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Cost Analysis of Euclidean and Manhattan Distances in Node Networks

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Abstract:
This Study compares the Euclidean and Manhattan distance metric in a network of nodes by calculating the cost of assignment using the Hungarian method. The Hungarian method is then applied to determine the optimal assignment and total cost for each distance metric. The results highlight the differences in total cost between Euclidean and Manhattan distances, providing insights into how distance measures affect optimization in network problems. The results clearly demonstrate that the Euclidean distance metric leads to a significantly lower optimal cost compared to the Manhattan metric. Since Euclidean distance reflects the true shortest path between nodes, it ensures higher cost efficiency in the assignment process.

Keywords: Euclidean distance, Manhattan distance, Networks, Hungarian Method.

1. Introduction:

Distance plays a key role in the analysis of node networks, where nodes represent specific points and edges represent the connections between them. The weight assigned to each edge depends on how the distance between two nodes is measured. Different distance metrics can change the overall cost of the network and influence the outcome of optimization methods. Euclidean distance measures the straight-line distance between two points, Manhattan distance measures only horizontal and vertical directions. Because of these differences, the values

Produced by each metric vary, and this variation affects cost calculations and optimal assignments.

To study this effect, a node network is created by assigning coordinates to each node, and distances are calculated using both Euclidean and Manhattan formulas. These distance form two separate cost matrices. The Hungarian Method is then applied to find the minimum-cost assignment for each matrix. By comparing the results, this study aims to identify which distance metric provides better cost efficiency in network based optimization.

2. Definitions:

2.1 Nodes:

In graph theory, a node is a fundamental element representing an object, entity, or points. A node is a specific point or location in a network where data, movement, or connections begin or end. Each node is represented using a pair of coordinates(x, y).

2.2 Edge:

An edge is a line or arc that connects two vertices (or nodes), representing a relationship or pathway between them. Edge is the connection or link between two nodes in a network. Edges are assigned weights based on the distance between the connected nodes.

2.3 Undirected graph:

An undirected graph is a set of nodes(Vertices) Connected by edges that have no direction, meaning connections are bidirectional; you can traverse from one node to another and back along the same edge, like a simple line between points, representing relationships such as friendships or roads where travel is possible both ways.

2.4 Cost Matrix:

A cost matrix is a table that represents the cost of moving from one point to another in a network. Each row of the matrix represents a starting node, and each column represents a destination node. The value placed at the intersection of a row and column shows the cost, distance, or time required to travel between those two nodes.

2.5 Optimization:

Optimization refers to the process of selecting the best possible solution from a set of available choices. It helps to identify the most effective route, method, or allocation so that overall outcomes becomes better than other alternatives. Optimization is used to determine the most cost-efficient path based on distance measurements.

3. Formulation of Node Network:

1. **Select nodes:** Decide which points represent the key locations or entities for the study and label them uniquely (e.g., A, B, C, D, E…).
2. **Assign coordinates:** For each node, provide a two-dimensional coordinate pair. Use actual positions or chosen sample values that reflect the intended layout.
3. **Choose graph type:** For this study, select an undirected complete graph where connections are mutual and distances are symmetric.
4. **Draw edges:** Represent possible connections between nodes as edges on the plotted coordinate plane.
5. **Compute pairwise distances:** For each node pair, compute the Euclidean distance and Manhattan distance. Record both results.
6. **Form cost matrices:** Create two NxN matrices with diagonal entries set to a large number or infinity to prevent self-assignment as needed by the algorithm.
7. **Assign weights:** Place computed distances into the matrices so that each entry is the edge weight for that pair.
8. **Proceed to optimization:** Use the cost matrices as input to your assignment or optimization method (Hungarian method) to obtain results.

4. Euclidean Distance Computation:

Euclidean distance represents the straight-line separation between two nodes in a network. Euclidean distance measures the shortest possible distance two nodes by considering a straight-line path connecting them. To calculate this, each node is first assigned coordinates on a two-dimensional plane. The distance can be computed directly by applying the Euclidean formula, which reflects the shortest possible path between two points.

$$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

The calculation begins by noting the difference between the X-coordinates of the two nodes and the differences between their Y-coordinates. The coordinates are written in the form(x, y), where X represents horizontal position and Y represents vertical position of the node. These differences indicate how far apart the nodes lie in both horizontal and vertical directions. One of the main advantages of Euclidean distance is that it models real-world straight-line movement accurately. After squaring the horizontal and vertical differences, the results are added together.

