International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Geodetic Domination on Adjacent Clique of Size-3

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Abstract:
The geodetic set and Geodetic dominating set of these graphs are obtained. We also studied and proved the results on minimum geodetic number and minimum geodetic domination number of these graphs. Also the results obtained are illustrated by graphs.

Keywords: Dominating set, Geodetic set, Geodetic dominating set, Domination number, Geodetic domination number.

1.1 Introduction

The theory of domination in graphs introduced by Ore [7] and Berge [2] is an emerging area of research in graph theory today. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et.al. [4,5]. Many graph theorists, to mention some of them Allan and Laskar.[1], Cockayne and Hedetniemi [3], and others have studied various types of domination parameters of graphs. M Reddappa, C Jaya Subba Reddy, and B Maheswari [8] are studied on " Roman domination on adjacent clique of size-3".

1.2 Preliminaries

A set S of vertices in a graph G is uniform if the distance between every two distinct vertices of S is the same fixed number. A geodetic set is essential if for every two distinct vertices u,v ∈ S, there exists a third vertex w of G that lies in some u-v geodesic but in no x-y geodesic for x, y ∈ S and {x,y} ≠ {u,v}. A clique of a simple graph G is S of V such that $< S >$ is complete.

Dominating set: Let $G(V,E)$ be a graph. A subset D of V is said to be a dominating set of G if every vertex in $V-D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set is called as the domination number and is denoted by $\gamma(G)$.

Dominating Function of a graph: Let $G(V,E)$ be a graph. A function $f:V\rightarrow[0,1]$ is called a dominating function of G if

$$f(N[v]) = \sum_{u\in N[v]} f(u) \geq 1, for each v \in V,$$

where $N[v]$ is the closed neighbourhood of the vertex v.

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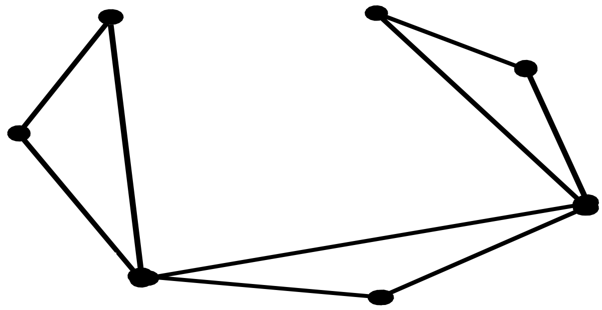
1.3 Geodetic domination in a graph with adjacent clique of Size 3



In what follows we consider graphs of this type **G**. We denote this type of interval graph by **G**. The domination and the Geodetic domination is studied in the following for the graph **G**.

Theorem 1.3.1: Let **G** be the adjacent clique of size3 with n vertices, where $n \geq 5$. Then the geodetic number of **G** is

$g(\mathbf{G}) = m + 3$ for $n = 2m + 3, 2m + 4,$ where $m = 1, 2, 3, \ldots \ldots \ldots$ respectively.

Proof: Let **G** be the adjacent clique of size3 with n vertices, where $n \geq 5$.

Case1: Suppose $n = 2m + 3$, where $m = 1, 2, 3, \ldots \ldots$

Let $V = \{v_1, v_2, v_3, \ldots \ldots . v_n\}$ be the vertices of **G**.

Let us consider $S = \{v_1, v_2, v_4 \ldots \ldots . v_{n-3}, v_{n-1}, v_n\}$ be the subset of V.

We have to show that S is a geodetic set of **G**.

That is $I[S] = V(\mathbf{G})$

Let $S_i = I[S] = \cup_{v_i, v_n \in S} I[v_i, v_n]$, where $i = 1, 2, 4 \ldots \ldots n - 3, n - 1$

Sub Case(i): Suppose $n = 5$, we have

$V = \{v_1, v_2, v_3, v_4, v_5\}$ and $S = \{v_1, v_2, v_4, v_5\}$

Now
$$S_1 = I[v_1, v_5] = \{v_1, v_3, v_5\}$$
$$S_2 = I[v_2, v_5] = \{v_2, v_3, v_5\}$$
$$S_4 = I[v_4, v_5] = \{v_4, v_5\}$$

So, $I[S] = S_1 \cup S_2 \cup S_4$
$$= I[v_1, v_5] \cup I[v_2, v_5] \cup I[v_4, v_5]$$
$$= \{v_1, v_3, v_5\} \cup \{v_2, v_3, v_5\} \cup \{v_4, v_5\}$$
$$= \{v_1, v_2, v_3, v_4, v_5\}$$

$= V(\mathbf{G})$.

