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The Market Capitalization of Internet Stocks Using in Birth –Death Process

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ABSTRACT
This paper attempts to demonstrate that the high volatility of share prices can nevertheless be used in building a model that leads to a particular cross-sectional size distribution. The model focuses on both transient and steady-state behavior of the market capitalization of the stock, which in turn is modeled as a birth–death process.

Keywords: Biotechnology and internet stocks; asset pricing; convergence rate; volatility

1. INTRODUCTION
Issuing stocks is arguably the most important way for growth companies to finance their projects, and in turn helps transfer new ideas into products and services for the society. Although the content of growth stocks may change over time perhaps consisting of railroad and utility stocks in the early 1900s, and biotechnology and internet stocks in 2000 studying the general properties of growth stocks is essential for understanding financial markets and economic growth.
However, uncertainty is manifest for growth stocks. For example, as demonstrated in the recent market from 1999 to 2002, (a) growth stocks tend to have low or even negative earnings;(b) the volatility of growth stocks is high both their daily appreciation and depreciation rates are high; (c) it is difficult to predict the future growth rates and earnings. Consequently, it poses a great challenge to derive a meaningful mathematical model within the classical valuation framework, such as the net-present-value method which relies on current earnings and the prediction of future earnings.
Indeed, since it appears that the only thing that we are sure about growth stocks is their uncertainty, we may wonder whether there is much more to say about them. The current paper attempts to illustrate that a mathematical model for growth stocks can, nevertheless, be built via birth–death processes, mainly by utilizing the *high volatility* of their share prices.
One motivation of the current study comes from a report on internet stocks in the researchers at Credit Suisse First Boston observed that 'there is literally a mathematical relationship between the ranking of the internet stock and its capitalization'. This observation is summarized later in a research report by Mauboussin and Schay (2000), it is suggested that a linear downward pattern emerges when the market capitalizations of internet stocks are plotted against their associated ranks on a log–log scale, with rank one being the largest market capitalization. Also reported that this phenomenon does not seem to hold

for nongrowth stocks. The article challenges people to investigate whether such a phenomenon happens simply by chance or if there is certain mechanism behind it.

2. MATHEMATICAL MODEL

In modeling growth stocks, instead of working on the price of a growth stock, it makes more sense to study the market capitalization, defined as the product of the total number of outstanding shares and the market price of the stock, because growth stocks tend to have frequent stock splits, which immediately makes the price drop significantly but has little effect on the market capitalization.

Consider at time t a growth stock with a total market capitalization M(t). We postulate that

$M(t) = \Theta(t)X(t)$,

Where Θ (t) represents the overall economic and sector trend and *X(t)* represents each individual variation within the sector. Hence, Θ (t) is the same for all firms within the same industry sector, and the individual variation term *X(t)* varies for different firms within the sector.

The individual variation term *X(t)* is modeled as a birth–death process: given that *X(t)* is in state i, the instantaneous changes are $i \rightarrow i+1$, with rate $i\lambda + g$ for $i \geq 0$, and $i \rightarrow i+1$, with rate $i\mu + h$ for $i \geq 1$, where the parameters are such that $\lambda, \mu > 0$, $g > 0$, $h \geq 0$, $\lambda < \mu$. The unit of *X(t)* could be, for example, millions or billions of dollars.

Under the standard notation, *X(t)* is a birth–death process with the birth rate λ_i and the death rate μ_i satisfying

$$\lambda_i = i\lambda + g, \qquad \mu_i = i\mu + h, \quad i \geq 1,$$
$$\lambda_0 = g, \qquad \mu_0 = 0,$$

and the infinitesimal generator of *X(t)* is given by the infinite matrix

$$\begin{pmatrix} -g & g & 0 & 0 & \cdots \\ \mu + h & -\lambda-\mu-g-h & \lambda+g & 0 & \cdots \\ 0 & 2\mu + h & -2\lambda-2\mu-2g-2h & 2\lambda+g & \cdots \\ \vdots & \vdots & \ddots & \ddots & \ddots \end{pmatrix}$$

In the model , the state 0 only signifies that the size of X(t) is below a certain minimal level. It does not imply, for example , that the company goes bankrupt .

The two parameters λ and μ represent the instantaneous appreciation and depreciation rates of *X(t)* due to market fluctuation; the model assumes that they influence *X(t)* proportionally to the current value. In general, because of the difficulty of predicting the instantaneous upward and downward price movements (partly thanks to the efficient market hypothesis), for both growth stocks and nongrowth stocks λ and μ must be quite close, $\lambda/\mu \approx 1$; in addition, for growth stocks, both λ and μ must be large, because of their high volatility. The requirement that $\lambda < \mu$ is postulated here to ensure that the birth–death process *X(t)* has a steady-state distribution.

