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An Improved Method for Assignment and Transportation Problems Using Minima-Based Optimization

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Abstract:
In this paper, we present a new approach to solving assignment problems using row and column minima. The assignment problems are solved without row or column reduction, similar to the Hungarian method. The concept of making zeros in each row and column was avoided. We have find the analysis demonstrates the potential advantages of method in terms of optimality and computational efficiency, providing a valuable addition to the toolkit for solving transportation problems

Keywords: Assignment Problem, minimum cost,optimality

Introduction:
The Assignment problem is similar to a game where tasks must be assigned to people in a way that minimizes the overall effort or cost. Each task must be performed by exactly one person, and each person can only perform one task. The goal is to find the best way to match tasks with people such that the total effort or cost is as low as possible. The assignment model deals with matching workers with varying skills to jobs. Skill Variation affects the cost of completing a job. The Hungarian Method is a combinatorial optimization algorithm that efficiently solves assignment problems in mathematical optimization. It was developed by Hungarian mathematicians D. Kuhn and J. Munkres and others in the 1950s. The main objective of the assignment problem is to find the optimal assignment of a set of tasks to a set of resources available at minimum cost.
The Hungarian Method to solveassignment problem has first discovered in the mid-20th century and was developed by Hungarian mathematicians Dénes Kőnig, Jenő Egerváry, and János Konrád in the 1930s. However, the method gained widespread recognition when it was rediscovered and popularized by American mathematician Harold W. Kuhn and James Munkres in the 1950s..

Difference between Assignment Problem & Transportation Problem
The assignment problem is a special case of transportation problem in which each job(origin) is associated with one and only one person(destination). In such a case, m=n, where m is supply and n is demand, and supply and demand are all equal to one, that is, each task needs one worker, and each worker is assigned to one task. The assignment problem is a specific type of optimization problem, and

its primary objective is to efficiently allocate tasks to resources in a way that minimizes the total cost or time. The main objectives of the assignment problem are as follows:

Minimization of the Total Cost or Time

The primary goal is to minimize the overall cost or time associated with completing a set of tasks by assigning them to resources. This optimization is essential for improving efficiency and resource utilization.

Objective function: The objective function is to minimize the total cost or maximize the total profit of the assignment. This is achieved by appropriately selecting the assignment based on the cost/profit values.

- **Constraints:** The main constraint in the assignment problem is that each resource is assigned exactly one task and each task is assigned to one person. This constraint ensures a one-to-one assignment.
- **Optimality Criteria:** In the assignment problem, the optimality criteria refer to the conditions that must be satisfied for an assignment to be considered optimal. The objective is to minimize the total cost or maximize the total profit of the assignment. The optimality criteria are based on finding the most efficient way to allocate resources to tasks.

The assignment problem can be categorized based on the objective function, which determines whether the goal is to minimize or maximize a specific criterion. Here are the types you mentioned:

Minimized Assignment Problem:

In the minimized assignment problem, the objective is to minimize the total cost or time associated with the assignments. The main goal was to find the most cost-effective way to assign tasks to resources.

Maximized Assignment Problem:

The maximization in the assignment problem deals with the objective of maximizing the total profit or benefiting from assignments.

The Hungarian Method [4] is a systematic algorithm for solving the assignment problem. It involves several steps, including row and column reduction, line drawing, and determining the optimal assignment. Each step is as follows:

Row Reduction:

The goal of row reduction is to create zeroes in each row of the cost matrix. This process involves subtracting the smallest element in each row from all the elements in that row.

- **Subtract the Minimum in Each Row:**

Find the minimum element in each row and subtract it from the corresponding row of the matrix.

Column Reduction:

Column reduction creates zeros in each column of the cost matrix. The process involves subtracting the smallest element in each column from all the elements of that column.

Line Drawing:

The line drawing step involves drawing lines (either horizontal or vertical) to cover all zeros in the cost matrix while minimizing the number of lines. Draw the minimum number of lines to cover all zeros in the rows and columns horizontally or vertically, not diagonally.

Optimal Assignment:

The optimal assignment is determined based on the number of covered zeros. In this step find the minimum number of lines required to cover all zeros and set the matrix to get the optimal assignment.

Minimum Number of Lines:

- If the drawn minimum number of lines are equal to the order of matrix i.e. matrix size, an optimal assignment is possible. The next step is as follows:
- If not, proceed to the minimum covering step.

Minimum Covering:

- Find the smallest uncovered element and subtract it from all the uncovered elements and add it to the elements at the point of intersection of the two lines. The other Covered elements remain unchanged.
- Go back to The line drawing step is then repeated.