Hence $S = \{v_1, v_2, v_4, v_5\}$ is a minimum geodetic set of **G**.

So, the geodetic number is $g(\mathbf{G}) = 4$.

Sub Case(ii): Suppose $n = 7$.

Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $S = \{v_1, v_2, v_4, v_6, v_7\}$

Now $S_1 = I[v_1, v_7] = \{v_1, v_3, v_5, v_7\}$
$$S_2 = I[v_2, v_7] = \{v_2, v_3, v_5, v_7\}$$
$$S_4 = I[v_4, v_7] = \{v_4, v_5, v_7\}$$
$$S_6 = I[v_6, v_7] = \{v_6, v_7\}$$

So, $I[S] = S_1 \cup S_2 \cup S_4 \cup S_6$

$$= I[v_1, v_7] \cup I[v_2, v_7] \cup I[v_4, v_7] \cup I[v_6, v_7]$$
$$= \{v_1, v_3, v_5, v_7\} \cup \{v_2, v_3, v_5, v_7\} \cup \{v_4, v_5, v_7\} \cup \{v_6, v_7\}$$
$$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

$= V(\mathbf{G})$.

Thus $S = \{v_1, v_2, v_4, v_6, v_7\}$ is a minimum geodetic set of $\mathbf{G}$. So the geodetic number is $g(\mathbf{G}) = 5$.

Sub Case (iii): Suppose $n = 9$.

Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and $S = \{v_1, v_2, v_4, v_6, v_8, v_9\}$

Now $S_1 = I[v_1, v_9] = \{v_1, v_3, v_5, v_7, v_9\}$

$$S_2 = I[v_2, v_9] = \{v_2, v_3, v_5, v_7, v_9\}$$
$$S_4 = I[v_4, v_9] = \{v_4, v_5, v_7, v_9\}$$
$$S_6 = I[v_6, v_9] = \{v_6, v_7, v_9\}$$
$$S_8 = I[v_8, v_9] = \{v_8, v_9\}$$

So, $I[S] = S_1 \cup S_2 \cup S_4 \cup S_6 \cup S_8$

$$= I[v_1, v_9] \cup I[v_2, v_9] \cup I[v_4, v_9] \cup I[v_6, v_9] \cup I[v_8, v_9]$$
$$= \{v_1, v_3, v_5, v_7, v_9\} \cup \{v_2, v_3, v_5, v_7, v_9\} \cup \{v_4, v_5, v_7, v_9\} \cup \{v_6, v_7, v_9\} \cup \{v_8, v_9\}$$
$$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$$

$= V(\mathbf{G})$.

Hence $S = \{v_1, v_2, v_4, v_6, v_8, v_9\}$is a minimum geodetic set of $\mathbf{G}$. So the geodetic number is $g(\mathbf{G}) = 6$.

Thus $g(\mathbf{G}) = 4$ for $n = 5$

$= 5$ for $n = 7$

$= 6$ for $n = 9$

Generalizing, we get that the general form of geodetic set of $\mathbf{G}$ as

$S = \{v_1, v_2, v_4 \ldots \ldots . v_{n-3}, v_{n-1}, v_n\}$ for $n = 5, 7, 9 \ldots \ldots \ldots.$

So, the geodetic set can be written as

$g(\mathbf{G}) = \{v_1\} + \{v_2, v_4, v_5\}$ for $n = 5$

$= \{v_1, v_2\} + \{v_4, v_6, v_7\}$ for $n = 7$

$= \{v_1, v_2, v_4\} + \{v_6, v_8, v_9\}$ for $n = 9$

And so on

$$= \{v_1, v_2, v_4 \ldots \ldots v_{n-5}\} + \{v_{n-3}, v_{n-1}, v_n\}$$

$= m + 3$, where $m = \{v_1, v_2, v_4 \ldots \ldots v_{n-5}\}$

Thus $g(\mathbf{G}) = m + 3$, for $n = 2m + 3$, where $m = 1, 2, 3, \ldots$ ....respectively.

