Square Sum Slither Labeling of Certain Graphs

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Abstract:
Labeling is a part of graph theory that has greater applications in various spheres of science and technology. Apart from science and technology, they play a vital role in day to day life commuting. Hence the concept of labeling possesses rich gamut of applications even far beyond human intellect. In this paper, an attempt has been made to prove that certain graphs admit square sum slither labeling.

Keywords: Square sum labeling, Slither labeling, Tessellation of K_4, Mirror Comb graph, Tessellation of Triangular Ladder graph and Double Book graph.

Introduction:
The graphs that are considered here are special graphs as they are easy to construct and have a spate of applications in their repertoire. These graphs do not require special attention to construct and comprehend. The extended grid graph play vital role in the area of interconnected networks. Starting with the Tessellation of K_4 that has wide range of applications in the inter-connected networks and other fields of science and technology, the Mirror comb graph is a special case of the path graph, Tessellation of triangular ladder graph and the double book graph, a special formation of K_3 also has wide applications.

Basic Definitions:
Square Sum Labeling:
The square sum labeling considered in this paper is due to Mitesh J. Patel and G.V. Ghodasara [2] who gave the definition as follows; "A graph G having p vertices and q edges is said to be a square summable if it is possible to define a bijective map given by $g:V(G) \rightarrow \{0,1,2,..p-1\}$ such that the induced function $g^*:E(G) \rightarrow \mathbb{N}$ given by $g^*(uv) = (g(u))^2 + (g(v))^2$, for every $uv \in E(G)$ is an injective map". J.A. Gallian [1] effectively compiled the different types of labeling for different graphs which remains as benchmark for graph theoretic researchers to come up with manipulations of labeling types and apply them for a variety of graphs.

Slither Labeling:
The labeling of the vertices or edges or both that is done in the slither formation is known as the slither labeling. Those graphs that admit the square sum slither labeling are said to be square sum slither labelled graphs.

Tessellation of K_4:
A K_4 graph is a complete graph on four vertices. A cycle on four vertices in which the diagonals are joined to form K_4 graph. Tessellation is the concept wherein two or more geometrical shapes are joined together

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without overlapping. Tessellation of K_4 denoted by T(1, n) is obtained when two or more K_4 graphs joined together without overlapping. The number of vertices in the graph is 2n + 2 and the number of edges is 5n +1. R. Sreenivasan and M.S. Paulraj [4] [5] proved that the tessellation of K_4 and the protracted tessellation of K_4 admit vertex antimagic edge labeling.

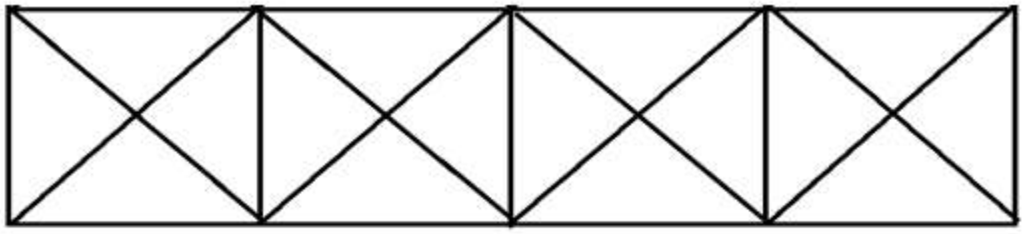

Fig 1. Tessellation of K_4 T(1,4)

Mirror Comb Graph:
The comb graph is a special case of a path wherein there is pendant vertex attached to each vertex in the path. The mirror comb graph, MC_n is a comb graph having the mirror image of the comb graph and attached together such that the path of the comb graph and its mirror image is common.

