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Fractional Solid Transportation Problem under Fermatean Fuzziness: An SDR Decomposition and Optimization Algorithm

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Abstract
This paper presents a structured optimization framework for the Fermatean Fuzzy Fractional Solid Transportation Problem (FFFSTP), a three-dimensional decision model involving sources, destinations, and routing alternatives under uncertainty. The proposed methodology initially ensures model feasibility through balance verification and necessary adjustments. All Fermatean fuzzy parameters are systematically transformed into crisp equivalents using an appropriate grade function to facilitate computational tractability. The fractional objective function is decomposed into numerator and denominator components, each optimized independently using a refined Source–Destination–Routing (SDR) tableau approach. The optimization process incorporates reduction techniques and iterative adjustment mechanisms to maintain feasibility across all constraints. The final solution is obtained by integrating the optimized components, resulting in an efficient and consistent allocation strategy. The proposed approach provides a robust framework for solving complex transportation problems in advanced fuzzy environments.

Keywords: Fermatean fuzzy sets; Fractional solid transportation problem; SDR tableaux; Decomposition algorithm; Three-dimensional transportation.

1. Introduction

The foundation of transportation theory can be traced to the early work of Hitchcock (1941), who first formulated the classical transportation problem for distributing commodities from multiple sources to various destinations. This was further strengthened by Koopmans (1949), who analyzed the optimal utilization of transportation systems within an economic framework. These early models were deterministic and primarily linear in structure. The introduction of fuzzy set theory by Zadeh (1965) marked a significant turning point in decision-making under uncertainty. Soon after, Bellman and Zadeh (1970) incorporated fuzzy concepts into decision-making environments, enabling mathematical models to account for imprecision and vagueness. Zimmermann (1978) and Verdegay (1984) subsequently extended fuzzy principles to mathematical programming, establishing the theoretical basis for fuzzy optimization models. Kaufmann and Gupta (1991) further strengthened the computational framework through fuzzy arithmetic concepts.

Parallel to these developments, Charnes and Cooper (1962) introduced fractional programming, allowing objective functions expressed as ratios to be systematically optimized. This advancement later motivated research into fuzzy fractional programming, including contributions by Toksarı (2007) and Ebrahimnejad and Verdegay (2018), who developed solution methodologies for fractional models under fuzzy environments. The solid transportation problem, a three-dimensional extension involving sources, destinations, and conveyances (or routes), evolved from the classical model. Bit, Biswal, and Alam (1993) incorporated fuzziness into solid transportation structures, while Jiménez and Verdegay (1998) proposed solution procedures for fuzzy solid transportation problems. Later, Pandian and Natarajan (2010) introduced improved computational techniques, and Gupta and Kumar (2012) provided a comprehensive review of developments in solid transportation modeling. Further advancements focused on ranking and solution techniques for fuzzy transportation systems, including the work of Kaur and Kumar (2014). Das and Chakraborty (2015) extended fractional transportation models to fuzzy settings, integrating ratio-based objectives with uncertainty modeling. More recently, Yager (2013) proposed Pythagorean fuzzy sets as an extension of intuitionistic fuzzy theory, enhancing the flexibility of uncertainty representation. Building upon this foundation, Senapati and Yager (2019, 2020) introduced Fermatean fuzzy sets and associated aggregation operators, offering greater expressive power for handling higher degrees of membership and non-membership. These developments have created new opportunities for modeling complex optimization problems under advanced fuzzy environments.

Despite the substantial progress in fuzzy transportation and fractional programming, limited research has addressed the integration of Fermatean fuzzy theory with fractional solid transportation structures. This gap motivates the development of structured solution methodologies capable of handling three-dimensional transportation systems with ratio-based objectives under Fermatean fuzzy uncertainty.

Section 1 outlines the background of transportation problems under uncertainty and motivates the development of a Fermatean fuzzy fractional solid framework. Section 2 presents essential preliminaries on Fermatean fuzzy sets and the ranking mechanisms used for defuzzification. Section 3 describes the proposed SDR-based optimization algorithm for numerator and denominator components. Section 4 demonstrates the procedure through a numerical example, and Section 5 concludes with key findings and future research directions.