These steps constitute the Hungarian Method for solving the assignment problem, providing an optimal assignment of tasks to resources with a minimum total cost or time. The iterative nature of the algorithm ensures convergence to the optimal solution.

Hungarian Method

Example :- Minimize the cost by Assignment problem

Job

	J_1	J_2	J_3	J_4	J_5
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

.

Solution:-

Step 1 :- Subtracting the smallest element of each row from every element of the corresponding row, we get the following matrix:

Job

	J_1	J_2	J_3	J_4	J_5
A	1	8	1	6	0
B	5	7	6	5	0
C	3	5	4	2	0
D	3	1	6	2	0
E	4	2	8	4	0

Step 2 :- Subtracting the smallest element of each Column from every element of the corresponding Column, we get the following matrix:

Job

	J_1	J_2	J_3	J_4	J_5
A	0	7	0	4	0
B	4	6	5	3	0
C	2	4	3	0	0
D	2	0	5	0	0
C	3	1	7	2	0

Step 3 :- Make the assignment to the reduced matrix:

Job

	J_1	J_2	J_3	J_4	J_5
A	[0]	7	0	4	0
B	4	6	5	3	[0]
C	2	4	3	[0]	~~0~~
D	2	[0]	5	~~0~~	~~0~~
E	3	1	7	2	~~0~~

Since the number of order of matrix is 5 and here minimum number of assignment is equal to 4 which is not equal to the order of matrix. So we will proceed to the next step.

Step 4:-Draw the minimum numbers of lines (horizontal and vertical) to cover all zeros in the reduced matrix obtained in step 3.

To do first marks (✓) row E as having no assignment and column (V) as having zeros in row E. Next marks (✓) rows B as these contain assignment in the marked column (V) . Now draw lines through each unmarked row and marked column:

Job

	J_1	J_2	J_3	J_4	J_5	
A	…….[0]……. .	…….7…….	……~~0~~….	…….4…….	……~~0~~……	
B	4	6	5	3	[0]	✓
C	……2…….	…. ...4…….	…….3… ….	…..[0]….	……~~0~~…..	
D	…....2…….	……[0]…..	…....5…… .	………~~0~~…..	……~~0~~… ….	
E	3	1	7	2	~~0~~	✓

Step 5 :- Here the number of lines is not equal to order of matrix. Now we can choose the smallest element from the uncovered elements and add it to the point of intersection and the new matrix is

A	0	7	0	4	1
B	3	5	4	2	0
C	2	4	3	0	1
D	2	0	5	0	1
E	2	0	6	1	0

Now we will repeat step 3 i.e. we will make the assignment to the reduced matrix :

Job

	J_1	J_2	J_3	J_4	J_5
A	……[0]…….	……7….....	.......~~0~~…...	…..4……	.......1…….
B	3	5	4	2	[0]
C	2	4	3	[0]	1
D	2	[0]	5	~~0~~	1
E	2	0	6	1	~~0~~

Again here number of lines is 4 which is not equal to order of matrix so we will repeat the step 5. The new matrix will be

Job

	J_1	J_2	J_3	J_4	J_5
A	~~0~~	9	[0]	6	3
B	1	5	2	4	[0]
C	[0]	4	1	~~0~~	1
D	~~0~~	~~0~~	3	[0]	1
E	~~0~~	[0]	4	1	~~0~~

Hence the optimal time taken.

Person:	A	B	C	D	E
Jo Assigned:	J_3	J_5	J_1	J_4	J_2
Time Taken:	2	1	4	3	3

Minimum Time taken = 2+1+4+3+3= 13 hrs.

Proposed Methodology (New Formulation)

The method for solving assignment problem through new formulation and the procedure is as follows:-

Step 1:- Prepare the cost matrix [C_{ij}].

Select the minimum element of each row and taken in the circle ◯ or box □.

Step 2 :- Select the minimum element of each column and taken in the circle ◯ or box □.

Step 3 :- Now combine all the row minima's and column minima's and obtain the modified matrix.

Step 4 :- Draw the minimum number of lines (Horizontal or Vertical but not diagonal) to cover all the minima's in modified matrix.

If number of lines is equal to order of matrix then make the assignment to get required solution.

Step 5 :- If number of minimum lines is not equal to order of matrix then optimality is not reached then go to next step.

Step 6 :- Find the smallest element of whole matrix which is not covered by lines. Subtract this smallest element with all other 'not covered' elements by lines and add the element at point of intersection of lines. Leave the elements covered by lines as it is.

Step 7 :- Obtain new modified matrix and now select the minimum element of each row and each column and taken in circle or in box and combine the all the row minima and column. Obtain new modified matrix and now repeat this step until you get order of matrix = minimum number of lines.