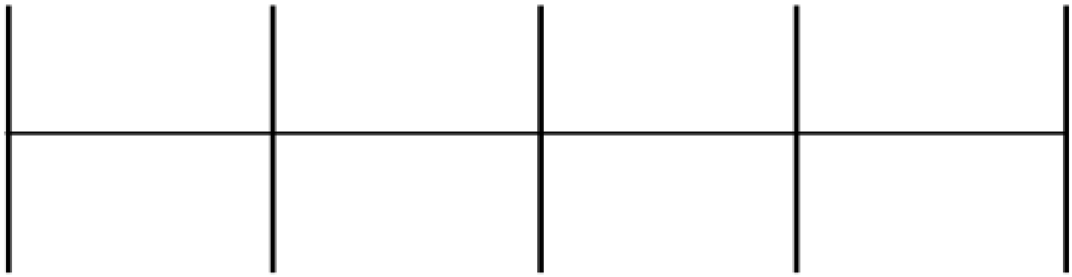

Fig 2. Mirror Comb graph MC_5

Tessellation of Triangular Ladder Graph:
R. Sreenivasan and M.S. Paulraj [6] defines the triangular ladder graph L_n as the one that is derived from the ladder graph with the addition of the connecting vertex u_i with the vertex v_{i+1}. In this paper the tessellation of triangular ladder graph is denoted by $T(L_{m,n})$. The number of edges in the graph is 3mn + m + n and the number of vertices is mn + m + n + 1. R. Sreenivasan and M.S. Paulraj [6] proved that the tessellation of triangular ladder graph admits vertex antimagic edge labeling.

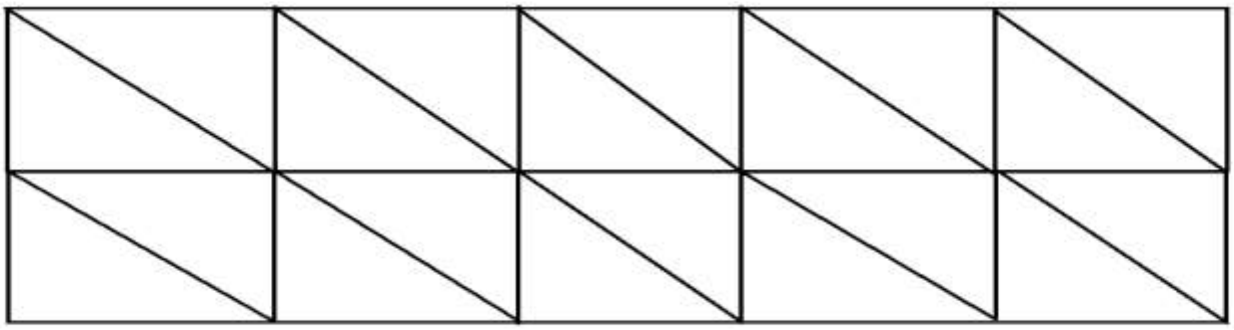

Fig 3. Tessellation of Triangular Ladder Graph $T(L_{2,5})$

Double Book Graph:

The book graph as defined by B.N. Kavitha and Indrani Kelkar [2] is the Cartesian product of a star and an edge and is denoted by B_n. A double book graph is a graph that is formed by sharing a common edge, the pivot edge, between the equal number of multiple pages and is denoted by $B_{n,n}$. The number of vertices in the double book graph is 2(n+1) and the number of edges is 4n + 1. R. Sreenivasan [8] proved that a book graph admits vertex antimagic edge slither labeling.

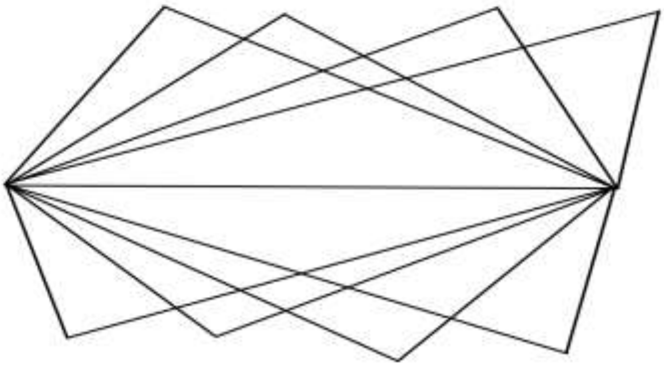

Fig 4. Double Book Graph $B_{4,4}$

Main Results:

Theorem – 1: The tessellation of K_4 graph T(1,n) admits square sum slither labeling for $n \geq 2$.