2. Preliminaries

2.1 Definition: Consider a universal set X. A Fermatean fuzzy set is characterized by membership and non-membership functions defined on X, satisfying a cubic constraint condition that extends intuitionistic fuzzy theory. $\tilde{F} = \{\langle x, \alpha_{\tilde{F}}(x), \beta_{\tilde{F}}(x)\rangle : x \in X\}$ where $\alpha_{\tilde{F}}(x) : X \to [0, 1]$ and $\beta_{\tilde{F}}(x) : X \to [0, 1]$ which satisfies the relation $0 \le (\alpha_{\tilde{F}}(x))^3 + (\beta_{\tilde{F}}(x))^3 \le 1, \forall x \in X$. The number $\alpha_{\tilde{F}}(x)$ and $\beta_{\tilde{F}}(x)$ are the degree of membership and non-membership of the element $x \in X$ in the FFS $\tilde{F}$.

For any FFS $\tilde{F}$ and $x \in X$, the degree of indeterminacy is represented by $\pi_{F}(x) = \sqrt[3]{1 - (\alpha_{\tilde{F}}(x))^3 + (\beta_{\tilde{F}}(x))^3}$. It is to be noted that, for simplicity, we shall denote the object $\tilde{F}=\langle\alpha_{\tilde{F}}(x), \beta_{\tilde{F}}(x)\rangle$ instead of $\tilde{F} = \{\langle x, \alpha_{\tilde{F}}(x), \beta_{\tilde{F}}(x)\rangle : x \in X\}$.

2.2 Definition: Let $\tilde{F}_1=\langle\alpha_{\tilde{F}_1}(x), \beta_{\tilde{F}_1}(x)\rangle$ and $\tilde{F}_2=\langle\alpha_{\tilde{F}_2}(x), \beta_{\tilde{F}_2}(x)\rangle$ be two FFSs. Then the basic arithmetical operations of Fermatean fuzzy sets $\tilde{F}_1$ and $\tilde{F}_2$ are defined as follows:

(1) Addition: $\tilde{F}_1 \oplus \tilde{F}_2 = \langle \sqrt[3]{(\alpha_{\tilde{F}_1})^3 + (\alpha_{\tilde{F}_2})^3 - (\alpha_{\tilde{F}_1})^3(\alpha_{\tilde{F}_2})^3}, \beta_{\tilde{F}_1}\, \beta_{\tilde{F}_2}\rangle$.

(2) Multiplication: $\tilde{F}_1 \otimes \tilde{F}_2 = \langle \alpha_{\tilde{F}_1}\alpha_{\tilde{F}_2}, \sqrt[3]{(\beta_{\tilde{F}_1})^3 + (\beta_{\tilde{F}_2})^3 - (\beta_{\tilde{F}_1})^3(\beta_{\tilde{F}_2})^3} \rangle$

(3) Scalar Multiplication: $\lambda \odot \tilde{F} = \langle \sqrt[3]{1-(1-(\alpha_{\tilde{F}})^3)^\lambda}, \beta_{\tilde{F}}{}^{\lambda} \rangle$ provided $\lambda > 0$

(4) Exponent: $\tilde{F}^\lambda = \langle (\alpha_{\tilde{F}})^\lambda, \sqrt[3]{1-(1-(\beta_{\tilde{F}})^3)^\lambda} \rangle$.

2.3 Definition: Let $\tilde{F}_1 = \langle \alpha_{\tilde{F}_1}(x), \beta_{\tilde{F}_1}(x) \rangle$ and $\tilde{F}_2 = \langle \alpha_{\tilde{F}_2}(x), \beta_{\tilde{F}_2}(x) \rangle$ be two FFSs. Then their set operations are defined as follows:

(1) Union: $\tilde{F}_1 \cup \tilde{F}_2 = \langle \max(\alpha_{\tilde{F}_1}(x), \alpha_{\tilde{F}_2}(x)), \min(\beta_{\tilde{F}_1}(x), \beta_{\tilde{F}_2}(x)) \rangle$.

