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Numerical Stability Investigation of Oscillatory MHD Flow with Radiation Effects Using Von Neumann Method

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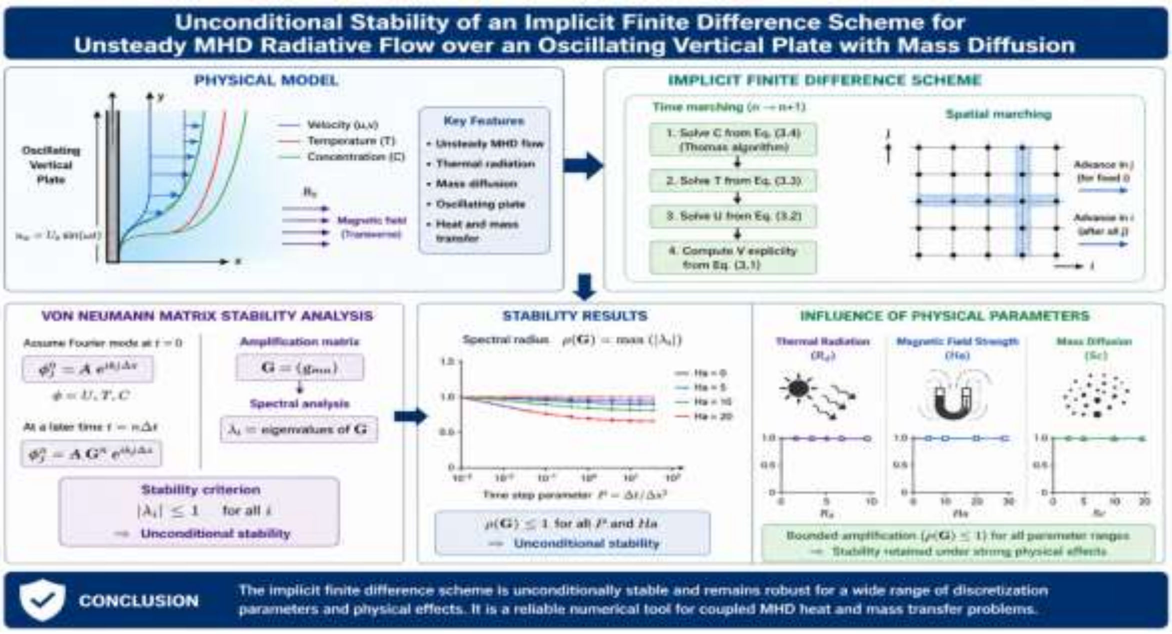
Abstract:
This work examines the numerical stability of an implicit finite difference approach applied to unsteady magnetohydrodynamic (MHD) radiative flow over an oscillating vertical plate in the presence of mass diffusion. Stability is assessed using a Von Neumann matrix analysis, where the amplification matrix is formulated and its spectral characteristics are evaluated. The findings indicate that the numerical scheme remains unconditionally stable, maintaining bounded amplification across all choices of discretization parameters. Additionally, the influence of key physical factors such as thermal radiation, magnetic field strength, and diffusion is investigated, demonstrating that the stability of the method is retained even under significant physical effects. Overall, the implicit scheme proves to be a robust and dependable tool for modeling coupled MHD heat and mass transfer phenomena.

Graphical Abstract:



Keywords: MHD flow; Thermal Radiation; Oscillating Vertical Plate; Implicit Finite Difference; Von Neumann Stability; Amplification factor; Numerical convergence.

Introduction:

Magnetohydrodynamic (MHD) flow and heat transfer over vertical surfaces have attracted considerable attention due to their extensive applications in engineering and industrial processes such as nuclear energy systems, geothermal engineering, metallurgical operations, cooling of electronic equipment, polymer processing, crystal growth, chemical reactors, and astrophysical fluid dynamics. The interaction between magnetic fields and electrically conducting fluids significantly alters the transport characteristics of momentum, heat, and mass transfer. In many practical situations, thermal radiation, chemical reactions, thermal diffusion, and oscillatory motion further influence the behavior of the boundary layer flow. Consequently, the investigation of unsteady MHD convective flow over vertical plates under the combined effects of radiation and mass transfer has become an important area of research in computational fluid dynamics and heat transfer analysis.

