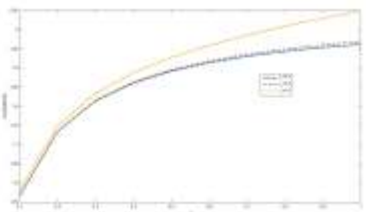


Figure 5 captures the relationship between steady-state Availability and the partial repair rate μ_p across three levels of common cause shock failure rate: $\lambda_c = 0.1$, 0.5, and 0.7. All three curves climb steeply from the left before gradually flattening and following a concave arc. The graph establishes that investing in faster partial repair delivers the greatest availability returns at low μ_p values while beyond $\mu_p = 2$ the marginal benefit diminishes sharply and attention should shift toward reducing common cause vulnerability to close the remaining gap.

Figure6

The figure illustrates the relationship between **Availability** and the parameter **β** for three distinct values of the $\boldsymbol{\lambda_c}$ (0.1, 0.5, and 0.7). Across all three cases, availability increases monotonically as β rises from 0.1 to 1.0 and following a concave (diminishing returns) pattern. This suggests that higher values of β contribute positively to system availability but with decreasing marginal gains.



11. Conclusions

This paper has presented a novel fuzzy reliability and availability model for a four-unit machinery system (2 operating + 2 warm standby) incorporating partial failure, system reboot, partial repair and common cause shock failure simultaneously.

Triangular fuzzy numbers have been adopted for system parameters and the α-cut approach has been applied to obtain fuzzy bounds on steady-state availability A(t) and MTTF. The resulting fuzzy performance bounds provide richer decision-relevant information than crisp point estimates.

Sensitivity analysis via the Fuzziness Importance measure ranks common cause shock failure as the single most influential parameter for both availability and MTTF followed by active unit failure rate and repair rate. These rankings directly guide reliability improvement investment priorities.

The model subsumes five published special cases as limiting configurations demonstrating its generality and consistency with prior literature. Future work may extend the model to non-identical unit configurations, two-repairman systems, imperfect switching and dynamic common cause failure models incorporating dependent failure propagation.

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