(2) Intersection: $\tilde{F}_1 \cap \tilde{F}_2 = \langle \min(\alpha_{\tilde{F}_1}(x), \alpha_{\tilde{F}_2}(x)), \max(\beta_{\tilde{F}_1}(x), \beta_{\tilde{F}_2}(x)) \rangle$.

(3) Compliment: $(\tilde{F}_1)' = \langle \beta_{\tilde{F}_1}(x), \alpha_{\tilde{F}_1}(x) \rangle$.

2.4 Definition: Let $\tilde{F} = \langle \alpha_{\tilde{F}}(x), \beta_{\tilde{F}}(x) \rangle$ be any FFS then score function (grade function) of $\tilde{F}$ denoted by $S_F(\tilde{F})$ and is defined by $S_F(\tilde{F}) = \langle \alpha_{\tilde{F}}{}^3 - \beta_{\tilde{F}}{}^3) \rangle$. Here the score function $S_F(\tilde{F}) \in [-1,1]$.

But we have defined some score functions $S_F(\tilde{F}), S_F(\tilde{F}) \in [0,1]$ which are as follows:

(1) $S_F(\tilde{F}) = \frac{1}{2}(1 + \alpha_{\tilde{F}}{}^3 - \beta_{\tilde{F}}{}^3)$

(2) $S_F(\tilde{F}) = \frac{1}{3}(1 + 2\alpha_{\tilde{F}}{}^3 - \beta_{\tilde{F}}{}^3)$

(3) $S_F(\tilde{F}) = \frac{1}{2}(1 + \alpha_{\tilde{F}}{}^2 - \beta_{\tilde{F}}{}^2)|\alpha_{\tilde{F}} - \beta_{\tilde{F}}|$

2.5 Definition: Let $\tilde{F} = \langle \alpha_{\tilde{F}}, \beta_{\tilde{F}} \rangle$ be any FFS then accuracy function of $\tilde{F}$ denoted by $H_F(\tilde{F})$ and is defined by $H_F(\tilde{F}) = \alpha_{\tilde{F}}{}^3 + \beta_{\tilde{F}}{}^3$

2.6 Definition: Let $\tilde{F}_1 = \langle \alpha_{\tilde{F}_1}(x), \beta_{\tilde{F}_1}(x) \rangle$ and $\tilde{F}_2 = \langle \alpha_{\tilde{F}_2}(x), \beta_{\tilde{F}_2}(x) \rangle$ be two FFSs. Then ranking or order relations of Let $\tilde{F}_1$ and $\tilde{F}_2$ are defined as follows:

(1) $\tilde{F}_1 >_{max} \tilde{F}_2$ iff either $(S_F(\widetilde{F_1}) > S_F(\widetilde{F_2}))$ or $(S_F(\widetilde{F_1}) = S_F(\widetilde{F_2}))$ and $(H_F(\widetilde{F_1}) > H_F(\widetilde{F_2}))$

(2) $\tilde{F}_1 <_{min} \tilde{F}_2$ iff either $(S_F(\widetilde{F_1}) < S_F(\widetilde{F_2}))$ or $(S_F(\widetilde{F_1}) = S_F(\widetilde{F_2}))$ and $(H_F(\widetilde{F_1}) < H_F(\widetilde{F_2}))$

(3) $\tilde{F}_1 \cong_{equal} \tilde{F}_2$ iff either $(S_F(\widetilde{F_1}) = S_F(\widetilde{F_2}))$ and $(H_F(\widetilde{F_1}) = H_F(\widetilde{F_2}))$

2.7 Definition: The problem includes constraints on Fermatean fuzzy supply at each source, Fermatean fuzzy demand at each destination, and Fermatean fuzzy capacity constraints on each mode of transportation. Formally, it can be modeled as:

Minimize $Z = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{c}^F{}_{ijk} * x_{ijk}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} \tilde{d}^F{}_{ijk} * x_{ijk}}$

subject to constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \widetilde{a_i}^F; k = 1,2,\dots,l.$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = \widetilde{b_j}^F; i = 1,2,\dots,m.$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = \widetilde{e_k}^F; j = 1,2,\dots,n.$$

$$x_{ijk} \geq 0$$

where m is the number of sources, n is the number of destinations, k is the number of transportation modes, $\tilde{c}^F{}_{ijk}$ is the Fermatean fuzzy profit coefficient, $\tilde{d}^F{}_{ijk}$ is the Fermatean fuzzy cost coefficient, $\widetilde{a_i}^F$ is the

Fermatean fuzzy supply available at source i, $\widetilde{b_j}^F$ is the Fermatean fuzzy demand required at destination j, $\widetilde{e_k}^F$ is the maximum Fermatean fuzzy capacity for transportation from i to j via k.

