Perishable Inventory Model for Deteriorating Items with Time-Dependent Demand, Partial Backlogging, and Salvage Value

Kiran Thakur[1], Harshvardhan Singh[2], Jaya Kushwah[3]

[1,2]Researcher, *Department of Mathematics Vikrant University, Gwalior- 474006 M.P India.*
[3]Assistant Professor, *Department of Mathematics Vikrant University, Gwalior- 474006 M.P India.*

Abstract
In perishable inventory model present for deteriorating items with time-dependent demand and holding cost . The proposed model reflects several realistic features commonly observed in practical inventory systems. Item deterioration occurs continuously over time, which makes the management of perishable or high-value products particularly challenging. Demand is assumed to vary with time, capturing real-world market behavior more accurately than constant demand assumptions. This study the model proposed model considers realistic inventory features such as partial backlogging during shortages and includes salvage value for remaining stock. A comprehensive total cost function covering holding, purchasing, backlogging, lost sales, and salvage return is developed, and optimal policy is obtained by minimizing average cost. The model numerical experiment effectiveness of the proposed model extends existing deteriorating-item inventory systems and offers useful insights for managing perishable or high-value products.

Keywords-Perishable inventory, Time-dependent demand, Partial backlogging, Holding cost, Salvage value.

I. Introduction

Inventory management plays a vital role in supply chain and operations management, particularly for products that deteriorate over time such as perishable food items, pharmaceuticals, blood products, chemicals, and high-technology goods. Deterioration results in a gradual loss of product quantity or quality, leading to reduced usability and economic value. As a consequence, improper inventory control of such items can cause significant financial losses due to increased holding costs, spoilage, and wastage. Therefore, developing effective inventory models, including the Economic Order Quantity (EOQ) model, often assume constant demand and constant holding costs with no deterioration or shortages. However, these assumptions are rarely valid in real-world situations. In practice, demand for many products varies with time due to factors such as seasonality, market trends, promotional activities, and product life cycles. Similarly, holding costs may also be time-dependent as storage expenses, interest rates, energy costs, and warehousing conditions fluctuate over time. Early research EOQ-type models with constant demand and holding costs, assuming no shortages. Notable contributions in this area include the works of Covert and Philip [1]An EQO model items with constant deterioration and Ghiani Introduction to Logistics Systems Planning and Control which laid the foundation for perishable inventory modeling. However, these early models were limited in their ability to capture real-world complexities such as time-varying demand, fluctuating costs, and customer behavior during shortages. Goyal and Giri[2]Recent trends in modeling of deteriorating inventory models for deteriorating items under linearly and exponentially increasing demand, highlighting the importance of demand variability in inventory decision-making. Similarly, time-dependent holding costs were introduced in several studies, including the work of Dohi [4] to account for changes in storage and operational expenses over the planning horizon. While early models assumed either no shortages or complete backlogging, later studies recognized that partial backlogging more accurately reflects customer waiting behavior. Rafiquland Chaudhuri[5] Inventory model with partial backlogging for deteriorating items Furthermore, the inclusion of salvage value has been shown to significantly affect inventory decisions. Studies by Hwang and Ho[6] Inventory systems with salvage value and deteriorating items. and Padmanabhan and Pierskalla [7] Optimal policies for inventory models with salvage value and shortages. In recent years, researchers have increasingly focused on Tersine, & Sandifer[9]Inventory models for perishable items with shortages. developing integrated inventory models V. K. Mishra and L. S. singh[10] deterioration, time-dependent demand and holding costs, partial backlogging, and salvage value. S. Kumar and R. Kumar [16]A deterministic inventory model for perishable item with time dependent demand and shortages. Dr. S. Kumar [19] An inventory model with time varying holding cost, exponential decaying demand and constant deterioration . R.P. Tripathi, D. Singh and S. Aneja [22]Inventory models for stock- dependent demand and time varying holding cost under different trade credits. To address this gap the present study develops a comprehensive models provide more realistic and practical solutions for managing perishable and high-value products. The present study proposes an inventory model that incorporates with a single optimization framework The objective of the model is to determine the optimal replenishment and shortage policies. The results of this study offer valuable managerial insights and practical guidelines for effective inventory control in realistic economic and dealing with market condition.

