International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

A Hybrid Variance-Possibility Ranking Approach for Fuzzy Critical Path Analysis

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Abstract

The success of a project hinges on effective planning, scheduling, and task management to minimize completion time. The Critical Path Method (CPM) is a cornerstone technique for identifying critical tasks and constructing optimal schedules. However, traditional CPM relies on crisp activity durations, which are often impractical due to real-world uncertainties. This ambiguity introduces fuzziness in time estimates, necessitating the use of fuzzy numbers to model activity durations. Numerous researchers have proposed fuzzy CPM techniques ranging from defuzzification to possibility theory. Existing methods often overlook the interplay between magnitude, uncertainty, and risk sensitivity. To bridge this gap, a hybrid approach is proposed that integrates adaptive centroid, variance and possibility in ranking index.This hybrid approach enables robust critical path identification in fuzzy project networks, enhancing decision-making under uncertainty.

Keywords: Critical path, fuzzy number, risk index, possibility theory.

1. Introduction

The aim of Critical Path Method is to find the shortest time in which the project can be completed. A project involves a large number of interrelated activities that must be completed on or before a specified time limit, in a specified sequence, with specified quantity and minimum cost of using resources such as money, material, facilities and space. Network representations are widely used for problems in areas such as production, distribution, project planning, resource management and financial planning.The successful implementation of CPM requires the availability of clear determined time duration for each activity. Traditional CPM requires exact activity durations, but real-world projects face uncertainty due to some delays and incomplete data. Fuzzy CPM addresses this by modelling activity times as fuzzy numbers. By using sophisticated ranking techniques, managers can identify robust critical paths and make informed decisions under ambiguity.

Ordering fuzzy numbers is a non-trivial task because fuzzy numbers do not have a natural order like real numbers. Since fuzzy numbers overlap, traditional ordering methods like greater than or less than do not apply directly. Various methods have been proposed in the literature to rank fuzzy numbers, and these methods often consider multiple factorsincluding centroid, variance, possibility measure etc.,.In this work, a hybrid approach is proposed that integrates adaptive centroid, variance and possibility in ranking index.

In 1965, Zadeh introduced the concept of fuzzy sets, later extended tovariance measures in fuzzy numbers. In 1981,Yager[1] introduced a ranking index combining mean and spread .Özlem

ÇomaklıSökmen [2] found critical path analysis and projectcompletion time using possible, optimistic and pessimistic values, later Yager's ranking method was used for ranking. Narayanamoorthy Samayan and Maheswari Sengottaiyan [3] applied a ranking procedure based on Hexagonal fuzzy numbers to find a criticalpath in a fuzzy project network. Phani Bhushan Rao, P., &Nowpada [4], used lexicographic ordering oftrapezoidal fuzzy numbers in ranking.Ravi Shankar et al., [5] proposed a metric distance ranking method for fuzzy numbers to a critical path method for fuzzy project network.Yao& Lin [6] applied a signed distance ranking method for fuzzynumbers. Yao &Lin [7] used decomposition principle and the crisp ranking systemon R and constructed a new ranking system for fuzzy number.

2. Basic Terminologies:

Definition 2.1: Triangular Fuzzy Number

A triangular fuzzy number $\bar{A}$ can be defined by a triplet (a,b,c) whose membership function is defined as

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{x-a}{b-a} & x \in [a,b) \\ 1 & x = b \\ \frac{c-x}{c-b} & x \in (b,c] \\ 0 & x \notin [a,c] \end{cases}$$

Definition 2.2: Fuzzy Activity Time

The time duration of an activity represented by a fuzzy number is called fuzzy activity time FAT_{ij}. If the activity is a triangular fuzzy number it is denoted by FAT_{ij}= (a,b,c)

Definition 2.3: Fuzzy Earliest Start and Earliest Finish Time

Fuzzy earliest start time $FES_{ij}$of an activity i-j is the earliest occurrence of the event 'i' and earliest finish time $FEF_{ij}$of an activity i-j is the sum of the duration of the activity and earliest start of i-j.

FEF_j=Max [FES_{ij}+ FAT_{ij}] ----- (1)

Definition 2.4: Fuzzy Latest Start and Latest Finish Time

Fuzzy latestfinish time $FLF_{ij}$of an activity i-j is the latest occurrence of the event 'j' and latest start time of an activity. FLS_{ij}i-j is the latest finish of that activity minus duration of the activity.