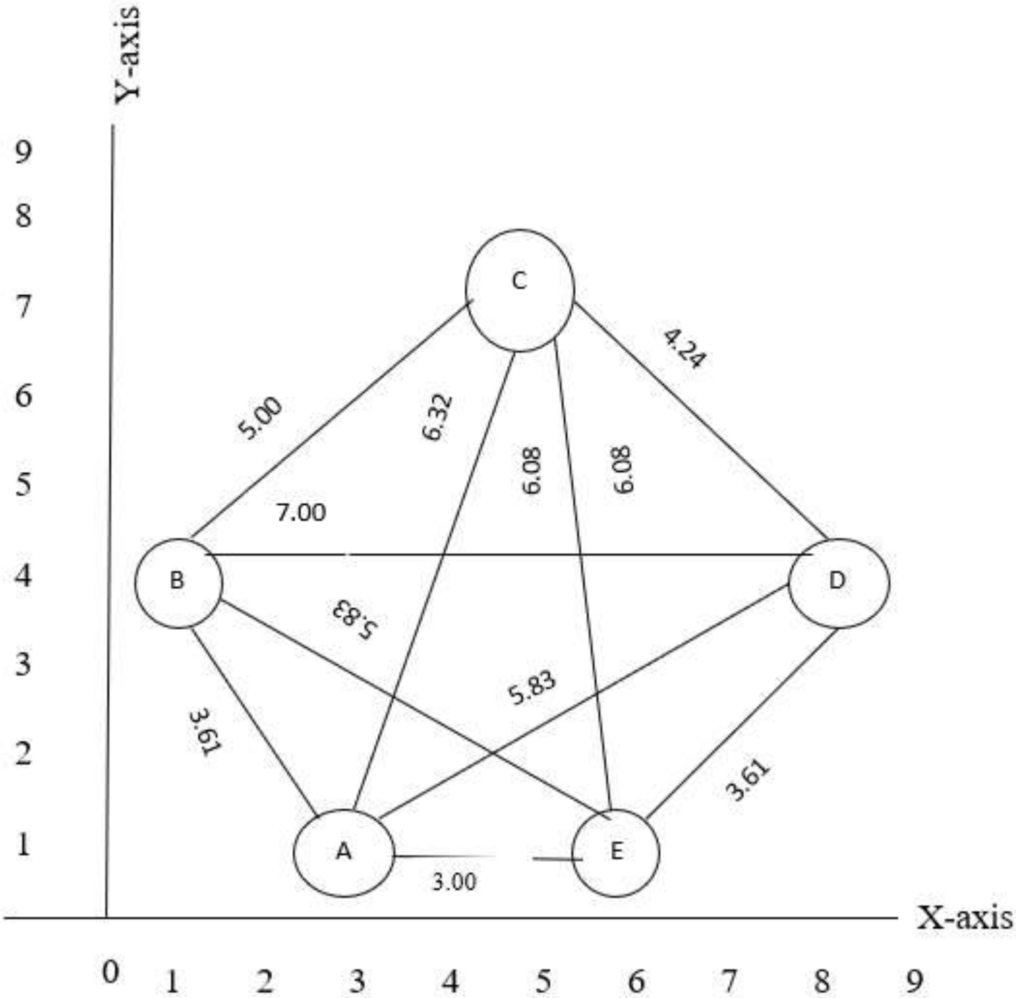
This sum represents the combined separation of the nodes in two dimensions. By taking square root of this value, the final Euclidean distance is obtained. Whether comparing two locations on a map or analyzing points on a graph, this method gives a clear indication of how close or far nodes are from each other. In this journal, Euclidean distance serves as the weights for the edges connecting nodes.

These weights help in determining the total cost of movement across the network. By comparing these values with Manhattan distances, the study can identify which distance measure results in lower overall cost and provides better efficiency for the given layout.