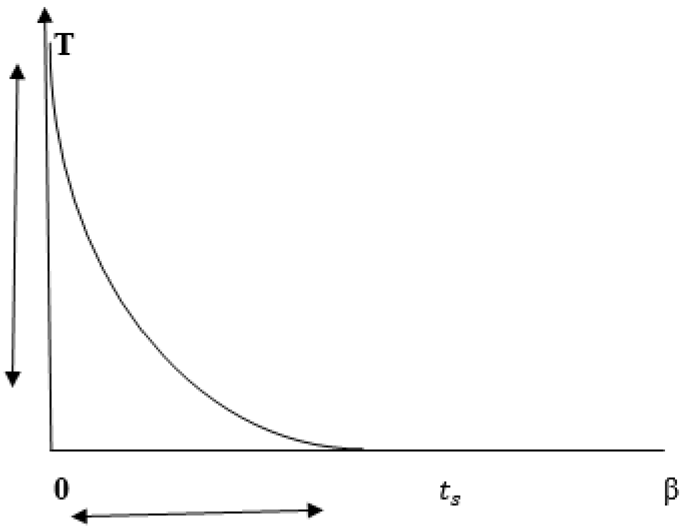
II. Assumptions

1. θ = Inventory item deteriorate at a constant rate.
2. D(t)=Demand rate is time-dependent t ,is defined as D(t)= r+ k_t r>0,k>0,
3. h(t)= Holding cost is time-varying,
4. β = Shortages are partially backlogged
5. CS= salvage value s is recovered for leftover or unsold items at the end of the cycle.
6. B(t) = The backlogging rate and demand that is backordered at time t. B(t)=$\frac{1}{1+\theta(T-t)}$ $t \leq t_s \leq T$,
7. Inflation is incorporated using Continuous discounting: $e^{-\theta t}$
8. The remaining fraction (1- β) is considered lost sales.
9. The lead time is negligible and instantaneous replenishment.
10. Costs considered include: purchasing, holding, backlogging, lost sales and salvage are inflation adjusted and measure in present Value terms.

III. Notations

Q = Order quantity per cycle

T = Length of the replenishment cycle
ts = Inventory depletion time (no shortage period ends)
D(t) = Time-dependent demand rate
h(t) = Time-dependent holding cost rate
θ = Deterioration rate of inventory (demand growth parameter)
β = Fraction of unmet demand that is backlogged .
I(t) = Inventory level at time t
B(t) = The backlogging rate and demand that is backordered at time t.
Oc = Ordering cost per cycle
Cp = Purchasing cost per cycle
Ch = Holding cost per cycle
Cb = Backlogging cost per cycle
Cl = Lost sales cost per cycle
Cs = Salvage value per cycle
Cd = Deterioration cost per cycle
Tc = Total cost per cycle



Mathematical formulation:

Case I:- During the inventory period $[0, t_s]$ the inventory diminishes due to the Combined effects of demand and deterioration, Consequently, it is described by the differential Equation ,

$\frac{dI(t)}{dt}$ =−D(t)−θI(t) when I(t)>0, , $0 \le t \le t_1$ — (1)

with the boundary condition $I(0) = I_0$ The solution of equation (1) is

$I(t) = -\frac{r}{\theta} + \frac{k}{\theta^2}(\theta t - 1) + (I_0 + \frac{r}{\theta} - \frac{k}{\theta^2})e^{-\theta t}$, $0 \le t \le t_s$ — (2)

Case II :- During the backorder period $[t_s\ , T]$ the demand is undergoes partial backlogging. The inventory level satisfied

$\frac{dI_1(t)}{dt} = B(t) \cdot D(t) = \frac{r+kt}{1+\theta(T-t)}$, $t_s \le t \le T$ —(3) with

the boundary condition t= t_s ,T

$$I_1(t) = I_s - \frac{r+kT+\frac{k}{\theta}}{\theta}\log\frac{(1+\theta(T-t)}{1+\theta(T-ts)} + \frac{k[\theta(t_s-t)]}{\theta^2}\ ,\ t_s \le t \le T$$