FLSi= Min [FLF_j - FAT_{ij}] ---- (2)

Definition 2.5: Total Float

Total float of an activity is the amount of time by which that particular activity may be delayed without affecting the duration of the project. It is calculated by the difference between the latest finish and the earliest finish of the activity or the difference between the latest start and the earliest start of the activity.

Definition 2.6: Critical Path

The path connecting the initial node to the terminal node of longest duration in any network is called the critical path.

Definition 2.7: Operations on Fuzzy Activities

Let FAT_1= (a_1,b_1,c_1) and FAT_2=(a_2,b_2,c_2) be the duration of two fuzzy activities then

FAT_1+ FAT_2= (a_1+ a_2,b_1+ b_2,c_1+ c_2)

FAT_1- FAT_2= (a_1-c_2,b_1- b_2,c_1-a_2)

3. Proposed Ranking

Comparing the variances of fuzzy numbers can be a part of the process for ordering fuzzy numbers, but it is not typically the sole criterion. In fuzzy set theory, a possibility measure is used to quantify the degree to which a fuzzy event is possible. Hence, in this work, a ranking method combining centroid, variance and possibility measure is proposed to rank the activities which are represented by triangular fuzzy number to find earliest start time latest finish time and critical path.

Instead of simple centroid, risk- weighted mean is considered with a penalty term λ for uncertainty. The parameter λ let's project manager tune risk sensitivity. This method of ranking balances risk and magnitude.

$$R(\bar{A}_i) = C(\bar{A}_i).Poss(\bar{A}_i \geq \bar{A}_R) - N.Var(\bar{A}_i) - - - (3)$$

$$\text{where} C(\bar{A}_i) = \frac{a_i+2b_i+c_i}{4} + \lambda\left(b_i - \frac{a_i+c_i}{2}\right)$$

$$Var(\bar{A}_i) = \frac{(c_i - a_i)^2}{24}$$

$$N\,Var(\bar{A}_i) = \frac{Var(\bar{A}_i)}{Max(Var(\bar{A}_1), Var(\bar{A}_2) \dots \dots Var(\bar{A}_n))}$$

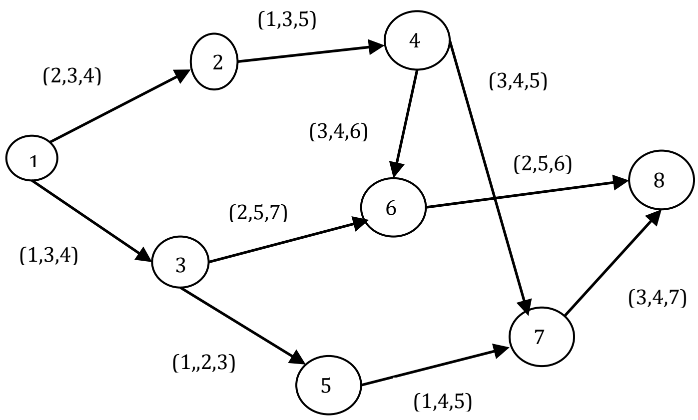
$$Poss(\bar{A}_i > \bar{A}_j) = \underbrace{Sup}_{x\geq y}(Min(\mu_{\bar{A}_i}(x), \mu_{\bar{A}_j}(y))$$

4. Procedure:

- Calculate earliest start and latest finish of each activity using (1) and (2).
- Use proposed ranking approach (3) to find the maximum and minimum of fuzzy activities
- Find fuzzy slack time $FTS_{ij}$with respect to each activity using
- $FTS_{ij} = FLF_j - (FES_i$ + duration of the activity)
- Find all possible paths from initial to terminal node and estimate its duration.
- Estimate the ranking value of each path by proposed ranking function.
- Identify the fuzzy critical path, the path with longest duration.

To explain the proposed approach in a better way, a numerical example is presented. For validation, a project network discussed in [8] is considered and fuzzy critical path is found using the proposed ranking function.

Consider the network



To find $FEST_{ij}$ for activity 6-8

Max[(3,8,11),(6,10,15)]

Let $\bar{A}_1 = (3,8,11)$ and $\bar{A}_2 = (6,10,15)$ and $\lambda = 0.5$

$$C(\bar{A}_1) = \frac{3+16+11}{4} + (0.5)(8-7) = 8$$

$$\text{Similarly } C(\bar{A}_2) = 11.5;$$

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E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

$$\therefore \bar{A}_R = \bar{A}_2$$
$$Var(\bar{A}_1) = 2.666; Var(\bar{A}_2) = 3.375; N.Var(\bar{A}_1) = 0.7899;$$
$$N.Var(\bar{A}_2) = 1$$
$$Poss(\bar{A}_1 > \bar{A}_R) = 0.66; Poss(\bar{A}_2 > \bar{A}_R) = 1$$
$$R(\bar{A}_1) = 8(0.66) - 0.7899 = 4.49$$
$$R(\bar{A}_2) = 11.5(1) - 1 = 10.5$$

Hence , Max [(3,8,11),(6,10,15)] = (6,10,15).