5. Euclidean Distance Node Network For 5x5:

The coordinates are A=(3,1) B=(1,4) C=(5,7) D=(8,4) E=(6,1)

By applying Euclidean distance formula for the given network coordinates



6. Assignment Problem Using Euclidean Distance and Hungarian Method:

	A	B	C	D	E
A	-	3.61	6.32	5.83	3.00
B	3.61	-	5.00	7.00	5.83
C	6.32	5.00	-	7.00	5.83
D	5.83	7.00	4.24	-	3.61
E	3.00	5.83	6.08	3.61	-

The above box is original cost Matrix

Row reduction: Find the minimum element in each row and subtract it from every element of that row.

	A	B	C	D	E
A	-	0.61	3.32	2.83	0
B	0	-	1.39	3.39	2.22
C	2.08	0.76	-	0	1.84
D	2.22	3.39	0.63	-	0
E	0	2.83	3.08	0.61	-

Column reduction: Find the minimum element in each column and subtract it from every element from that column.

	A	B	C	D	E
A	-	0	2.69	2.83	0
B	0	-	0.76	3.39	2.22
C	2.08	0.15	-	0	1.84
D	2.22	2.78	0	-	0
E	0	2.22	2.45	0.61	-

Row wise & Column wise assignment: Assign zeros such that each row and each column has only one assignment. Mark the selected zeros and cross out the remaining zeros. Draw the minimum number of horizontal and vertical lines to cover all zeros in the matrix.

	A	B	C	D	E
A	-	[0]	2.69	2.83	~~0~~
B	[0]	-	0.76	3.39	2.22

C	**2.08**	**0.15**	**-**	**[0]**	**1.84**
D	**2.22**	**2.78**	**[0]**	**-**	**0**
E	**0**	**2.22**	**2.45**	**0.61**	**-**

Lines are drawn through the 1st row & 1st Column, and 3rd row & 4th row, If the number of lines drawn is unequal the solution is not optimal. Select the smallest uncovered element. Subtract it from all uncovered elements and add it to elements covered by two lines. K=0.61

	A	**B**	**C**	**D**	**E**
A	**-**	**0**	**2.69**	**2.83**	**0**
B	**0**	**-**	**0.15**	**2.78**	**1.61**
C	**2.69**	**0.15**	**-**	**0**	**1.84**
D	**2.83**	**2.78**	**0**	**-**	**0**
E	**0**	**1.61**	**1.84**	**0**	**-**

Row wise & Column wise Assignment: Assign zeros that each row and each column has only one assignment. Mark the selected zeros and cross out the remaining zeros.

	A	**B**	**C**	**D**	**E**
A	**-**	**[0]**	**2.69**	**2.83**	**0**
B	**[0]**	**-**	**0.15**	**2.78**	**1.61**
C	**2.69**	**0.15**	**-**	**[0]**	**1.84**
D	**2.83**	**2.78**	**[0]**	**-**	**0**
E	**0**	**1.61**	**1.84**	**0**	**-**

Lines are drawn through the 1st row & 1st Column, and 4th row & 4th Column, If the number of lines drawn unequal to the order of the Matrix, the solution is not optimal select the smallest uncovered element k=0.15

	A	**B**	**C**	**D**	**E**
A	**-**	**0**	**2.69**	**2.98**	**0**
B	**0**	**-**	**0**	**2.78**	**1.46**
C	**2.69**	**0**	**-**	**0**	**1.69**
D	**2.98**	**2.78**	**0**	**-**	**0**
E	**0**	**1.46**	**1.69**	**0**	**-**

Row wise & Column wise Assignment: The number of lines is equal to the order of the matrix then the solution is optimal.

	A	B	C	D	E
A	-	[0]	2.69	2.98	0
B	0	-	[0]	2.78	1.46
C	2.69	0	-	[0]	1.69
D	2.98	2.78	0	-	[0]
E	[0]	1.46	1.69	0	-

Optimal Assignments: Make zero assignments. Calculate the minimum total cost using the original cost matrix.