Remark: A necessary and sufficient condition for existence of solution is $\sum_{i=1}^{m} \widetilde{a_i}^F = \sum_{j=1}^{n} \widetilde{b_j}^F = \sum_{k=1}^{m} \widetilde{e_k}^F$ i.e., the problem must be balanced. If the problem is unbalance, then it must be converted to balanced problem by introducing dummy source or origin.

3. A Decomposition Algorithm for Fermatean Fuzzy Fractional Solid Transportation Problems

To efficiently solve the Fermatean Fuzzy Fractional Solid Transportation Problem, a structured decomposition-based optimization approach is proposed. The methodology utilizes a Source–Destination–Routing (SDR) tableau framework combined with systematic reduction and iterative adjustment procedures to ensure feasibility and optimality.

Phase I: Model Preparation and Defuzzification

Step 1: Compute the total supply from all sources and compare it with the total demand across all destinations and routing levels. If the totals are equal, the model is balanced and feasible for further analysis.

Step 2: If the total supply does not equal the total demand, introduce appropriate dummy sources, destinations, or routing channels with zero transportation cost to restore equilibrium. Once balance is achieved, proceed to the transformation stage.

Step 3: Convert all Fermatean fuzzy parameters involved in the model into equivalent crisp numerical values using the Grade function of Fermatean fuzzy sets. This transformation enables the problem to be handled using conventional optimization techniques.

Phase II: Optimization of Numerator and Denominator Components

Step 4: Extract the numerator cost matrix from the fractional objective function and construct the three-dimensional Source–Destination–Routing (SDR) cost tableau.

Step 5: A normalization procedure is applied to the SDR tableau by eliminating row-wise and column-wise minimum values, resulting in a reduced cost structure. The feasibility of the reduced system is then verified using sub-tableaux representations.

Step 6: Transform the reduced SDR tableau into the Source Capacity–Destination Requirement (SC–DR) sub-tableau. Verify whether the row totals match the source capacities and the column totals match the destination requirements. If the condition is satisfied, proceed to the next step; otherwise, apply further adjustments.

Step 7: If inconsistencies arise, an iterative adjustment mechanism is employed in which zero-valued positions are strategically covered to minimize computational complexity. The smallest uncovered element is used to update the tableau by appropriate subtraction and addition operations. This process is repeated until all feasibility conditions are satisfied.

Step 8: Construct the Destination Requirement–Routing (DR–R) sub-tableau from the optimized SC–DR matrix and apply the same reduction and adjustment procedure until optimality is achieved.

Step 9: Form the Routing–Source Capacity (R–SC) sub-tableau from the optimized DR–R solution. Continue the iterative reduction and adjustment process until no further improvement is possible.

Step 10: Verify the global feasibility of the SDR allocation by ensuring consistency among supply, demand, and routing quantities.

Step 11: Extract the denominator cost matrix from the fractional model and repeat the optimization procedure described in Steps 5–10 to obtain the optimal denominator allocation.

Phase III: Allocation and Final Fractional Solution

Step 12: Identify the dimension (source, destination, or routing) containing the fewest zero-cost cells in the optimized tableau. Allocate the maximum feasible quantity to a zero-cost cell corresponding to the minimum original transportation cost. If multiple alternatives exist, apply an appropriate tie-breaking rule.

Step 13: Update the tableau by removing fully satisfied rows or columns and revising the remaining supply, demand, and routing values. Repeat the allocation and updating process until all constraints are fully satisfied.