$\boldsymbol{I_1}(t)=(\frac{\boldsymbol{r+kT}}{\boldsymbol{\theta}}+\frac{\boldsymbol{kt}}{\boldsymbol{\theta^2}})\log(1+\boldsymbol{\theta}\,(T-\boldsymbol{t_s})-\frac{\boldsymbol{kt}}{\boldsymbol{\theta(T-t_s)}}$ —(4)

If boundary condition is at t=T

$I_1(t)=\frac{r+kT}{\theta}\log[1+\theta(T-t)]-\frac{k}{\theta}(T-t)$

Consequently, the overall cost per replenishment cycle is composed of the following parts.

1. Purchase cost per cycle

$Cp=\int_0^{t_s} D(t)\,dt$

$=\int_0^{t_s} D(t)\,dt=\int_0^{t_s}(r+kt)\,dt$

$=(rt+\frac{kt^2}{2})$

$=[rt_s+\frac{kt_s^2}{2}]$

2. Holding cost per cycle

$Ch=\int_0^{t_s} h(t)I(t)\,dt$

$=h\int_0^{t_s} I(t)\,dt,$

$=-\frac{r}{\theta}\int_0^{t_s} dt+\frac{k}{\theta^2}\int_0^{t_s}(\theta t-1)\,dt+\int_0^{t_s}(I_0+\frac{r}{\theta}-\frac{k}{\theta^2})\left(e^{-\theta t_s}\right)dt$

$=\int_0^{t_s} h[-\frac{rt_s^2}{2\theta}+\frac{kt_s^3}{3\theta}-\frac{kt_s^2}{2\theta^2}+\frac{1}{\theta^2}(I_0+\frac{r}{\theta}-\frac{k}{\theta^2})(1-e^{-\theta t_s})\ (1+\theta t_s)]$

$=h[-\frac{rt_s}{\theta}+\frac{kt_s^2}{2\theta}-\frac{kt_s}{\theta^2}+\frac{1}{\theta}(I_0+\frac{r}{\theta}-\frac{k}{\theta^2})(1-e^{-\theta t_s})]$

3.Backlogging cost per cycle

$Cb=p\int_{t_s}^{T} B(t)\,dt$

$Cb=p\int_{t_s}^{T}\frac{1}{1+\theta(T-t)}dt$

$Cb=p\int_{1+\theta(T-ts)}^{1}\frac{1}{x}\left(-\frac{dx}{\theta}\right)$

$=\frac{p}{\theta}\int_{1+\theta(T-ts)}^{1}\left(-\frac{dx}{x}\right)$

$Cb=\frac{p}{\theta}\log(1+\theta(T-t_s))$

4. Lost Sales cost per cycle

$Cl=c\int_{t_s}^{T}(1-\beta)(r+kt)\,dt$

$Cl=c(1-\beta)\int_{t_s}^{T}(r+kt)dt$

$=c\,(1-\beta)\,[\int_{t_s}^{T} r\,dt+\int_{t_s}^{T} kt\,dt$

$=r(T-ts)+\frac{k}{2}(T^2-t_s^2)$

Cl=c(1−β)[r(T− t_s)+$\frac{k}{2}$(T^2−$t_s{}^2$)]

5. Deterioration cost per cycle

Cd=$\int_0^{t_s}$ θI(t) **dt**

$=\int_0^{t_s} -\ \frac{r}{\theta}+\ \frac{k}{\theta^2}(\theta t-1)+\left(I_{0+}\frac{r}{\theta}-\ \frac{k}{\theta^2}\right)e^{-\theta t},$

Cd=$\theta\int_0^{t_s}\int_0^{t_s} -\ \frac{r}{\theta}+\ \frac{k}{\theta^2}(\theta t-1)+\left(I_{0+}\frac{r}{\theta}-\ \frac{k}{\theta^2}\right)\ (1-e^{-\theta t})$