Natural and mixed convection flows over vertical plates are fundamental problems in fluid mechanics. Early investigations by Gebhart and Pera [19] established the importance of combined thermal and mass diffusion effects in buoyancy-driven convection flows. Their work demonstrated that temperature and concentration gradients jointly influence the flow structure and transport phenomena in boundary layer regions. Subsequently, several researchers extended these concepts to electrically conducting fluids subjected to magnetic fields and radiative heat transfer. Takhar et al. [9] analyzed unsteady flow and heat transfer on a semi-infinite plate with an aligned magnetic field and demonstrated the suppressing influence of magnetic forces on fluid velocity. Chamkha [8] further examined unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate and reported that magnetic fields significantly affect both thermal and concentration boundary layers.

The influence of thermal radiation on MHD flow problems has gained increasing importance because radiative heat transfer becomes dominant at high temperatures encountered in engineering systems. Uddin et al. [1] investigated MHD forced convective laminar boundary layer flow from a convectively heated moving vertical plate in the presence of radiation and transpiration effects. Their study revealed that radiation enhances thermal boundary layer thickness and significantly modifies the velocity distribution. Etwire and Seini [3] studied radiative MHD flow over a vertical plate with convective boundary conditions and highlighted the importance of radiative heat flux in controlling the temperature field. Rajput and Shareef [4] examined MHD free convective flow along a vertical oscillatory plate with radiative heat transfer and observed that oscillatory motion and radiation jointly alter the transient behavior of the fluid flow.

Oscillatory plate problems are particularly important because they model numerous engineering applications involving periodic motion such as vibrating machinery, heat exchangers, petroleum transport systems, and oscillating cooling devices. Muthucumaraswamy et al. [5][22] investigated chemical reaction, thermal radiation, and MHD effects on accelerated and oscillatory vertical plate flows using finite difference techniques and variable temperature conditions.Their work demonstrated that increasing magnetic field strength suppresses fluid velocity, whereas radiation enhances thermal diffusion within the boundary layer. Veeresh et al. [6] investigated Joule heating and thermal diffusion effects on MHD radiative Casson fluid flow past an oscillating vertical plate and found that thermal diffusion significantly influences both concentration and temperature distributions. Rao et al. [30] further analyzed unsteady MHD free convection Casson fluid flow past an exponentially accelerated vertical plate and reported strong coupling between acceleration, radiation, and magnetic field parameters.

Recent investigations have focused on advanced fluid models, porous media, nanofluids, and thermal radiation effects in MHD systems. Sudarmozhi et al. [2] studied magneto-radiative and heat convective boundary layer flow in Maxwell fluid across a porous inclined vertical plate and emphasized the role of viscoelastic properties in modifying the flow characteristics. Goud et al. [24] examined thermal radiation effects on stratified MHD fluid flow through an accelerated vertical porous plate and reported that radiation significantly increases temperature distribution within the boundary layer. Abbas et al. [25] analyzed the combined effects of thermal radiation and thermophoresis on mixed convection boundary layer flow over a vertical plate and showed that thermophoretic transport strongly affects nanoparticle concentration profiles. Similarly, Yasin et al. [26] investigated ferrofluid MHD convection heat transfer over a vertical plate and concluded that magnetic fields substantially improve heat transfer performance in ferrofluids.