Step 14: Combine the optimal numerator and denominator allocations to determine the overall optimal solution of the Fermatean Fuzzy Fractional Solid Transportation Problem.

4. Numerical Illustration

To demonstrate the Fermatean Fuzzy Fractional Solid Transportation Problem (FFFSTP), consider a system with three sources (SC_1, SC_2, SC_3), three destinations (DR_1, DR_2, DR_3), and three routing alternatives (R_1, R_2, R_3). Shipments are permitted from every source to every destination through each route. Due to uncertainty in transportation cost and operational efficiency, all parameters are expressed as Fermatean fuzzy numbers (μ,ν).

The numerator of the fractional objective represents fuzzy transportation costs defined for each source–destination–route combination, along with fuzzy source capacities and destination requirements. The denominator represents corresponding fuzzy performance or efficiency measures for the same combinations. For modeling consistency, the capacities and requirements in both components are assumed identical, ensuring a balanced solid transportation structure.

The goal is to determine shipment allocations that minimize the ratio of total fuzzy transportation cost to total fuzzy performance, subject to source, destination, and routing constraints.

Routing	R_1			R_1			R_1			
		R_2			R_2			R_2		
			R_3			R_3			R_3	
	DR_1			DR_2			DR_3			Source capacity
SC_1	(0.86,0.46)	(0.84,0.49)	(0.81,0.53)	(0.88,0.43)	(0.79,0.58)	(0.76,0.62)	(0.73,0.66)	(0.70,0.70)	(0.66,0.73)	(0.88,0.46)
SC_2	(0.62,0.76)	(0.58,0.79)	(0.53,0.81)	(0.46,0.84)	(0.36,0.86)	(0.87,0.41)	(0.85,0.43)	(0.82,0.44)	(0.78,0.49)	(0.79,0.66)
SC_3	(0.80,0.46)	(0.69,0.60)	(0.64,0.56)	(0.75,0.70)	(0.78,0.66)	(0.70,0.75)	(0.82,0.58)	(0.92,0.36)	(0.89,0.39)	(0.60,0.84)
Destination requirement	(0.86,0.53)			(0.73,0.66)			(0.70,0.76)			

Table 1. Fermatean fuzzy cost matrix for the numerator component

Routing	R_1			R_1			R_1			(0.82,0.54)
		R_2			R_2			R_2		(0.79,0.66)
			R_3			R_3			R_3	(0.70,0.76)
	DR_1			DR_2			DR_3			Source capacity
SC_1	(0.80,0.30)	(0.75,0.40)	(0.82,0.25)	(0.70,0.45	(0.66,0.50)	(0.88,0.20)	(0.90,0.18)	(0.78,0.35)	(0.72,0.42)	(0.88,0.46)
SC_2	(0.85,0.28)	(0.60,0.55)	(0.92,0.15)	(0.74,0.38)	(0.67,0.48)	(0.83,0.30)	(0.76,0.33)	(0.89,0.22)	(0.64,0.52)	(0.79,0.66)
SC_3	(0.69,0.46)	(0.95,0.10)	(0.72,0.40)	(0.80,0.32)	(0.77,0.37)	(0.58,0.56)	(0.62,0.53)	(0.91,0.17)	(0.87,0.26)	(0.60,0.84)
Destination requirement	(0.86,0.53)			(0.73,0.66)			(0.70,0.76)			

Table 2. Fermatean fuzzy cost matrix representing the denominator component of the fractional model

Balance is first verified by comparing total supply and total demand across all destinations and routing levels. If inequality occurs, suitable dummy sources, destinations, or routes with zero cost are introduced to achieve equilibrium.

Total Source Capacity = 0.80 + 0.60 + 0.35 = 1.75

Total Destination Requirement = 0.75 + 0.55 + 0.45 = 1.75

Total Routing Capacity = 0.78 + 0.60 + 0.37 = 1.75

Since all totals are equal, the model is balanced. Subsequently, all Fermatean fuzzy parameters are converted into crisp values using the grade function to facilitate computation.