The parameter g > 0 models the rate of increase in X(t) due to nonmarket factors, such as the effect of additional shares being issued through public offerings or the effect of warranties on the stock being exercised .The parameter h attempts to capture the rate of decrease in X(t) due to nonmarket factors, such as the effect of dividend payments. For most growth stocks, h ≈0, as no dividends are paid.

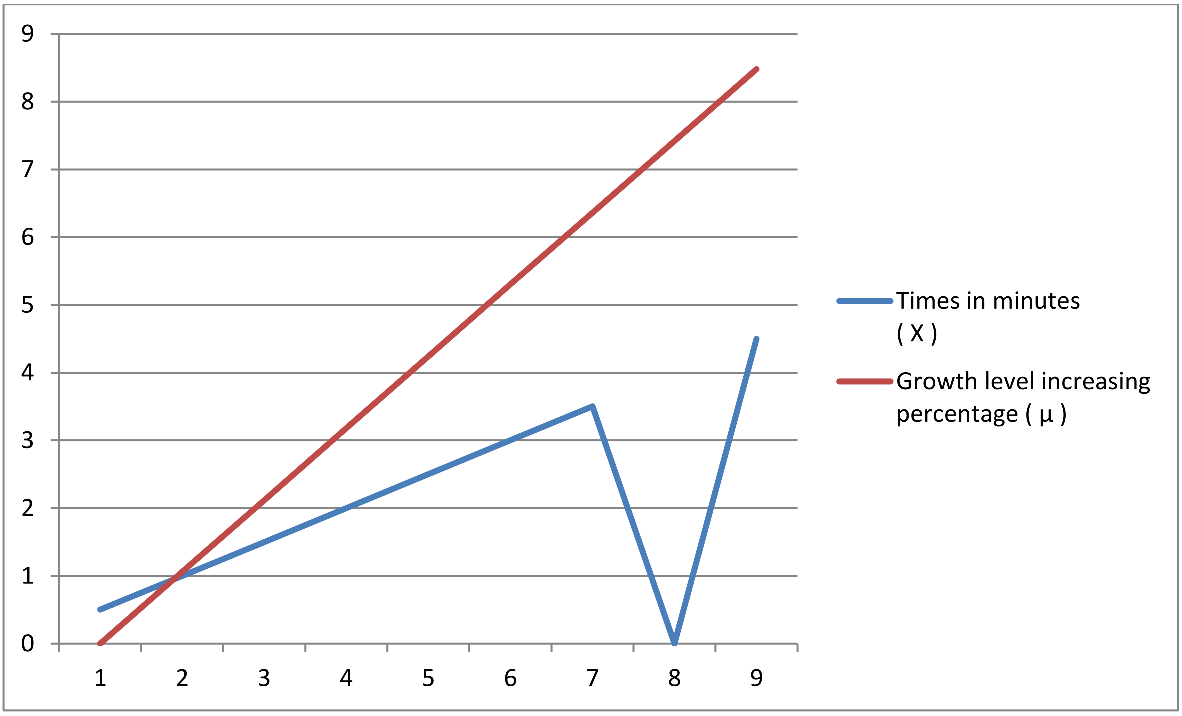
The parameter of the value in exponential distribution is μ =0.06 and λ = 0.07.

Duration of the time intervals $= \frac{8.5\ minutes}{17\ samples}$ = 0.5 minutes, therefore 1 sample to produces a minimum time taken as 0.5 minutes implies 0.56 % (Growth level increasing percentage upward)

Table -2.1

Times in minutes (X)	0.5	1	1.5	2	2.5	3	3.5	4.0	4.5
Growth level increasing percentage (μ)	0	1.06	2.12	3.18	4.24	5.3	6.36	7.42	8.48

Graph -2.2



Birth Process

Table -2.3

Times in minutes (X)	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5
Non Growth level decreasing percentage (λ)	1.7	2.77	3.84	4.91	5.98	7.05	8.12	9.19	10.26

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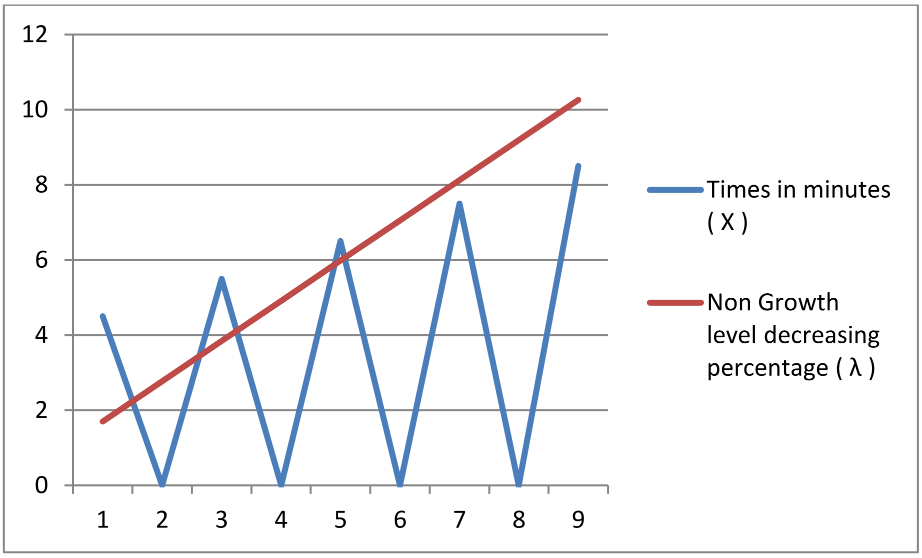
Graph -2.4