Case2: suppose $n = 2m + 4$, where $m = 1, 2, 3, \ldots$ ....

Let $V = \{v_1, v_2, v_3, \ldots \ldots . v_n\}$ be the vertices of $\mathbf{G}$.

Let us consider $S = \{v_1, v_2, v_4 \ldots \ldots . v_{n-4}, v_{n-2}, v_n\}$ be the subset of V.

We have to show that S is a geodetic set of $\mathbf{G}$.

That is$I[S] = V(\mathbf{G})$

Let $S_i = I[S] = \cup_{v_i, v_n \in S} I[v_i, v_n]$, where $i = 1, 2, 4 \ldots \ldots n-4, n-2$

Sub Casa(i): Suppose $n = 6$, we have

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $S = \{v_1, v_2, v_4, v_6\}$

Now $S_1 = I[v_1, v_6] = \{v_1, v_3, v_5, v_6\}$

$$S_2 = I[v_2, v_6] = \{v_2, v_3, v_5, v_6\}$$
$$S_4 = I[v_4, v_6] = \{v_4, v_5, v_6\}$$

So, $I[S] = S_1 \cup S_2 \cup S_4$

$$= I[v_1, v_6] \cup I[v_2, v_6] \cup I[v_4, v_6]$$
$$= \{v_1, v_3, v_5, v_6\} \cup \{v_2, v_3, v_5, v_6\} \cup \{v_4, v_5, v_6\}$$
$$= \{v_1, v_2, v_3, v_4, v_5, v_6\}$$

$= V(\mathbf{G})$.

Hence $S = \{v_1, v_2, v_4, v_6\}$is a minimum geodetic set of **G**.

So, the geodetic number is $g(\mathbf{G}) = 4$.

Sub Case(ii): Suppose $n = 8$.

Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $S = \{v_1, v_2, v_4, v_6, v_8\}$

Now $S_1 = I[v_1, v_8] = \{v_1, v_3, v_5, v_7, v_8\}$

$$S_2 = I[v_2, v_8] = \{v_2, v_3, v_5, v_7, v_8\}$$
$$S_4 = I[v_4, v_8] = \{v_4, v_5, v_7, v_8\}$$
$$S_6 = I[v_6, v_8] = \{v_6, v_7, v_8\}$$

So, $I[S] = S_1 \cup S_2 \cup S_4 \cup S_6$

$$= I[v_1, v_8] \cup I[v_2, v_8] \cup I[v_4, v_8] \cup I[v_6, v_8]$$
$$= \{v_1, v_3, v_5, v_7, v_8\} \cup \{v_2, v_3, v_5, v_7, v_8\} \cup \{v_4, v_5, v_7, v_8\} \cup \{v_6, v_7, v_8\}$$
$$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$$

$= V(G_2)$.

Thus $S = \{v_1, v_2, v_4, v_6, v_8\}$is a minimum geodetic set of **G**. So the geodetic number is $g(\mathbf{G}) = 5$.

Sub Case(iii): Suppose $n = 10$.

Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ and $S = \{v_1, v_2, v_4, v_6, v_8, v_9\}$

Now $S_1 = I[v_1, v_{10}] = \{v_1, v_3, v_5, v_7, v_9, v_{10}\}$

$$S_2 = I[v_2, v_{10}] = \{v_2, v_3, v_5, v_7, v_9, v_{10}\}$$
$$S_4 = I[v_4, v_{10}] = \{v_4, v_5, v_7, v_9, v_{10}\}$$
$$S_6 = I[v_6, v_{10}] = \{v_6, v_7, v_9, v_{10}\}$$
$$S_8 = I[v_8, v_{10}] = \{v_8, v_9, v_{10}\}$$

So, $I[S] = S_1 \cup S_2 \cup S_4 \cup S_6 \cup S_8$

$$= I[v_1, v_{10}] \cup I[v_2, v_{10}] \cup I[v_4, v_{10}] \cup I[v_6, v_{10}] \cup I[v_8, v_{10}]$$
$$= \{v_1, v_3, v_5, v_7, v_9, v_{10}\} \cup \{v_2, v_3, v_5, v_7, v_9, v_{10}\} \cup \{v_4, v_5, v_7, v_9, v_{10}\}\{v_6, v_7, v_9, v_{10}\} \cup \{v_8, v_9, v_{10}\}$$
$$= \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$$

$= V(\mathbf{G})$.