Proof: The tessellation of K_4 is formed from a cycle on four vertices in which the diagonals are joined to form K_4 graph. Consider the graph T(1,4), a graph formed on arbitrary value of n = 4. The labeling of the vertices are done in the slither form using non-negative integers. The label of an edge is the sum of squares of the labels of the vertices that are incident with it. The vertex slither labeling in T(1,4) is given by

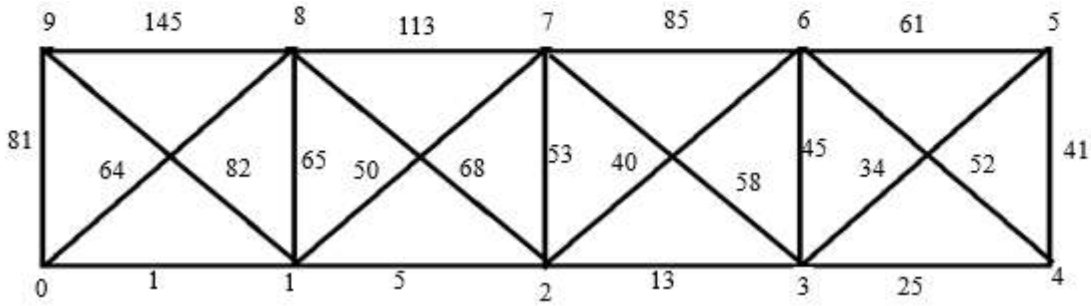

Fig 5. Square Sum Slither Labeling in T(1,4)

In the above graph, it is evident that the labels of the vertices are in slither formation with non-negative integers and one of the labels recur. The label of an edge is the sum of squares of labels of the vertices that are incident with it and they are distinct as well. Hence the graph admits square sum slither labeling. Since n = 4 is arbitrary, the above slither labeling of vertices and the square sum labeling of edges hold for all values of $n \geq 2$. Hence the graph is square sum slither labelled.

Theorem – 2: The mirror comb graph MC_n, $n \geq 2$is square sum slither labelled.

Proof: Consider the mirror comb graph MC_n that is formed on a path of n vertices. From each vertex of the path arises a pendant vertex on the either side. Slither label the vertices of the graph with non-negative integers such that none of the labels of the vertex repeats. The label of an edge is the sum of squares of labels of the vertices. Let n = 5 be arbitrary. The slither labeling of the vertices are given as follows;

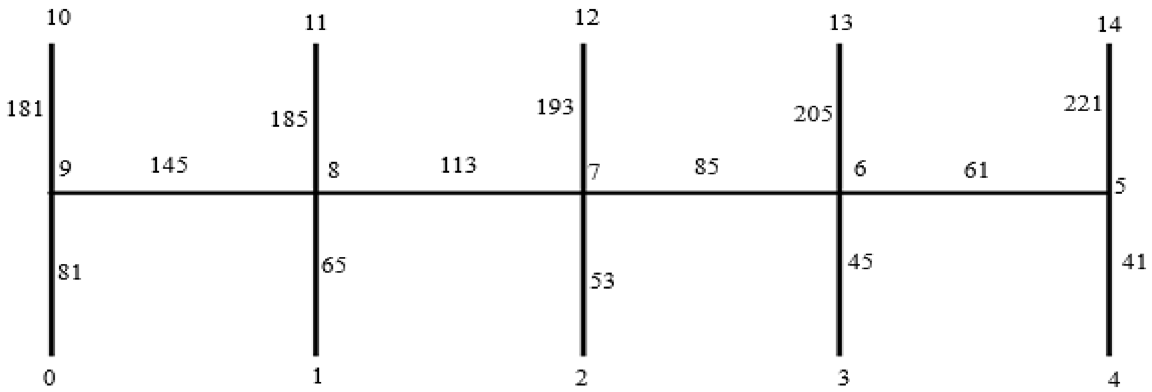

Fig 6. Square Sum Slither Labeling in MC_5

In the above graph, it is apparent that the labels of the vertices are in slither pattern assigned with non-negative integers and one of the labels repeat. The label of an edge is the sum of squares of labels of the vertices that are incident with it and they are distinct too. Hence the graph admits square sum slither labeling. As n = 5 is arbitrary, the above slither labeling of vertices and the square sum labeling of edges hold for all values of $n \geq 2$. Hence the graph is square sum slither labelled.