In recent years, nanofluid-based MHD heat transfer studies have received significant attention due to their enhanced thermal conductivity and industrial relevance. Ellahi et al. [32] studied aggregation effects on water-based nanofluid flow over a vertical plate with MHD and reported improved thermal transport characteristics in the presence of nanoparticles. Sheikholeslami [33], [36] numerically simulated nanofluid flow considering magnetic field and thermal radiation effects and demonstrated that radiation increases heat transfer rates while magnetic fields reduce fluid velocity. Kumar et al[37]. and Dawar et al[34]. investigated thermal radiation, heat source, and chemically reactive MHD nanofluid flows over vertical plates.

The effects of thermal diffusion and mass diffusion have also been extensively investigated in convective MHD flows. Anil Kumar et al. [28] studied Soret and Dufour effects on MHD natural convection flow past an accelerated vertical plate and concluded that thermal diffusion significantly affects concentration boundary layers. Shankar et al. [35] examined MHD flow with Soret and Dufour effects over a vertical surface with thermal radiation and found that cross-diffusion mechanisms considerably influence heat and mass transfer rates. Sahoo et al. [31] investigated entropy generation in radiative MHD nanofluid flows with chemical reactions and highlighted the importance of irreversibility analysis in thermal system optimization. Arulmozhi et al. [23] analyzed radiative and chemical reactive effects on MHD nanofluid flow over a vertical plate and reported enhanced heat transfer due to radiation and nanoparticle interactions.

Apart from analytical investigations, numerical methods play a crucial role in solving nonlinear coupled governing equations associated with unsteady MHD radiative flows. Classical finite difference schemes developed by Crank and Nicolson [10] remain widely used due to their unconditional stability and second-order accuracy. The mathematical foundations of finite difference techniques were systematically presented by Richtmyer and Morton [11], Smith [12], Ames [18], Strikwerda [16], and LeVeque [17]. Fletcher [13] provided comprehensive computational techniques for fluid dynamics problems involving convection–diffusion equations. More recently, Fazio and Jannelli [14] proposed a non-standard finite difference scheme for MHD boundary layer flow to improve numerical stability and convergence characteristics. Yu et al. [15] developed free-stream preserving finite difference schemes for ideal MHD equations and demonstrated improved computational accuracy in magnetohydrodynamic simulations. These numerical approaches have become essential for studying complex transient MHD flows involving radiation, oscillation, chemical reactions, and mass transfer effects.

Although substantial research has been carried out on MHD convective flow over vertical plates, limited attention has been devoted to the combined influence of oscillatory motion, thermal radiation, magnetic field, and mass diffusion using stable finite difference approaches for transient analysis. Many earlier studies primarily focused either on steady-state configurations or simplified fluid models without comprehensive numerical stability considerations. Therefore, there remains a need for detailed investigation of unsteady radiative MHD flow past oscillating vertical plates using accurate and stable numerical schemes capable of capturing the coupled transport mechanisms effectively.

Motivated by these observations, the present study focuses on the numerical investigation of unsteady MHD radiative flow past an oscillating semi-infinite vertical plate with heat and mass transfer effects. The governing dimensionless momentum, energy, and concentration equations are solved using an implicit finite difference method based on the Crank–Nicolson scheme. The study aims to analyze the influence of important physical parameters such as magnetic field parameter, thermal radiation parameter, thermal Grashof number, mass Grashof number, Schmidt number, and oscillation frequency on the velocity, temperature, and concentration distributions. The numerical approach adopted in this work ensures stability and computational efficiency while providing physically realistic solutions relevant to engineering and industrial applications involving MHD transport phenomena.

Problem Formulation and Governing Equations:

Unsteady laminar buoyancy-driven flow of a viscous, incompressible fluid along an oscillating semi-infinite vertical plate maintained at constant temperature is investigated, incorporating the effects of thermal radiation. The fluid is modeled as a dilute binary mixture, wherein the concentration of the diffusing species is sufficiently low relative to the carrier fluid; accordingly, its impact on the thermophysical properties of the mixture is neglected, except through its contribution to concentration gradient–induced transport.

