Development of an Enhanced Algorithm for solving the Assignment problem with Optimal Efficiency

S. Yoga[1], S. Abinaya[2]

[1]Assistant Professor of Mathematics, Sakthi College of Arts and Science for Women, Oddanchatram, Palani Main Road, Dindigul – 624 619, Tamil Nadu, India.

[2]PG Scholar, PG and Research Department of Mathematics, Sakthi College of Arts and Science for Women, Oddanchatram , Palani Main Road, Dindigul–624 619,TamilNadu, India.

ABSTRACT:

The assignment problem is a well-established mathematical model with significant applications in industrial operations and real-world optimization scenarios. The proposed approach follows a structured set of procedural steps designed for ease of implementation and computational efficiency. A numerical algorithm was developed to enhance the practical applicability of the method and the results were evaluated using two effective techniques. The first technique, known as the Subtract Column and Add-One Method, involves column-wise minimization followed by the strategic addition of one to guide optimal assignments. The second, more refined technique is the Subtract Row and Column and Add-One Method, where both column and row reductions are performed prior to applying the Add-one strategy. These methods were critically assessed for their consistency, accuracy and reliability, demonstrating the robustness and clarity of the proposed solution approach.

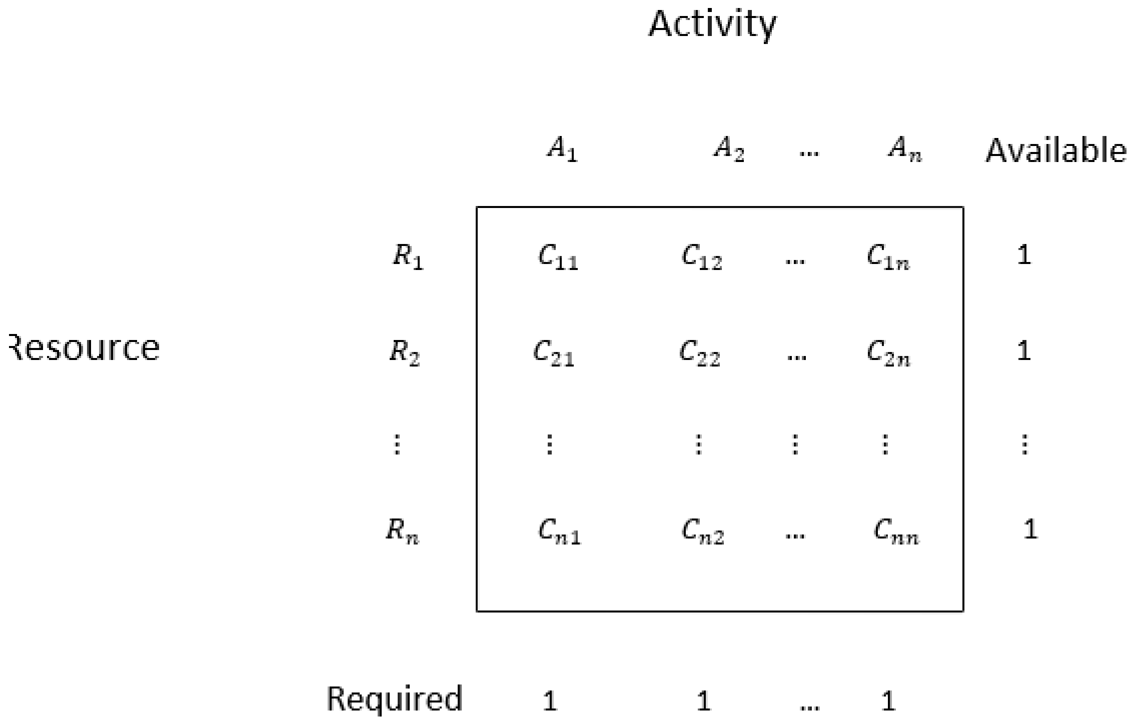
KEYWORDS: Assignment problem, Hungarian assignment method (HA-method), Divide column and Subtract one method, Proposed method 1 and 2, optimization

I. INTRODUCTION

The Assignment problem is a classical and widely studied optimization problem in the domain of operations research. It involves assigning a set of agents to a set of tasks in a one-to-one manner, such that each agent is assigned to exactly one task and each task is handled by exactly one agent. The objective is to either minimize the total cost or maximize the total profit or efficiency associated with the assignment. This problem arises in various real-world scenarios, including staff scheduling, machine-task allocation, transportation planning and resource management. Although well-established algorithms such as the Hungarian Method exist and provide optimal solutions, they may appear complex or computationally intensive, especially in educational or small-scale practical settings. To provide a simpler yet systematic approach, this study introduces two methods – the Subtract Column and Add-One Method and the Subtract Row and Column and Add-One Method. These techniques operate by reducing the cost matrix using column-wise and row-wise subtraction, followed by the application of an “Add-one” step to identify optimal zero positions and determine feasible assignments. The proposed methods aim to enhance clarity, reduce computational complexity and offer a user-friendly solution path, making them particularly suitable for beginners, students and educators. The algorithms and procedural steps of these methods are detailed in the subsequent sections.