Similarly, FES_{ij}, FLF_{ij} and total slack time of all activities were found and tabulated in table 1 and possible paths from initial to terminal node and their rank are presented in table2.

Table 1: Total Fuzzy Slack Time of Each Activity of the Network

Activity	Duration	FES_{ij}	FLF_{ij}	FTS_{ij}
1-2	(2,3,4)	(0,0,0)	(-9,3,15)	(-13,0,13)
1-3	(1,3,4)	(0,0,0)	(-7,5,16)	(-11,2,15)
2-4	(1,3,5)	(2,3,4)	(-4,6,16)	(-13,0,13)
3-5	(1,2,3)	(1,3,4)	(-4,7,17)	(-11,2,15)
3-6	(2,5,7)	(1,3,4)	(2,10,19)	(-9,2,16)
4-6	(3,4,6)	(3,6,9)	(2,10,19)	(-13,0,13)
4-7	(3,4,5)	(3,6,9)	(1,11,18)	(-13,1,12)
5-7	(1,4,5)	(2,5,7)	(1,11,18)	(-11,2,15)
6-8	(2,5,6)	(6,10,15)	(8,15,21)	(-13,0,13)
7-8	(3,4,7)	(6,10,14)	(8,15,21)	(-13,1,12)

Table 2: Path Length and the Corresponding Rank

S.No.	**Path**	**Path Length(P_i)**	**R(P_i)**	**Rank**
1.	1-2-4-7-8	(-52,2,50)	0.903	2
2.	1-3-5-7-8	(-46,7,57)	6.02	3
3.	1-2-4-6-8	(-52,0,52)	-1	1
4.	1-3-6-8	(-33,4,44)	3.3199	4

From the table 2, the critical path is **1-2-4-6-8** and the duration of the path is **(8, 15, 21).**To compare the result, total float of all activities were calculated using earliest start, earliest finish, latest start and latest finish time and shown in table 3.

Table 3: Total Float of Each Activity

Activity	Duration	Earliest Start Time	Earliest Finish Time	Latest Start Time	Latest Finish Time	Total Float
1-2	(2,3,4)	(0,0,0)	(2,3,4)	(-13,0,13)	(-9,3,15)	(-13,0,13)
1-3	(1,3,4)	(0,0,0)	(1,3,4)	((-11,2,15)	(-7,5,16)	(-11,2,15)

2-4	(1,3,5)	(2,3,4)	(3,6,9)	(-9,3,15)	(-4,6,16)	(-13,0,13)
3-5	(1,2,3)	(1,3,4)	(2,5,7)	(-7,5,16)	(-4,7,17)	(-11,2,15)
3-6	(2,5,7)	(1,3,4)	(3,8,11)	(-5,5,17)	(2,10,19)	(-9,2,16)
4-6	(3,4,6)	(3,6,9)	(6,10,15)	(-4,6,16)	(2,10,19)	(-13,0,13)
4-7	(3,4,5)	(3,6,9)	(6,10,14)	(-4,7,15)	(1,11,18)	(-13,1,12)
5-7	(1,4,5)	(2,5,7)	(3,9,12)	(-4,7,17)	(1,11,18)	(-11,2,15)
6-8	(2,5,6)	(6,10,15)	(8,15,21)	(2,10,19)	(8,15,21)	(-13,0,13)
7-8	(3,4,7)	(6,10,14)	(9,14,21)	(1,11,18)	(8,15,21)	(-13,1,12)

From table 3, the activities **1-2, 2-4, 4-6, 6-8** are critical activities, hence the critical path is **1-2-4-6-8** and found to be same.

5. Conclusion

Fuzzy CPM enables practical project scheduling underuncertainty, and the proposed method for ranking enhances itby integrating risk-aware ranking directly into the critical pathanalysis.Also it contributes to this field by balancing magnitude,uncertainty, and risk sensitivity, offering a practical tool forfuzzy project networks.

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