A Cartesian coordinate system (x,y) is introduced such that the x-axis is aligned vertically along the plate in the upward direction, while the y-axis is oriented normal to the surface and directed into the fluid domain. The plate is considered semi-infinite and executes oscillatory motion within its own plane, thereby inducing an unsteady boundary-layer flow in the adjacent fluid. Thermal radiation effects are incorporated by modeling the fluid as a gray, absorbing–emitting, and non-scattering medium.The physical configuration of the problem, along with the coordinate system and relevant boundary layer development, is schematically illustrated in Figure 1.

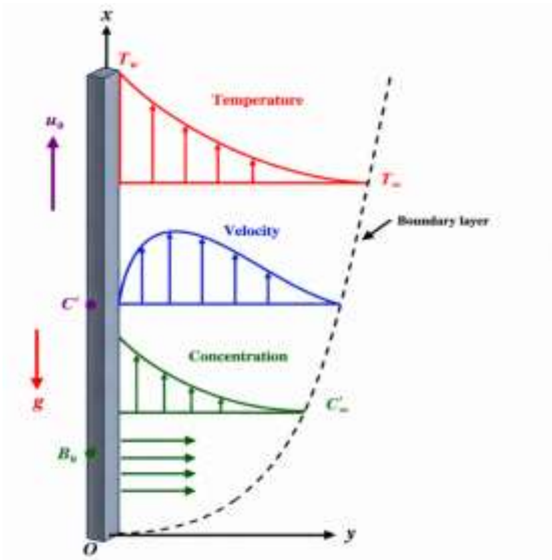


Figure:1 Physical Model of the Problem

Initially, both the fluid and the plate are assumed to be in a state of thermal and species equilibrium, with uniform temperature and concentration fields throughout the domain. At time t>0, the plate is set into oscillatory motion in its own plane with a specified angular frequency ω, under the influence of gravity. Concurrently, the thermal condition T_w at the plate is altered by raising its temperature, while the concentration of the diffusing species at the plate surface is increased uniformly. These imposed boundary variations generate temperature and concentration gradients in the surrounding fluid, thereby initiating coupled buoyancy-driven flow due to thermal and solutal effects.

The fluid is modeled as a gray medium that absorbs and emits thermal radiation without scattering effects, while viscous dissipation is assumed to be sufficiently small to be neglected. Under these simplifying assumptions, and adopting the Boussinesq approximation to account for density variations only in the buoyancy terms, the governing equations of motion are reduced to the boundary-layer form. These equations represent the conservation of mass, momentum, energy, and species concentration for the unsteady free-convective flow. The resulting mathematical formulation can be expressed as follows (Gebhart and Pera) [19]:

$$u_x + u_y = 0 \qquad (2.1)$$

$$u_{t'} + uu_x + vu_y = g\beta(T' - T_\infty) + g\beta^*(C' - C'_\infty) + \nu u_{yy} - \frac{\sigma B_\circ^2}{\rho} u \qquad (2.2)$$

$$\rho C_p\left(T'_{t'} + uT'_x + vT'_y\right) = kT'_{yy} - (q_r)_y \qquad (2.3)$$

$$C'_{t'} + uC'_x + vC'_y = DC'_{yy} \qquad (2.4)$$

The initial and boundary conditions are

$$\begin{array}{llllllll} t' \le 0: & u = 0, & v = 0, & T' = T_\infty, & C' = C'_\infty & & & \\ t' > 0: & u = u_0 \cos\omega' t', & v = 0, & T' = T_w, & C' = C'_w & at & y = 0 & \\ & u = 0, & T' = T_\infty, & C' = C'_\infty & & at & x = 0 & (2.5) \\ & u \to 0, & T' \to T_\infty, & C' \to C'_\infty & & as & y \to \infty & \end{array}$$