Cd=$[-rt_s+\frac{kt_s{}^2}{2}-\frac{kt_s}{\theta}\left(I_{0+}\frac{r}{\theta}-\ \frac{k}{\theta^2}\right)\ (1-e^{-\theta t})]$

6. Salvage cost per cycle

CS=v I(t_s) $=\int_t^T\frac{r+kt}{1+\theta(T-t)}dt$

CS = $\frac{r-\frac{k}{\theta}}{\theta}$ log(1+ θ(T−t_s))+$\frac{k}{\theta}(T-t)$

7. Ordering cost per cycle

Oc =c_0

Total cost per cycle

Tc= Oc+ Cp+ Ch+ Cb + Cl-Cs + Cd

Tc $=c_0+[\ rt_s+\frac{kt_s{}^2}{2}]+h[-\frac{rt_s}{\theta}+\frac{kt_s{}^2}{2\theta}\ -\frac{kt_s}{\theta^2}+\frac{1}{\theta}(I_0+\frac{r}{\theta}-\frac{k}{\theta^2})\ (1-e^{-\theta t_s})]+\frac{p}{\theta}$log (1+ θ(T−$t_s$))+ c(1−β)[r(T− t_s)+$\frac{k}{2}$(T^2−$t_s{}^2$)]- $\frac{r-\frac{k}{\theta}}{\theta}$ log(1+ θ(T−t_s))+$\frac{k}{\theta}(T-t)$+ $[-rt_s+\frac{kt_s{}^2}{2}-\frac{kt_s}{\theta}\left(I_{0+}\frac{r}{\theta}-\ \frac{k}{\theta^2}\right)\ (1-e^{-\theta t})]$

Numerical example

Let consider an inventory system with the given characterized by data provided in units, then

r =50, k= 0.8, θ=0.1, β=0.6, , t_s=6, , c=10, p=4,cl=6, s=0, $Ch=900$,Cp=15,Cs=2.5 baseTc=1469.90

Table-I

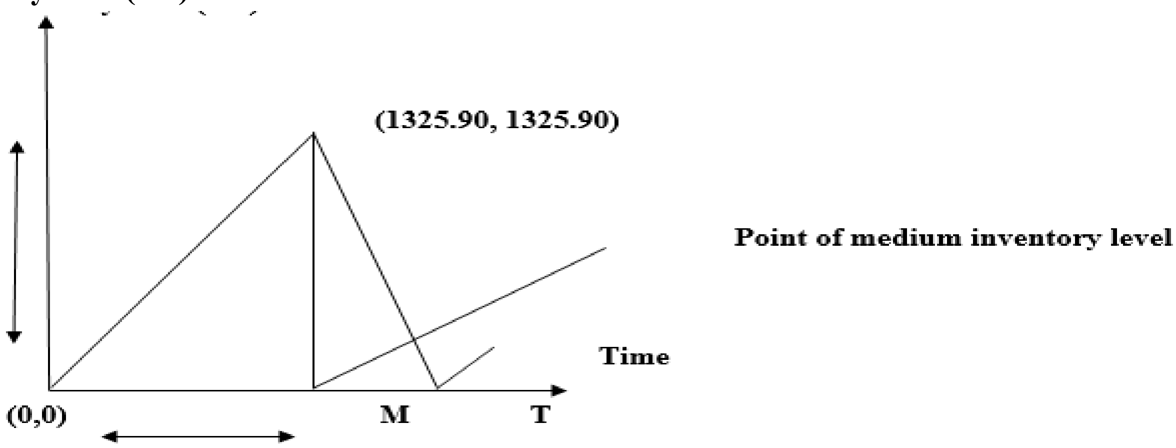
Effect of changing deteriorating rate θ.