	A	B	C	D	E
A	-	[0]	2.69	2.98	0
B	0	-	[0]	2.78	1.46
C	2.69	0	-	[0]	1.69
D	2.98	2.78	0	-	[0]
E	[0]	1.46	1.69	0	-

Optimal Solution: The Minimum Cost of Euclidean Distance is 19.46

WORK	JOB	COST
A	B	3.61
B	C	5.00
C	D	4.24
D	E	3.61
E	A	3.00
	TOTAL	19.46

7. Manhattan Distance Computation:

Manhattan distance measures the path between two nodes by moving only in horizontal and vertical directions. This method treats the network like a grid, where movement is possible along straight lines that turn at right angles. Because it avoids diagonal travel, the Manhattan measure often gives a longer but more realistic distance for grid- based layouts.

To compute Manhattan distance, the absolute difference between the x-coordinates of the two nodes is taken first. This value shows how many units the movement occurs horizontally. Then, the absolute difference between the y-coordinates is calculated to represent the vertical shift needed to reach the second node.

$$d = |x_2 - x_1| + |y_2 - y_1|$$

Once the horizontal and vertical separations are identified, they are added together. This simple addition gives the final Manhattan distance. No squaring or square root steps are required, which makes this method straightforward and easy to apply, especially when analyzing grid-like networks where diagonal movement is not possible.

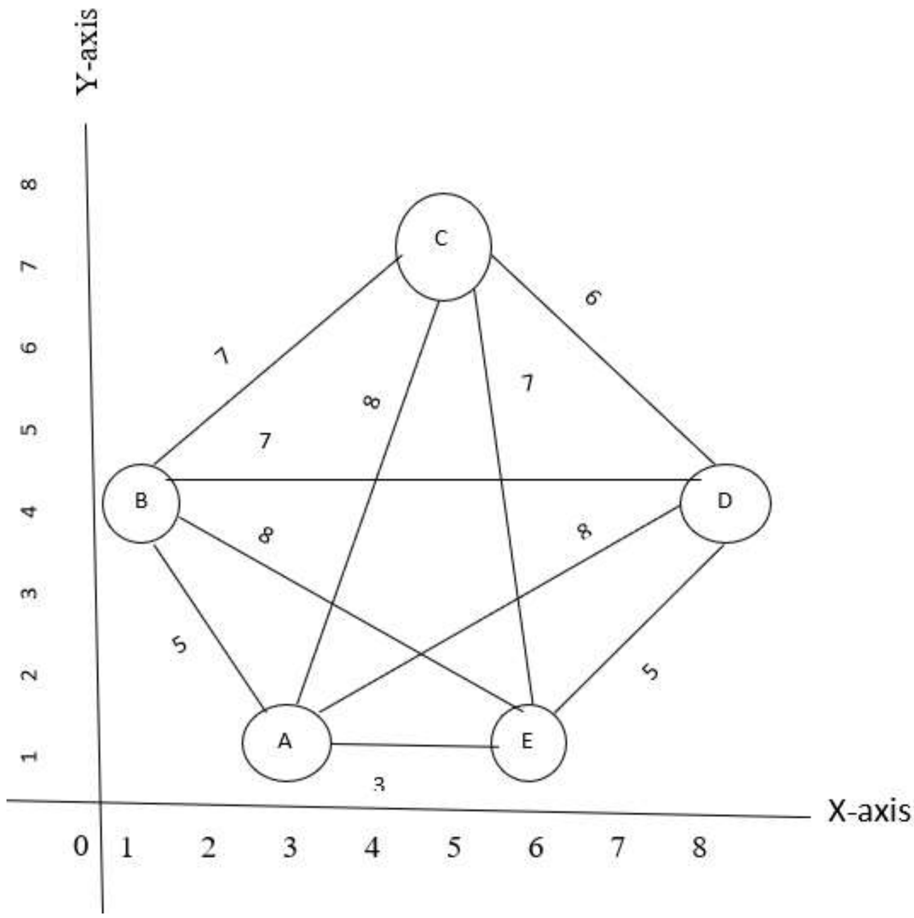
The Manhattan measure is useful in situations where movement follows structured paths, such as city blocks, circuit layouts, or grid-based networks. Because every step must follow a fixed direction, this distance reflects how the actual paths might be used in real systems. It also helps identify patterns in movement that the Euclidean measure may overlook.