Step 8 :- Make the assignment one at time in a position. Begin with row and column that have only one minima. Mark ✓ on the assigned minima and put [X] over all the minima lying in that row or column. So that they can't be considered for future assignment. Continue in this manner until every row and column has exactly one assignment.

Step 9 :- If number of given assignments is equal to order of matrix then optimal solution is reached.

Example :-

Job

	J_1	J_2	J_3	J_4	J_5
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Step 1 :- Row minima

Select the minimum element of each row and taken in the circle ◯ or box □:-

Job

	J_1	J_2	J_3	J_4	J_5
A	2	9	2	7	(1)
B	6	8	7	6	(1)
C	4	6	5	3	(1)

D	4	2	7	3	1
E	5	3	9	5	1

Step 2 :- Column minima

Select the minimum element of each Column and taken in the circle or box

Job

	J1	J2	J3	J4	J5
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Step 3 :- Combine all minima

Combine all minima the row minima's and column minima's and obtain the modified matrix: -

Job

	J1	J2	J3	J4	J5
A	2	9	2	7	1
B	6	8	7	6	1
C	4	6	5	3	1
D	4	2	7	3	1
E	5	3	9	5	1

Step 4 :- Draw minimum numbers of lines to cover all minimas: -

Job

	J1	J2	J3	J4	J5
A	…..2…….	…….9……..	….2….. ..	…….7……..	….1…..
B	6	8	7	6	1
C	……….4………. .	……..6…….	…….5…..	…..3…..	….1…..
D	……..…4………. .	…2…..	…….7…...	……..3…….	….1…..
E	5	3	9	5	1

Here number of line is 4 which is not equal to the order of matrix. Now we choose the minimum element from uncovered element 3 and subtract and at the point of intersection.
Now new matrix will be :-

A	2	9	2	7	4
B	3	5	4	3	1
C	4	6	5	3	4
D	4	2	9	5	4
E	2	0	6	2	1

Repeat all the Steps as follows :-

Step I :- Row Minima
Job

	J_1	J_2	J_3	J_4	J_5
A	2	9	2	7	4
B	3	5	4	3	1
C	4	6	5	3	4
D	4	2	9	5	4
E	2	0	6	2	1

Step II :- Column Minima
Job

	J_1	J_2	J_3	J_4	J_5
A	2	9	2	7	4
B	3	5	4	3	1
C	4	6	5	3	4
D	4	2	9	5	4
E	2	0	6	2	1

Step III :- Combine All Minima

Job

	J_1	J_2	J_3	J_4	J_5
A	(2)	9	(2)	7	4
B	3	5	4	3	(1)
C	4	6	5	(3)	4
D	4	(2)	9	5	4
E	(2)	(0)	6	(2)	(1)

Step IV :- Draw Minimum number of lines to cover the all minimas: -

Job

	J_1	J_2	J_3	J_4	J_5
A	……..(2)……… .	……..9…… .	……(2)….	……..7…….	..…...4… …
B	3	5	4	3	(1)
C	4	6	5	(3)	4
D	4	(2)	9	5	4
E	……(2)……	……(0)… .	…….6…..	……(2)…..	……(1)….

Therefore number of minimum number of lines are equal to the order of matrix,

Job

	J_1	J_2	J_3	J_4	J_5
A	(2) ✗	9	(2) ✓	7	4
B	3	5	4	3	(1) ✓
C	4	6	5	(3) ✓	4
D	4	(2) ✓	9	5	4
E	(2) ✓	(0) ✗	6	(2) ✗	(1) ✗

Assignment of jobs

Therefore A→J_3, B→ J_5 , C→ J_4 , D→ J_2 , E→ J_1

Hence Total Cost = 2+1+3+2+5=13

Conclusion

In conclusion, the Hungarian method is a highly efficient algorithm for solving assignment problems, providing an optimal solution in polynomial time. Its simplicity and effectiveness make it a popular choice in various fields, including operations research, computer science, and engineering, for solving assignment and matching problems in these fields. The method discussed in the project new formulation is a new approach to solve the assignment problem, which will save/reduce the time for row reduction/column reduction. In addition, the method discussed provides the same optimal solution as that obtained by the Hungarian method. The method's ability to handle complex scenarios with ease and guarantee an optimal solution makes it a valuable tool in optimization and decision-making processes.

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10. Publications and conferences by mathematical societies such as the American mathematical Society (AMS) and the European optimisation Mathematical Society (EMS) often include theoretical and applied research on problems, including the assignment problem.