Death Process

Two sample paths of the birth –death process with as

Graph -1 X(0) =150 , μ =0.06 , h =8

Graph -2 X(0) = 150, λ = 0.07 , g =9

1. Although the individual stock's valuation x(t) is assumed to have a steady – state distribution the over all economic trend Θ (t) can have a positive drift.
2. In general neither x(t) nor Θ (t) is likely to be observed directly in the market. Instead only M(t) is directly observed in the market.
3. We Provide a detailed analysis of both transient and steady-state behavior not just the steady-state analysis. The transient analysis not only presents some mathematical challenges, but also is essential to understand, why the theory of the size distribution is useful for growth stocks but not for nongrowth stocks.

In graph-1 provides an illustration of the model by showing the sample paths of two realizations of the birth-death process x(t) for about 8.5 minutes. In graph-1 the instantaneous jump rates λ and μ are small , while in graph-2 λ and μ are large.

The sample path suggests two points:

- For reasonably large λ and μ the jumps of the birth-death processes are almost unnoticeable and the overall sample paths fit in well with only intuition of market fluctuation.
- Although λ < μ , the sample paths of x(t) may still have some strong upward movements if λ is close to μ. For example, In graph-1 , x(t) increases from about 0.5 to 8.5 (more than 17 samples) within a short period. (about 0.5 minutes)

3. THE STEADY-STATE DISTRIBUTION

The steady-state measure of a birth-death process is given by

π_0=1, $\pi_n := \frac{\lambda_o \lambda_1 \dots \lambda_{n-1}}{\mu_1 \dots \mu_n}$, n =1,2,….

Normalizing { π_n} provides the steady-state distribution of the birth –death process:

$\lim_{t \to \infty} P(X(t) = n) = \frac{\pi_n}{S}$, $S := \sum_{n=0}^{\infty} \pi_n$

Below for the finiteness of S under the setting of in our case,

$$\pi_n = \left(\frac{\lambda}{\mu}\right)^n \frac{\left(\frac{g}{\lambda}\right)\left(1+\frac{g}{\lambda}\right)\left(2+\frac{g}{\lambda}\right)\ldots\left((n-1)+\frac{g}{\lambda}\right)}{\left(1+\frac{h}{\mu}\right)\left(2+\frac{h}{\mu}\right)\ldots\left(n+\frac{h}{\mu}\right)}, \; n \geq 1.$$

Using the gamma function , it can be succinctly expressed as

$$\pi_n = \frac{\Gamma\left(1+\frac{h}{\mu}\right)}{\Gamma(g/\lambda} \left(\frac{\lambda}{\mu}\right)^n \frac{\Gamma(n+g/\lambda)}{\Gamma\left(n+1+\frac{h}{\mu}\right)}, \; n \geq 0.$$

4. TRANSIENT MEAN AND VARIANCE

Only provides steady-state properties of X(t) , which might be relevant id the birth-death process X(t) has been run for a long times, ie, the stock has been traded in the market for a long period. However , it is quite possible that the parameters λ ,μ ,g, and h may have changed during the period, thus altering the steady-state distribution. Therefore, practically the steady-state properties are relevant only if the convergence from the transient states to the steady states is fast enough, ie, if the convergence can be observed in a timely fashion.

There are several ways to judge the convergence speed, we shall focus on the mean and variance of the transient distribution, which can lead to a measure of the convergence rate, a more accurate measure is the convergence rate for the transition probabilities , which attempts to capture the convergence of the whole distribution rather than just the first two moments.