In the case of an optically thin gray gas the local radiant absorption is expressed by

$$(q_r)_y = -4a^* \sigma\left(T_\infty^4 - T'^4\right) \qquad (2.6)$$

It is assumed that the temperature variations within the flow field are sufficiently small to allow the linearization of the radiative heat flux term. In particular, T^4 is approximated as a linear function of temperature by performing a Taylor series expansion about the ambient temperature T∞ and retaining only the leading-order terms, while neglecting higher-order nonlinear contributions. Accordingly, the expression for T^4 is simplified as follows:

$$T'^4 \cong 4T_\infty^3 T' - 3T_\infty^4 \qquad (2.7)$$

By using equations (2.6) and (2.7), equation (2.3) reduces to

$$\rho C_p \left(T'_{t'} + uT'_x + vT'_y\right) = kT'_{yy} + 16a^* \sigma T_\infty^3 (T_\infty - T') \quad (2.8)$$

On introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, V = \frac{v}{u_0}, t = \frac{t' u_0^2}{\nu}, X = \frac{x u_0}{\nu} Y = \frac{y u_0}{\nu}, T = \frac{T' - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gc = \frac{\nu g\beta^*(C'_w - C'_\infty)}{u_0^3}, \omega = \frac{\omega' \nu}{u_0^2}, \quad (2.9)$$

$$R = \frac{16a^* \nu^2 \sigma T_\infty^3}{k u_0^2}, Pr = \frac{\nu}{\alpha}, Sc = \frac{\nu}{D}, M = \frac{\sigma B_\circ^2 \nu}{\rho u_\circ^2}$$

Equations (2.1) to (2.4) are reduced to the following non-dimensional form

$$U_X + V_Y = 0 \quad (2.10)$$

$$U_t + UU_X + VU_Y = Gr\,T + Gc\,C + U_{YY} \quad (2.11)$$

$$T_t + UT_X + VT_Y = \frac{1}{\Pr} T_{YY} - \frac{R}{\Pr} T \quad (2.12)$$

$$C_t + UC_X + VC_Y = \frac{1}{Sc} C_{YY} \quad (2.13)$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$t \le 0: \quad U = 0, \; V = 0, \; T = 0, \; C = 0$$

$$t > 0: \quad U = Cos\,\omega t, \quad V = 0, \; T = 1, \; C = 1 \quad \text{at} \quad Y = 0$$

$$U = 0, \; T = 0, \; C = 0 \quad \text{at} \quad X = 0 \quad (2.14)$$

$$U \to 0, \; T \to 0, \; C \to 0 \quad \text{as} \quad Y \to \infty$$

Numerical Discretization via Finite Difference Method:

To obtain a numerical solution of the unsteady, nonlinear, and coupled governing equations (2.10)–(2.13) subject to the boundary conditions (2.14), an implicit finite difference formulation of Crank–Nicolson type is adopted. This scheme is chosen due to its second-order accuracy in both time and space and its unconditional stability characteristics for parabolic-type problems. The governing partial differential equations are discretized using a time-centered and space-centered averaging procedure, resulting in a system of algebraic equations at each time level. The corresponding finite difference representations of equations (2.10)–(2.13) are derived as follows:

$$\frac{\left[U_{n+1}(i,j) - U_{n+1}(i-1,j) + U_n(i,j) - U_n(i-1,j) + U_{n+1}(i,j-1) - U_{n+1}(i-1,j-i) + U_n(i,j-1) - U_n(i-1,j-1)\right]}{4\Delta X}$$

$$+ \frac{\left[V_{n+1}(i,j) - V_{n+1}(i,j-1) + V_n(i,j) - V_n(i,j-1)\right]}{2\Delta Y} = 0$$