II. FORMULATION OF THE ASSIGNMENT PROBLEM

The Assignment Problem is an important optimization model used in Operations Research to allocate tasks to available resources in the best possible way. The major aim of the problem is to find the most efficient assignment of n persons to n activities such that the overall cost of performing the tasks becomes minimum, or in some cases, the overall profit becomes maximum. In this model, every task must be given to only one person, and each person should handle only one task. The cost matrix (C_{ij}) is given as under:



Activity

$$\begin{array}{cc|cccc|c} & & A_1 & A_2 & \cdots & A_n & \text{Available} \\ \hline & R_1 & C_{11} & C_{12} & \cdots & C_{1n} & 1 \\ \text{Resource} & R_2 & C_{21} & C_{22} & \cdots & C_{2n} & 1 \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & R_n & C_{n1} & C_{n2} & \cdots & C_{nn} & 1 \\ \hline & \text{Required} & 1 & 1 & \cdots & 1 & \end{array}$$

Let Required:

- n be the number of agents (and tasks)
- C_{ij} be the cost of assigning agent i to task j
- X_{ij} be a decision variable, where

X_{IJ} = {1, if agent i is assigned to task j
0, otherwise}

Mathematical Formulation of the Assignment Problem

- **Objective Function:**

Minimize $Z = \sum_{i=1}^{n} \sum_{i=1}^{n} c_{ij} \, . \, X_{IJ}$

This function represents the total cost of assignment.

- **Subject to the constraints:**

1. Each agent is assigned to only one task:

$\sum_{J=1}^{n} x_{ij} = 1$ for i = 1,2,….n

2. Each task is assigned to only one agent:

$\sum_{i=1}^{n} x_{ij} = 1$ for j = 1,2,….n

3. Binary nature of decision variables:

$X_{ij} \in \{0,1\}$ for i, j

Notes:

- The problem assumes that the number of agents is equal to the number of tasks (i.e., a balanced assignment problem).
- If the problem is unbalanced, dummy rows or columns with zero cost can be added to make it balanced.

III. NEW APPROACH FOR SOLVING ASSIGNMENT PROBLEM

In this section, a new approach for solving the assignment problem is proposed using the Subtract Column and Add-One method and Subtract Row and Column and Add-One method. As an example, the traditional Hungarian Algorithm is used to demonstrate this technique. Additionally, the Divide Column and Subtract-One method is also presented with a separate example to highlight its effectiveness. These simplified approaches aim to reduce complexity and improve the clarity of solution steps. Presently we consider the assignment lattice where is the expense of allotting employment to machine.

Cost Matrix Example:

$$\begin{array}{c|cccc|} & 1 & 2 & \cdots & n \\ \hline 1 & C_{11} & C_{12} & \cdots & C_{1n} \\ 2 & C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n & C_{n1} & C_{n2} & \cdots & C_{nn} \\ \hline \end{array}$$

IV. COMPARISON OF EXISTING METHODS WITH PROPOSED METHOD

Proposed Method 1: Subtract Column and Add One Assignment Method

The proposed algorithm of proposed method is as follows:

Step 1: Form the cost matrix

Prepare the square cost matrix for the assignment problem.

If the matrix is not square, add dummy row or column to make it square.

Step 2: Subtract the minimum value in each column

For each column, find the minimum value and subtract it from every element in that column.

Step 3: Apply the Add one Method

Now add 1 to all elements and we get at least one in each row or column.

Step 4: Assign in terms of ones

Make assignment in terms of ones.

If there are some rows and columns without assignment then we cannot get the optimum solution. Then we go to the next step.

Step 5: Draw the minimum number of lines passing through all ones by using the following procedure:

1. Mark (✓) rows that do have assignments.
2. Mark (✓) columns that have crossed ones in that marked rows.
3. Mark (✓) rows that have assignments in marked columns.
4. Repeat (b) and (c) till no more rows or columns can be marked.
5. Draw straight lines through all unmarked rows and marked columns.

If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 6: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number, other numbers covered by lines remain unchanged. Then we get some new ones in rows and columns. Again make assignment in terms of ones.

Step7: If we cannot get the optimal assignment in each row and column. Then repeat (5)and(6) successively till an optimum solution is obtained.

1. Solve the following assignment problem using Proposed Method-1.

Find the assignment schedule time for the following table (4 × 3):

	I	II	III
A	9	26	15
B	13	27	6
C	35	20	15
D	18	30	20

Solution:

Step1: Here, number of rows not equal to number of columns.

So, add dummy column to balance matrix.