Denote the transition probability at time t by

$\boldsymbol{p_{i,j}}(t) := P(x(t) = j \mid X(0) = i)$

The transient expectation at time t by

$\boldsymbol{m_1}(t) := EX(t) = \sum_{j=0}^{\infty} \boldsymbol{j}\, \boldsymbol{p_{i,j}}(t)$,

and the second moment by

$\boldsymbol{m_2}(t) := EX^2(t) = \sum_{j=0}^{\infty} \boldsymbol{j}^2\, \boldsymbol{p_{i,j}}(t)$,

5. THE SIZE DISTRIBUTION FOR GROWTH STOCKS

In this section we shall apply the results on both the steady-state and the transient behavior of the model to study the size distribution of growth stocks since, for most growth stocks, there is no dividend payment, we shall assume from this section on that h = 0 , Basic transient and steady-state properties for h=0 the steady-state measure for X(t) is $\boldsymbol{\pi_n} = \frac{1}{\Gamma(g/\lambda} \left(\frac{\lambda}{\mu}\right)^n \frac{\Gamma(n+g/\lambda)}{n!}$

and

$F(n) = \sum_{k=n}^{\infty} \frac{\pi k}{S} = \frac{\pi_n F\left(n+\frac{g}{\lambda}, 1; n+1; \frac{\lambda}{\mu}\right)}{s}$

In addition ,

$S = \sum_{k=0}^{\infty} \boldsymbol{\pi_k} = F\left(\frac{g}{\lambda}, 1; 1; \frac{\lambda}{\mu}\right) = \left(1 - \frac{\lambda}{\mu}\right)^{-g/\lambda}$

Thanks to the following property of the hyper geometric function : $F(a, b; b; z) = (1-Z)^{-a}$. it gives

$F(n) = \lim_{t\to\infty} P(X(t) \geq n) \cong \frac{1}{\Gamma(g/\lambda} \left(1 - \frac{\lambda}{\mu}\right)^{g/\lambda - 1} \left(\frac{\lambda}{\mu}\right)^n n^{g/\lambda - 1}$

the moment –generating function of the steady –state distribution , under h = 0 , is

$\eta(\theta) = \left(\frac{\mu - \lambda e^{\theta}}{\mu - \lambda}\right)^{-g/\lambda}$

Thus for the steady-state distribution, the first two moments are

$m_1 = \eta'(0) = \frac{g}{\mu - \lambda}$,

$m_2 = \eta'(0) = \frac{g(\mu + g)}{(\mu - \lambda)^2}$,

and the variance is

$m_2 - m_1^2 = \frac{\mu g}{(\mu - \lambda)^2}$

For the properties of the transient behavior, the decay parameter which measures the speed of convergence to the steady state in an exponential way, is given by

$\gamma = \mu - \lambda$

Secondly ,

$m_1(t) = ie^{(\lambda-\mu)t} + \frac{g}{\mu - \lambda}\left[1 - e^{(\lambda-\mu)t}\right]$,

$m_2(t) = i^2 e^{2(\lambda-\mu)t} + i\,\frac{\lambda+\mu+2g}{\lambda-\mu}\left(e^{2(\lambda-\mu)t} - e^{(\lambda-\mu)t}\right)$

$+ \frac{g}{2(\mu-\lambda)}\left[1 - e^{2(\lambda-\mu)t}\right] + \frac{(\lambda+\mu+2g}{2(\mu-\lambda)^2}\left(1 - e^{(\lambda-\mu)t}\right)^2$

The exponents in $m_1(t)$ and $m_2(t)$ are all related to - μ , which also points out, from a different viewpoint, that μ - λ should affect thee speed of convergence in an exponential way. In addition, it is easily seen that

$\lim_{1\to\infty} m_1(t) = \frac{g}{\mu - \lambda} = m_1$

$\lim_{1\to\infty} m_2(t) = \frac{g(\mu+g)}{(\mu-\lambda)^2} = m_2$

6. THE SIZE DISTRIBUTION

Consider *N* growth firms within a particular sector, whose market capitalizations are governed by the model Suppose that, among these *N* firms, we observe the *K* largest Denote the market capitalization of the *K* observed stocks by $M_i(t)$, $1 \le i \le K$. Since all these *K* firms are from the same sector, we have

$M_i(t) = \Theta(t) X_i(t), \quad 1 \le i \le K,$

where *Θ(t)*, the overall economic and sector trend, is the same for all *K* stocks; but the individual variation terms $X_i(t)$ are different. Now suppose that we rank the market capitalizations such that $M_{(1)}(t) > M_{(2)}(t) > \ldots > M_{(K)}(t)$, where $M_{(1)}(t)$ denotes the largest firm, $M_{(2)}(t)$ the second largest firm, etc. Then we have $\log M_{(j)}(t) = \log \Theta(t) + \log(X_{(j)}(t))$

7. CONCLUSION

The main contribution of the current paper is that it provides an understanding of the size distribution for growth stocks, by building a stochastic model. There are three useful properties of the model. First, the model leads to cross-sectional equation for growth stocks, including both biotechnology and internet stocks. Second , the cross-sectional model only uses regression and relative ranks and thus easy to implement Third, the cross-sectional model remains valid irrespective to the market ups and downs, mainly because the model compares the relative value of a stock against the other stocks within its peer group.

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