Table 3. Defuzzified (crisp) cost matrix for the numerator component obtained using the grade function

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37
	DR_1			DR_2			DR_3			Source capacity
SC_1	0.742	0.678	0.767	0.625	0.581	0.836	0.861	0.715	0.649	0.80
SC_2	0.796	0.524	0.887	0.675	0.595	0.772	0.701	0.847	0.560	0.60
SC_3	0.928	0.654	0.739	0.702	0.615	0.509	0.544	0.874	0.820	0.35
Destination requirement	0.75			0.55			0.45			1.75

Table 4. Defuzzified (crisp) cost matrix for the denominator component obtained using the grade function

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37
	DR_1			DR_2			DR_3			Source capacity
SC_1	0.775	0.740	0.700	0.810	0.650	0.600	0.550	0.500	0.450	0.80
SC_2	0.400	0.350	0.300	0.250	0.200	0.800	0.770	0.745	0.680	0.60
SC_3	0.710	0.555	0.545	0.545	0.595	0.465	0.690	0.875	0.830	0.35
Destination requirement	0.75			0.55			0.45			1.75

From the Fermatean Fuzzy Fractional Solid Transportation framework, the numerator cost component is isolated to form the initial decision matrix. The corresponding three-dimensional Source–Destination–Routing (SDR) tableau is then subjected to a two-stage reduction process. First, the smallest element in each row is deducted from all entries of that row. Next, the smallest element in each column is deducted from the respective column entries. This procedure results in the reduced cost tableau used for further optimization.

Table 1. The numerator of the Fermatean Fuzzy Fractional Solid Transportation Problem.

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37
	DR_1			DR_2			DR_3			Source capacity
SC_1	0.742	0.678	0.767	0.625	0.581	0.836	0.861	0.715	0.649	0.80
SC_2	0.796	0.524	0.887	0.675	0.595	0.772	0.701	0.847	0.560	0.60
SC_3	0.928	0.654	0.739	0.702	0.615	0.509	0.544	0.874	0.820	0.35
Destination requirement	0.75			0.55			0.45			1.75

The reduced SDR matrix is next transformed into the Source Capacity–Destination Requirement (SC–DR) sub-tableau. The feasibility of this tableau is checked by ensuring that row totals correspond to the available source capacities and column totals satisfy the destination demands. If adjustments are required, the stepping-stone technique is employed: zero entries are covered using the fewest possible horizontal and vertical lines, the smallest uncovered element is identified and subtracted from all uncovered cells, and the same value is added to the elements located at the intersections of the covering lines.

Table 5. Source capacity–destination requirement (SC–DR) sub-tableau after initial reduction

	DR_1			DR_2			DR_3			Source capacity
SC_1	0	0.097	0	0	0	0.255	0.245	0	0.032	0.80
SC_2	0.111	0	0.177	0.107	0.071	0.248	0.142	0.189	0	0.60
SC_3	0.258	0.145	0.044	0.149	0.106	0	0	0.231	0.275	0.35
Destination requirement	0.75			0.55			0.45			1.75

Form the destination requirement–routing (DR–R) sub-tableau from the optimized SC–DR matrix and apply the stepping-stone method by covering all zero-cost cells with the minimum number of horizontal and vertical lines, subtracting the smallest uncovered value from all uncovered cells, and adding it to the cells at the intersections of the covering lines.

Table 6. Destination requirement–routing (DR–R) sub-tableau after iterative adjustment

	R_1			R_2			R_3			Destination requirement
DR_1	0	0.097	0	0	0	0.255	0.245	0	0.032	0.75
DR_2	0.111	0	0.177	0.107	0.071	0.248	0.142	0.189	0	0.55
DR_3	0.258	0.145	0.044	0.149	0.106	0	0	0.231	0.275	0.45
Routing	0.78			0.60			0.37			1.75

Build the routing–source capacity (R–SC) sub-tableau from the optimized DR–R matrix.

Table 7. Routing–source capacity (R–SC) sub-tableau representing intermediate allocation structure

	SC_1			SC_2			SC_3			Routing
R_1	0	0	0.245	0.111	0.107	0.142	0.258	0.149	0	0.78
R_2	0.097	0	0	0	0.071	0.189	0.145	0.106	0.231	0.60
R_3	0	0.255	0.032	0.177	0.248	0	0.044	0	0.275	0.37
Source capacity	0.80			0.60			0.35			1.75

Verify the overall feasibility of the combined SDR tableau. Ensure that the total supply, demand, and routing quantities are consistent with the allocations.