Hence $S = \{v_1, v_2, v_4, v_6, v_8, v_9\}$is a minimum geodetic set of **G**. So the geodetic number is $g(\mathbf{G}) = 6$.

Thus $g(\mathbf{G}) = 4$ for $n = 6$

$= 5$ for $n = 8$

$= 6$ for $n = 10$

Generalizing, we get that the general form of geodetic set of **G** as

$S = \{v_1, v_2, v_4 \dots \dots . v_{n-4}, v_{n-2}, v_n\}$ for $n = 6, 8, 10 \dots \dots \dots$

So, the geodetic set can be written as

$g(\mathbf{G}) = \{v_1\} + \{v_2, v_4, v_6\}$ for $n = 6$

$= \{v_1, v_2\} + \{v_4, v_6, v_8\}$ for $n = 8$

$= \{v_1, v_2, v_4\} + \{v_6, v_8, v_{10}\}$ for $n = 10$

And so on

$= \{v_1, v_2, v_4 \ldots\ldots v_{n-6}\} + \{v_{n-4}, v_{n-2}, v_n\}$

$= m + 3$, where $m = \{v_1, v_2, v_4 \ldots\ldots v_{n-5}\}$

Thus $g(\mathbf{G}) = m + 3$, for $n = 2m + 4$, where $m = 1, 2, 3, \ldots$ ....respectively.

From Case1, Case2, we have

$g(\mathbf{G}) = m + 3$ for $n = 2m + 3, 2m + 4$, where $m = 1, 2, 3, \ldots \ldots \ldots$ respectively.

Theorem 1.3.2: Let $\mathbf{G}$ be the adjacent clique of size3 with n vertices, where $n \geq 5$. Then the geodetic domination number of $\mathbf{G}$ is

$\gamma_g(G_2) = m + 3$ for $= 2m + 3, 2m + 4$, where $m = 1, 2, 3, \ldots \ldots \ldots$ respectively.

Proof: Let $\mathbf{G}$ be the adjacent clique of size3 with n vertices, where $n \geq 5$.

Suppose $m = 1$. Then $n = 5, 6$. For $n = 5, 6$, we can see that $GDS = \{v_1, v_2, v_4, v_5\}$; and $GDS = \{v_1, v_2, v_4, v_6\}$ are minimum geodetic dominating sets of G_2 respectively. Thus $\gamma_g(\mathbf{G}) = 4$ for $n = 5, 6$.

Similar is the case for $n = 7, 8$, where the geodetic dominating sets are respectively $GDS = \{v_1, v_2, v_4, v_6, v_7\}$; $GDS = \{v_1, v_2, v_4, v_6, v_8\}$and the geodetic domination number $\gamma_g(\mathbf{G}) = 5$.

Again for $n = 9, 10$, we see that $\gamma_g(\mathbf{G}) = 6$ and the geodetic dominating sets are $GDS = \{v_1, v_2, v_4, v_6, v_8, v_9\}$; $GDS = \{v_1, v_2, v_4, v_6, v_8, v_{10}\}$ respectively.

Thus $\gamma_g(\mathbf{G}) = 4$ for $n = 5, 6$

$= 5$ for $n = 7, 8$

$= 6$ for $n = 9, 10$

Generalizing, we get that the general form of a geodetic dominating sets of $\mathbf{G}$ is

$GDS = \{v_1, v_2, v_4, \ldots \ldots \ldots \ldots\ldots v_{n-3}, v_{n-1}, v_n\}$ for $n = 5, 7, 9, \ldots \ldots$ ....

$GDS = \{v_1, v_2, v_4, \ldots \ldots \ldots \ldots\ldots v_{n-4}, v_{n-2}, v_n\}$ for $n = 6, 8, 10 \ldots \ldots$ ....

and so on.