	I	II	III	IV
A	9	26	15	0
B	13	27	6	0
C	35	20	15	0

D	18	30	20	0

Step2: Select the minimum value and subtract it from every element in that column.

	I	II	III	IV
A	0	6	9	0
B	4	7	0	0
C	26	0	9	0
D	9	10	14	0

Step3: Add one to all elements.

	I	II	III	IV
A	1	7	10	1
B	5	8	1	1
C	27	1	10	1
D	10	11	15	1

Step4: Assign in terms of one.

	I	II	III	IV
A	$\boxed{1}$	7	10	$\cancel{1}$
B	5	8	$\boxed{1}$	$\cancel{1}$
C	27	$\boxed{1}$	10	$\cancel{1}$
D	10	11	15	$\boxed{1}$

Here we see that all ones are either assigned or crossed out.
That is, the total assigned ones is 4 which is equal to the number of rows or columns.
∴The optimum assignment is A→ I, B → III, C → II, D → IV

Hence the minimum total time of this assignment scheduled is,
Z = 9 + 6 + 20 + 0
Z = 35 hours.

Proposed Method 2: Subtract Row and Column and Add One Method

The proposed algorithm of proposed method is as follows:

Step 1: Form the cost matrix

Prepare the square cost matrix for the assignment problem. If the matrix is not square, add dummy row or column to make it square.

Step 2: Subtract the minimum value in each row

For each row, find the minimum value and subtract it from every element in that row.

Step 3: Subtract the minimum value in each column

For each column, find the minimum value and subtract it from every element in that column.

Step 4: Apply the Add-One Method

Now add 1 to all elements and we get at least one “1” in each row or column.

Step 5: Assign in terms of ones

Make assignment in terms of ones .If there are some rows and columns without assignment, then we cannot get the optimum solution. Then we go to the next step.

Step 6: Draw the minimum number of lines passing through all ones by using the following procedure

a) Mark (√) rows that do not have assignment.
b) Mark (√) columns that have crossed ones in that marked row.
c) Mark (√) rows that have assignment in marked column.
d) Repeat (b) and (c) till no more rows or columns can be marked.
e) Draw straight lines through all unmarked rows and marked columns.

If the number of lines drawn is equal to the number of rows or columns, then the current solution is optimal solution. Otherwise go to next step.

Step 7: Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered numbers by this smallest number. Other numbers covered by lines remain unchanged. Then we get some new ones in rows and columns. Again make assignment in terms of ones.

Step 8: If we cannot get the optimal assignment in each row and column, then repeat (6) and (7) successively till an optimum solution is obtained.

2. Solve the following assignment problem using Proposed Method-2

Find the assignment schedule time for the following table (4 × 3):

	I	II	III
A	9	26	15
B	13	27	6
C	35	20	15
D	18	30	20

Solution:

Step1: Here, number of rows not equal to number of columns.

So, add dummy column to balance matrix.

	I	II	III	IV
A	9	26	15	0
B	13	27	6	0
C	35	20	15	0
D	18	30	20	0

Step2: Select the minimum value and subtract it from every element in that row.

	I	II	III	IV
A	9	26	15	0
B	13	27	6	0
C	35	20	15	0
D	18	30	20	0

Step3: Select the minimum value and subtract it from every element in that column.

	I	II	III	IV
A	0	6	9	0
B	4	7	0	0
C	26	0	9	0
D	9	10	14	0

Step4: Add one to all elements.

	I	II	III	IV
A	1	7	10	1
B	5	8	1	1
C	27	1	10	1
D	10	11	15	1

Step5: Assign in terms of one.

	I	II	III	IV
A	$\boxed{1}$	7	10	$\cancel{1}$
B	5	8	$\boxed{1}$	$\cancel{1}$
C	27	$\boxed{1}$	10	$\cancel{1}$
D	10	11	15	$\boxed{1}$

Here we see that all ones are either assigned or crossed out.
That is, the total assigned ones is 4 which is equal to the number of rows or columns.
∴The optimum assignment is A→ I, B → III, C → II, D → IV
Hence the minimum total time of this assignment scheduled is,
Z = 9 + 6 + 20 + 0
Z = 35 hours.

3. Solving the following assignment problem using Hungarian Assignment Method.
Find the assignment schedule time for the following table 4 × 3:

	I	II	III
A	9	26	15
B	13	27	6
C	35	20	15
D	18	30	20

Solution:
Step1: Here, number of rows not equal to number of columns.
So, add dummy column to balance matrix.

	I	II	III	IV
A	9	26	15	0

B	13	27	6	0
C	35	20	15	0
D	18	30	20	0

Step2: Select the minimum value and subtract it from every element in that row.

	I	II	III	IV
A	9	26	15	0
B	13	27	6	0
C	35	20	15	0
D	18	30	20	0

Step3: Select the minimum value and subtract it from every element in that column.