Table 8. Optimal allocation table obtained for the numerator component

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37
	DR_1			DR_2			DR_3			Source capacity
SC_1	0	0.097	0(**0.25**)	0	0(**0.55**)	0.255	0.245	0	0.032	0.80
SC_2	0.111	0(**0.5**)	0.177	0.107	0.071	0.248	0.142	0.189	0(**0.1**)	0.60
SC_3	0.258	0.145	0.044	0.149	0.106	0	0(**0.35**)	0.231	0.275	0.35
Destination requirement	0.75			0.55			0.45			1.75

Total Minimum cost of the numerator = 0.5×0.524+ 0.25× 0.767 +0.55×0.581+0.35× 0.544 = 0.9637. Extract the denominator cost matrix from the Fermatean Fuzzy Fractional Solid transportation model. From the Fermatean Fuzzy Fractional solid transportation model, isolate the denominator cost matrix to form the initial decision (criterion) table.

Table 2. The denominator of the Fermatean Fuzzy Fractional Solid Transportation Problem.

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37
	DR_1			DR_2			DR_3			Source capacity
SC_1	0.75	0.740	0.700	0.810	0.650	0.600	0.550	0.500	0.450	0.80
SC_2	0.400	0.350	0.300	0.250	0.200	0.800	0.770	0.745	0.680	0.60
SC_3	0.710	0.555	0.545	0.545	0.595	0.465	0.690	0.875	0.830	0.35
Destination requirement	0.75			0.55			0.45			1.75

For the three-dimensional source–destination–routing (SDR) tableau, apply a two-step reduction by first subtracting the minimum value in each row from all row elements and then subtracting the minimum value in each column from all column elements, resulting in the reduced cost tableau.

Table 9. Reduced SDR tableau for the denominator component after normalization

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37

Convert the reduced SDR tableau into the source capacity–destination requirement (SC–DR) sub-tableau and verify that row sums satisfy source capacities while column sums meet destination requirements. Then apply the stepping-stone method by covering all zero-cost cells with the minimum number of horizontal and vertical lines, subtracting the smallest uncovered value from all uncovered cells, and adding it to the cells at the intersections of the covering lines.

Table 10. SC–DR sub-tableau for the denominator component after reduction

	DR_1			DR_2			DR_3			Source capacity
SC_1	0.125	0.14	0.15	0.31	0.2	0.035	0	0	0	0.80
SC_2	0	0	0	0	0	0.485	0.47	0.045	0.48	0.60
SC_3	0.16	0.055	0.055	0.145	0.245	0	0.24	0.025	0.48	0.35
Destination requirement	0.75			0.55			0.45			1.75

Construct the destination requirement–routing (DR–R) sub-tableau from the optimized SC–DR matrix.

Table 11. DR–R sub-tableau for the denominator component after adjustment

	R_1			R_2			R_3			Destination requirement
DR_1	0.12	0	0.45	0.14	0.06	0	0.7	0.06	0.75	0.75
DR_2	0.22	0	0.03	0.11	0	0.13	0.06	0.60	0	0.55
DR_3	0	0.11	0.215	0	0.54	0	0	0.57	0.455	0.45
Routing	0.78			0.60			0.37			1.75

Build the routing–source capacity (R–SC) sub-tableau from the optimized DR–R matrix.

Table 12. R–SC sub-tableau for the denominator component showing allocation consistency

	SC_1			SC_2			SC_3			Routing
R_1	0.12	0.22	0	0	0	0.11	0.45	0.03	0.215	0.78
R_2	0.14	0.11	0	0.06	0	0.54	0	0.13	0	0.60
R_3	0.7	0.06	0	0.06	0.60	0.57	0	0	0.455	0.37
Source capacity	0.80			0.60			0.35			1.75

Verify the overall feasibility of the combined SDR tableau. Ensure that the total supply, demand, and routing quantities are consistent with the allocations.