Thus $\gamma_g(\mathbf{G}) = m + 3$ for $= 2m + 3, 2m + 4$, where $m = 1, 2, 3, \ldots \ldots \ldots$ respectively.

Theorem 1.3.3: Let $\mathbf{G}$ be the adjacent clique of size3 with n vertices, where $n \geq 5$. Then show that $\gamma_g(\mathbf{G}) = g(\mathbf{G})$.

Proof: Let $\mathbf{G}$ be the adjacent clique of size3 with n vertices, where $n \geq 5$.

By Theorem 1.3.1, we have

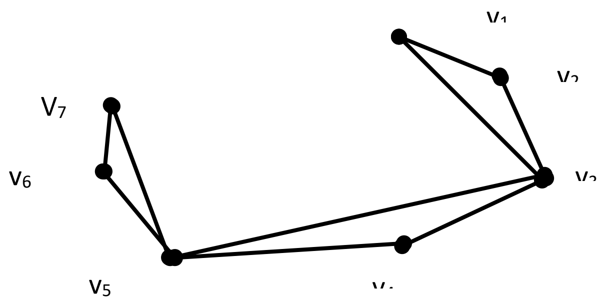
$g(\mathbf{G}) = m + 3$ for $n = 2m + 3, 2m + 4$, where $m = 1, 2, 3, \ldots \ldots \ldots$ respectively.

And by Theorem 1.3.2, we have

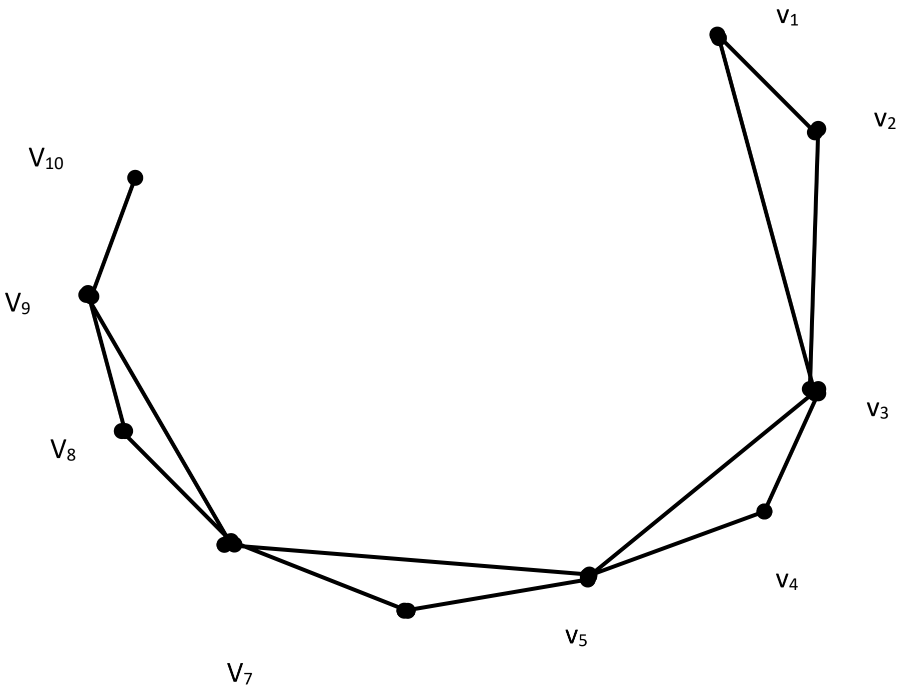
$\gamma_g(\mathbf{G}) = m + 3$ for $= 2m + 3, 2m + 4$, where $m = 1, 2, 3, \ldots \ldots \ldots$ respectively.

Clearly $\gamma_g(\mathbf{G}) = g(\mathbf{G})$.

1.4 Illustrations(i) n=7



$$GDS = \{v_1, v_2, v_4, v_6, v_7\}$$
$$\gamma_g(G_1) = 5$$



(ii) n=10

$$GDS = \{S = \{v_1, v_2, v_4, v_6, v_8, v_9\}$$

$$\gamma_g(G_1) = 6.$$

1.5. Conclusion:

In this paper we have to study geodetic and geodetic domination on consecutive clique of size-3. We have to use in these graphs are Computer net works, In Army and Navy etc.

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