Theorem – 3: The tessellation of triangular ladder graph $T(L_{1,\,n})$admits square sum slither labeling.

Proof: The tessellation of triangular ladder graph is derived from the ladder graph with the addition of the connecting vertex u_i with the vertex v_{i+1}. Each of the ladder in this tessellation is $K_4 – e$. Consider the graph $T(L_{1,4})$ where 4 is an arbitrary value. Slither label the vertices of the graph in such a way that the vertices are all distinct. The label of an edge is the sum of squares of labels of the vertices that are incident with it. The slither vertex labeling in the graph is given as follows;

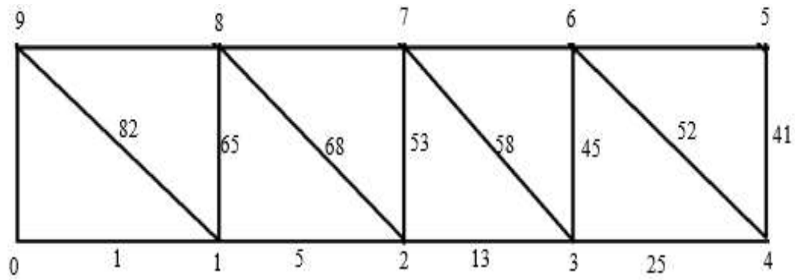

Fig 7. Square Sum Slither Labeling in $T(L_{1,\,4})$

In the above graph, it is palpable that the labels of the vertices are in slither pattern allocated with non-negative integers and one of the labels repeat. The label of an edge is the sum of squares of labels of the vertices that are incident with it and they are dissimilar too. Hence the graph admits square sum slither labeling. As n = 4 is arbitrary, the above slither labeling of vertices and the square sum labeling of edges hold for all values of $n \geq 2$. Hence the graph is square sum slither labelled.

Theorem – 4: The double book graph admits square sum slither labeling.

Proof: A double book graph is a graph that is formed by sharing a common edge, the pivot edge, between the equal number of multiple pages and is denoted by $B_{n,n}$. Label the vertices of the graphs with non-negative integers in a slither formation in such a way that the none of the vertices repeats. The label of an edge is the sum of square of label of the vertices that are incident with it. Let n = 4 be an arbitrary value. The square sum slither labeling in the double book graph is given by

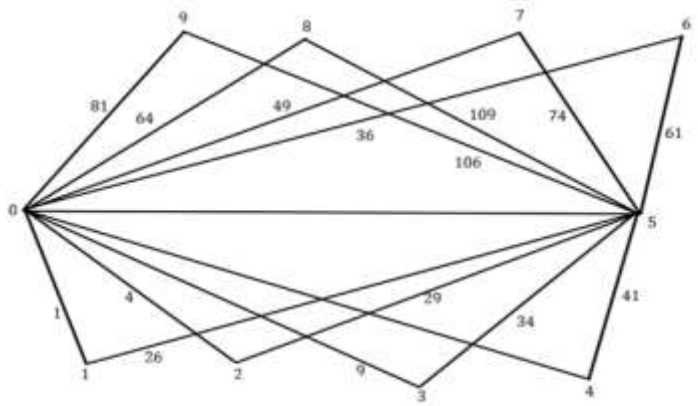

Fig 8. Square Sum Slither Labeling in Double Book Graph $B_{4,4}$

In the above graph, the labels of the vertices are in slither formation and they are distinct. The label of an edge is the sum of squares of labels of the vertices that are incident with it and they are distinct as well. Hence the graph admits square sum slither labeling. As n = 4 is arbitrary, it follows that the graph $B_{n,n}$ is square sum slither labelled for all n.

Conclusion:

In this paper, a special type of labeling the square sum slither labeling is considered for diverse graphs and it has been proved that the graphs admit the above-said property. Similarly, different graphs can be considered and they can be verified to admit the square sum slither labeling.

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