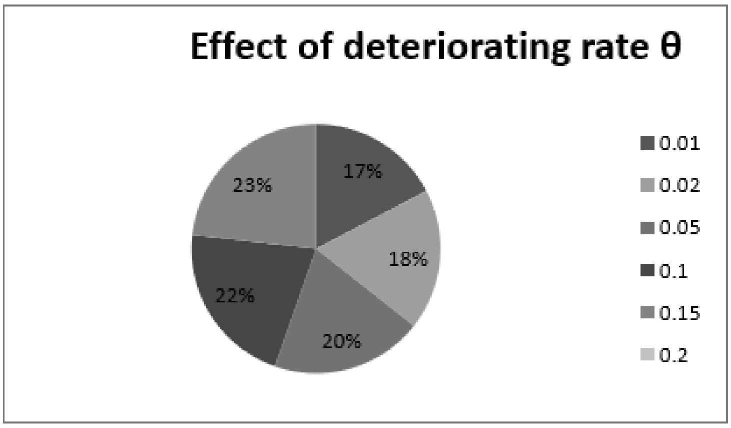
Θ	T_0	T_1	T	T_c
0.01	8.40	11.90	20.30	1072.70
0.02	8.60	12.20	20.80	1132.90
0.05	8.80	12.55	21.35	1224.00
0.10	9.10	13.00	22.10	1325.90
0.15	9.35	13.40	22.75	1437.00
0.20	9.60	13.80	23.40	1558.20

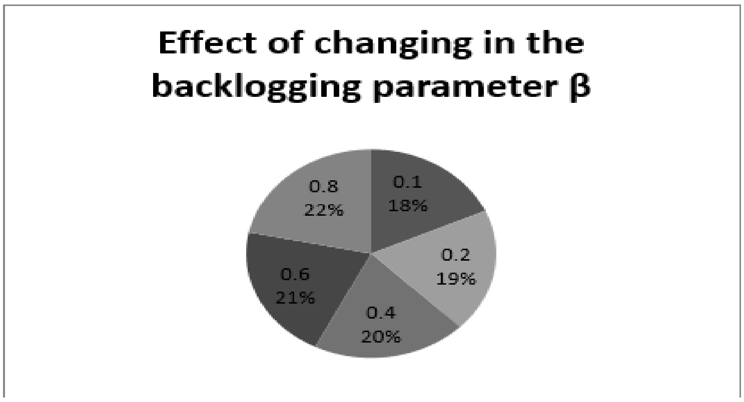
Table –II
Effect of changing in the backlogging parameter β

B	T_0	T_1	T	T_c
0.10	8.50	12.10	20.60	1162.00
0.20	8.70	12.35	20.90	1214.00
0.40	8.90	12.70	21.45	1268.70
0.60	9.10	13.00	22.10	1325.90
0.80	9.30	13.30	22.60	1379.70
0.90	9.40	13.45	22.85	1413.70

Inventory level (T c)







IV. Parametric Analysis

Beginning at Table I and Table II, we see the parametric to both the deterioration rate θ and the backlogging 0,1100,1200,1300,1400,1500, 1600,0.01,0.02,0.05,0.1,0.15,0.20, 0 ,1200,1250,1300 ,1350, 1400,1450, 0.1 0.2 0.4 0.6 0.8 ,0 .9, TC parameter β. An increase in θ intensifies inventory loss, when consequently raise the total cost and replenishment decisions. In contrast θ moderately influence system performance, higher Value duration and gradually increases the cost. Overall the parameter is more responsive θ than the parameter β.

This study develops a comprehensive inventory model for EQO framework for deteriorating items under time-dependent demand and holding cost, allowing partial backlogging and incorporating salvage value. The model captures several realistic features of modern inventory systems, particularly for perishable and high-value products, where deterioration and customer behavior during shortages play a crucial role in cost model. A mathematical formulation of the inventory dynamics is presented, and expressions for purchasing, holding, backlogging, lost sales, and salvage costs are derived. The total average cost per replenishment cycle is obtained and optimized with respect to the shortage starting time. The optimization results reveal the existence of an optimal shortage time that minimizes the total inventory cost by balancing holding and deterioration costs against shortage-related costs. The results further indicate that partial backlogging is more cost-effective than either no backlogging or full backlogging, as it reduces lost sales without excessively increasing backorder costs. Future studies may consider demand, variable deterioration rates, inflation effects multi-item inventory systems.