In this journal, Manhattan distance is assigned as edge weights between nodes to compare its performance with Euclidean distance. These values reveal how the total cost changes when only horizontal and vertical movement is allowed. By analyzing these weights, the study can determine whether Manhattan distance provides a more accurate or cost-effective measurements for the network structure.

8. Manhattan Distance Node Network For 5x5:

Coordinates For this given network are A=(3,1) B=(1,4) C=(5,7) D=(8,4) E=(6,1)

By applying Manhattan distance formula in the given network by using coordinates.



9. Assignment Problem Using Manhattan Distance and Hungarian Method:

	A	B	C	D	E
A	-	5	8	8	3
B	5	-	7	7	8
C	8	7	-	6	7
D	8	7	6	-	5
E	3	8	7	5	-

The above matrix is an original cost matrix
Row reduction: Find the minimum element in each row and subtract it from every element of that row.

	A	B	C	D	E
A	-	2	5	5	0
B	0	-	2	2	3
C	2	1	-	0	1
D	3	2	1	-	0
E	0	5	4	2	-

Column reduction: Find the minimum element in each column and subtract it from every element of that column.

	A	B	C	D	E
A	-	1	4	5	0
B	0	-	1	2	3
C	2	0	-	0	1
D	3	1	0	-	0
E	0	4	3	2	-

Row wise & Column wise Assignment: Assign zeros such that each row and each column has only one assignment. Mark the selected zeros and cross out the remain zeros. Draw minimum number of lines to cover all zeros in the matrix. Lines are drawn through1st row & 1st column, followed by lines through the 3rd &4th rows, in order to cover all zero entries in the matrix.

	A	B	C	D	E
A	-	1	4	5	[0]
B	[0]	-	1	2	3
C	2	[0]	-	0	1
D	3	1	[0]	-	0
E	0	4	3	2	-

If the number of lines drawn is unequal the solution is not optimal select the smallest uncovered elements and add it to elements covered by two lines. K=1

	A	B	C	D	E
A	-	1	4	5	0
B	0	-	0	1	2
C	3	0	-	0	1
D	4	1	0	-	0
E	0	3	2	1	-

Row wise & Column wise Assignment: Assign zeros and lines are drawn through 1st column ,3rd column,5th column and 3rd row. draw minimum number of lines the lines is unequal the solution is not optimum.

	A	B	C	D	E
A	-	1	4	5	[0]
B	[0]	-	0	1	2
C	3	[0]	-	0	1
D	4	1	[0]	-	0
E	0	3	2	1	-

Select the smallest uncovered element and subtract it from all uncovered elements and add it to elements covered by two lines. K=1

	A	B	C	D	E
A	-	0	4	4	0
B	0	-	0	0	2
C	4	0	-	0	2
D	4	0	0	-	0
E	0	2	2	0	-

Row wise & Column wise Assignment: The number of drawn lines equal to the order of the matrix the solution is optimum.

	A	B	C	D	E
A	-	[0]	4	4	0
B	0	-	[0]	0	2
C	4	0	-	[0]	2
D	4	0	0	-	[0]
E	[0]	2	2	0	-

Optimal Assignments: Make the final zero assignments and Calculate the minimum total cost using the original cost matrix.

	A	B	C	D	E
A	-	[0]	4	4	0
B	0	-	[0]	0	2
C	4	0	-	[0]	2
D	4	0	0	-	[0]
E	[0]	2	2	0	-

Optimal Solution: The minimum total cost of Manhattan distance is 26

WORK	JOB	COST
A	B	5

B	**C**	**7**
C	**D**	**6**
D	**E**	**5**
E	**A**	**3**
	TOTAL	**26**

10. Conclusion:

Euclidean and Manhattan distances were calculated using assigned node coordinates. The computed distances were converted into cost values and solved using the Hungarian method. This work focused on finding the cost efficiency of different distance measures in a node network. The results showed that Euclidean distance produced a lower total assignment cost. Euclidean distance is more cost efficient for assignment problems in networks.

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