	I	II	III	IV
A	0	6	9	0
B	4	7	0	0
C	26	0	9	0
D	9	10	14	0

Step4: Assign in terms of zeros.

	I	II	III	IV
A	$\boxed{0}$	6	9	0✕
B	4	7	$\boxed{0}$	0✕
C	26	$\boxed{0}$	9	0✕

D	9	10	14	$\boxed{0}$

Here we see that all ones are either assigned or crossed out.

That is, the total assigned ones is 4 which is equal to the number of rows or columns.

∴The optimum assignment is A→ I, B → III, C → II, D → IV

Hence the minimum total time of this assignment scheduled is,

$Z = 9 + 6 + 20 + 0$

$Z = 35$ hours.

4. Solving the following assignment problem using Divide column and subtract one method Assignment Method.

Find the assignment schedule time for the following table 4 × 3:

Solution:

Step1: Here, number of rows not equal to number of columns.

So, add dummy column to balance matrix.

	I	II	III	IV
A	9	26	15	0
B	13	27	6	0
C	35	20	15	0
D	18	30	20	0

Step2: Select the minimum element of each column. Then divide each column by its minimum elements.

	I	II	III	IV
A	1	1.3	2.5	0
B	1.44	1.35	1	0
C	3.88	1	2.5	0
D	2	1.5	3.33	0

Step3: Subtract 1 from all elements (expect zero).

	I	II	III	IV
A	0	0.3	1.5	0
B	0.44	0.35	0	0
C	2.88	0	1.5	0
D	1	0.5	2.33	0

Step4: Assign in terms of zeros.

	I	II	III	IV
A	$\boxed{0}$	0.3	1.5	$\xcancel{0}$
B	0.44	0.35	$\boxed{0}$	$\xcancel{0}$
C	2.88	$\boxed{0}$	1.5	$\xcancel{0}$
D	1	0.5	2.33	$\boxed{0}$

Here we see that all ones are either assigned or crossed out.
That is, the total assigned ones is 4 which is equal to the number of rows or columns.
∴ The optimum assignment is A → I, B → III, C → II, D → IV
Hence the minimum total time of this assignment scheduled is,
$Z = 9 + 6 + 20 + 0$
$Z = 35$ hours.

V. COMPARISON OF OPTIMAL VALUES OF FOUR METHODS

Problem	**HA-Method**	**Divide Column And Subtract one method**	**Proposed Method 1**	**Proposed Method 2**	**Optimum**
01	35	35	35	35	35

VI. CONCLUSION

In this paper, two new simplified methods — the Subtract Column and Add-Ones Method and the Subtract

Row and Column and Add-Ones Method — were proposed for solving assignment problems effectively. These methods were clearly explained and applied to numerical examples. In both cases, the optimal results matched those of classical methods like the Hungarian Assignment Method (HA-method) and Subtract row and Add ones Method (SRAO-method), confirming the accuracy and reliability of the proposed techniques. Compared to existing approaches, the new methods offer a more straightforward and time-efficient procedure, especially in unbalanced or complex assignment scenarios. Thus, this study demonstrates that the proposed approaches are not only easy to implement but also highly effective for practical applications in operations research.

REFERENCES

1. D.F. Votaw, 1952, A. Orden, The perssonel assignment problem, Symposium on Linear Inequalities and Programming, SCOOP 10, US Air Force, pp. 155-163.
2. H.W. Kuhn, 1955, The Hungarian method for the assignment problem, Naval Research Logistics Quarterly 2 (1&2) 83-97 (original publication).
3. M.S. Bazarra, John J. Jarvis, Hanif D. Sherali, 2005, Linear programming and network flows.
4. B.S. Goel, S.K. Mittal, 1982, Operations Research, Fifty Ed., 2405-2416.
5. Hamdy A. Tsaha, 2007, Operations Research, an introduction, 8th Ed..
6. HadiBasirzadeh 2012, Applied Mathematical Sciences, 6(47), 2345-2355.
7. Singh S., Dubey G.C., Shrivastava R. (2012) IOSR Journal of Engineering, 2(8), 01-15.
8. Turkensteen M., Ghosh D., Boris Goldengorin, Gerard Sierksma, 2008 European Journal of Operational Research, 189, 775-788.
9. Zimmermman H.J. (1996) Fuzzy Set Theory and its Applications, 3rd ed., Kluwer Academic, Boston.
10. Pentico D.W. (2007) European Journal of Operation Research, 176, 774-793.
11. Singh S. (2011) International Journal of Operation Research and Information System, 3(3), 87-97.
12. AnshumanSahu, RudrajitTapador, 2007, Solving the assignment problem using Genetic Algorithm And Simulated annealing, IJAM.
13. Shayle R. Searle, 2006, Matrix algebra useful for statistics, John Willey.