Table 13. Final optimal allocation table for the denominator component

Routing	R_1			R_1			R_1			0.78
		R_2			R_2			R_2		0.60
			R_3			R_3			R_3	0.37
	DR_1			DR_2			DR_3			Source capacity
SC_1	0.12	0.14	0(**0.78**)	0.22	0.11	0.06	0	0	0	0.80
SC_2	0	0.06	0.06	0	0(**0.60**)	0.60	0.11	0.54	0.57	0.60
SC_3	0.45	0	0(**0.02**)	0.03	0.013	0	0.215	0(**0.35**)	0.455	0.35
Destination requirement	0.75			0.55			0.45			1.75

Total Minimum cost of the denominator = $0.7\times0.7+0.5\times0.4+0.55\times0.2+0.1\times0.5+0.35\times0.875$ = 1.15625.

Total minimum cost of the given Fuzzy Fermatean Fractional Solid Transportation Problem = $\frac{0.9637}{1.15625}$.

Conclusion

This study introduces a systematic decomposition-based solution approach for the Fermatean Fuzzy Fractional Solid Transportation Problem. By integrating Fermatean fuzzy modeling with fractional optimization and three-dimensional transportation structures, the proposed framework effectively handles uncertainty in complex logistics systems. The transformation of fuzzy parameters into crisp equivalents, combined with SDR-based optimization and iterative refinement techniques, ensures feasible and efficient solutions. The methodology not only improves computational tractability but also enhances decision-making accuracy. Future work may focus on extending this approach to dynamic and multi-objective transportation models under advanced fuzzy environments.

References

1. Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. Management Science, 17(4), 141–164.
2. Bit, A. K., Biswal, M. P., & Alam, S. S. (1993). Fuzzy programming approach to solid transportation problems. Fuzzy Sets and Systems, 57(2), 183–194.
3. Charnes, A., & Cooper, W. W. (1962). Programming with linear fractional functionals. Naval Research Logistics Quarterly, 9, 181–186.
4. Das, S., & Chakraborty, D. (2015). Fractional transportation problems under fuzzy environments. Applied Mathematical Modelling, 39(10), 2816–2828.
5. Ebrahimnejad, A., & Verdegay, J. L. (2018). A new approach for fuzzy fractional programming. Fuzzy Optimization and Decision Making, 17(2), 179–200.
6. Gupta, A., & Kumar, A. (2012). Solid transportation problems: A review. International Journal of Mathematics in Operational Research, 4(3), 269–291.
7. Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics, 20, 224–230.
8. Jiménez, F., & Verdegay, J. L. (1998). Solving fuzzy solid transportation problems. Fuzzy Sets and Systems, 94(3), 311–318.
9. Kaufmann, A., & Gupta, M. M. (1991). Introduction to Fuzzy Arithmetic. Van Nostrand Reinhold.
10. Kaur, A., & Kumar, A. (2014). Ranking-based solution of fuzzy transportation problems. Applied Mathematical Modelling, 38(23), 5507–5520.
11. Koopmans, T. C. (1949). Optimum utilization of the transportation system. Econometrica, 17, 136–146.
12. Pandian, P., & Natarajan, G. (2010). A new method for solving solid transportation problems. Applied Mathematical Sciences, 4(38), 1853–1864.
13. Senapati, T., & Yager, R. R. (2019). Fermatean fuzzy sets. Journal of Ambient Intelligence and Humanized Computing, 11, 663–674.
14. Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy weighted averaging and geometric operators. International Journal of Intelligent Systems, 35(2), 206–232.

15. Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy aggregation operators. International Journal of Intelligent Systems, 35(2), 206–232.
16. Toksarı, M. D. (2007). Taylor series approach to fuzzy fractional programming. Information Sciences, 177(16), 3299–3314.
17. Verdegay, J. L. (1984). Fuzzy mathematical programming. Fuzzy Sets and Systems, 12(3), 223–229.
18. Yager, R. R. (2013). Pythagorean fuzzy subsets. IFSA–NAFIPS Proceedings, 57–61.
19. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353.
20. Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems, 1(1), 45–55.