References

1. Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with constant deterioration. Journal of Industrial Engineering, 24(1), 12–16.
2. Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. European Journal of Operational Research, 134(1), 1–16.
3. Ghiani, G., Laporte, G., & Musmanno, R. (2004). Introduction to Logistics Systems Planning and Control. Wiley.
4. Dohi, T., Tokui, K., & Maki, T. (1997). Inventory model with time-dependent holding cost and deteriorating items. International Journal of Production Economics, 49(1), 29–36
5. Rafiqul, I., & Chaudhuri, K. S. (2006). Inventory model with partial backlogging for deteriorating items. Computers & Industrial Engineering, 51(3), 343–356.
6. Hwang, H. S., & Ho, W. (1997). Inventory systems with salvage value and deteriorating item. International Journal of Production Research, 35(11), 3199–3212.
7. Padmanabhan, V. & Pierskalla, W. P. (1990). Optimal policies for inventory models with salvage value and shortages. European Journal of Operational Research, 45(1), 1–10.
8. Silver, E. A., Pyke, D. F., & Thomas, D. J. (2016). Inventory and Production Management in Supply Chains (4th Edition). CRC Press.
9. Tersine, R. J., & Sandifer, J. (1975). Inventory models for perishable items with shortages. Management Science, 21(9), 1069–1075.
10. V. K. Mishra and L. S. singh [2010] “Deteriorating inventory model with time dependent demand and partial backlogging”, Applied Mathematical Science, vol 4,p.p 3611-3619.
11. V. K. Mishra [2012]” Inventory model for time dependent holding cost and deterioration with salvage value and shortages”, International Journal for Research, vol 9, p.p – 296- 301.

12. V. K. Mishra, L. S. Singh, and R. kumar [2013]" An inventory model for deteriorating items with time – dependent demand and time varing holding cost under partial backlogging", Journal of Industrial Engineering International vol. 9(4), p.p 2-5.
13. V.K. Mishra [2014]"Control deterioration rate for time dependent demand and time – varying holding cost", Yugoslav Journal of Operation Research, vol 24(1), p.p 87-98.
14. D. Dutta, and P. Kumar [2015], "A partial backlogging inventory model for deteriorating items with time varying and holding cost: an interval number approach", International Journal of Mathematics in Operation Research , vol 7 p.p – 281-286.
15. S. Sharma, and A. K. Sharma [2015] "A deterministic inventory model with selling price dependent demand rate, quadratic holding cost and quadratic time varying deteriorating rate", Amity University, vol 4 p.p – 85-90
16. S. Kumar and R. Kumar [2015] "A deterministic inventory model for perishable item with time dependent demand and shortages", International Journal of Mathematics and Its Application, vol 4,5p.p – 105-111
17. S. Kumar [2016] "Optimization of deteriorating item inventory model with price and time dependent demand", AmityJournal of Operation Management, vol 4, p.p 38-54
18. N. A. Khan, V.S. Verma, and V. Kumar [2017] "An inventory model for comparts deteriorating items with time varying holding cost and price dependent demand", International Journal for Mathematics, vol 49
19. Dr. S. Kumar [2017] "An inventory model with time varying holding cost, exponential decaying demand and constant deterioration" A Peer Reviewed Refereed International Research Journal, vol 1 ,p.p – 01-09,
20. Vinod. [2018] "An inventory model with time – dependent deterioration and ramp-type demand rate complete and partial backlogging". Applied Mathematical Sciences, Vol. 4, no. 72, 3611 – 3619
21. P. Mishra and N. Shah [2018] "Inventory management of time dependent deteriorating items with salvage value", International Journal of Mathematics, Engineering and Management., vol 5, p.p - 544-555
22. R.P. Tripathi, D. Singh and S. Aneja [2018] "Inventory models for stock- dependent demand and time varying holding cost under different trade credits", Yugoslav Journal of Operation Research, vol 28, p.p 139-151,