$$\frac{[U_{n+1}(i,j)-U_n(i,j)]}{\Delta t}+U_n(i,j)\frac{[U_{n+1}(i,j)-U_{n+1}(i-1,j)+U_n(i,j)-U_n(i-1,j)]}{2\Delta X}$$
$$+V_n(i,j)\frac{[U_{n+1}(i,j+1)-U_{n+1}(i,j-1)+U_n(i,j+1)-U_n(i,j-1)]}{4\Delta Y}$$
$$=\frac{Gr}{2}[T_{n+1}(i,j)+T_n(i,j)]+\frac{Gc}{2}[C_{n+1}(i,j)+C_n(i,j)C_{i,j}^{n}]-\frac{M}{2}[U_{n+1}(i,j)+U_n(i,j)]$$
$$+\frac{[U_{n+1}(i,j-1)-2U_{n+1}(i,j)+U_{n+1}(i,j+1)+U_n(i,j-1)-2U_n(i,j)+U_n(i,j+1)]}{2(\Delta Y)^2}$$

$$\frac{[T_{n+1}(i,j)-T_n(i,j)]}{\Delta t}+U_n(i,j)\frac{[T_{n+1}(i,j)-T_{n+1}(i-1,j)+T_n(i,j)-T_n(i-1,j)]}{2\Delta X}$$
$$+V_n(i,j)\frac{[T_{n+1}(i,j+1)-T_{n+1}(i,j-1)+T_n(i,j+1)-T_n(i,j-1)]}{4\Delta Y}$$
$$=\frac{1}{Pr}\frac{[T_{n+1}(i,j-1)-2T_{n+1}(i,j)+T_{n+1}(i,j+1)+T_n(i,j-1)-2T_n(i,j)+T_n(i,j+1)]}{2(\Delta Y)^2}-\frac{R(T_{n+1}(i,j)+T_n(i,j))}{2Pr}$$

$$\frac{[C_{n+1}(i,j)-C_n(i,j)]}{\Delta t}+U_n(i,j)\frac{[C_{n+1}(i,j)-C_{n+1}(i-1,j)+C_n(i,j)-C_n(i-1,j)]}{2\Delta X}$$
$$+V_n(i,j)\frac{[C_{n+1}(i,j+1)-C_{n+1}(i,j-1)+C_n(i,j+1)-C_n(i,j-1)]}{4\Delta Y}$$
$$=\frac{1}{Sc}\frac{[C_{n+1}(i,j-1)-2C_{n+1}(i,j)+C_{n+1}(i,j+1)+C_n(i,j-1)-2C_n(i,j)+C_n(i,j+1)]}{2(\Delta Y)^2}$$

The computational domain is defined as a rectangular region with dimensions X_{max}=1and Y_{max}=20, where Y_{max} is chosen sufficiently large to ensure that it lies well beyond the momentum and thermal boundary layer thicknesses. This truncation of the infinite physical domain is justified through preliminary numerical experiments, confirming that the far-field boundary conditions (2.14) are satisfied within a prescribed accuracy of 10^{-5}.

A grid independence study is carried out to ensure numerical reliability. After examining several discretization levels, the mesh is fixed with spatial step sizes ΔX=0.05 and ΔY=0.25, along with a time increment Δt=0.01. To further verify grid adequacy, computations are repeated with the mesh refined by 50% in one direction and subsequently in both directions, and the results are compared. It is observed that refinement in the Y-direction alters the solution only in the fifth decimal place, while refinement in the X-direction or in both directions yields agreement up to three decimal places. Based on these convergence checks, the selected grid resolution is deemed sufficient for accurate numerical simulation.

The coefficients appearing in the discretized finite difference equations are assumed to remain constant within each time level. In the computational grid, the index i denotes the spatial discretization along the X-direction, j corresponds to the Y-direction, and k represents the temporal level t. The dependent variables U, V, and T are initialized across all grid points at t=0 using the prescribed initial conditions, thereby providing the starting solution for the time-marching procedure.

At the computational level, the solution advances from time step n to n+1 in a sequential, semi-implicit manner using previously computed field variables. All dependent variables U, V, T, and C at the new time level are updated based on known values at the preceding time level, ensuring temporal marching stability and consistency.

For each fixed spatial index i, the governing finite difference formulation corresponding to equation (3.4) is discretized over the internal nodal points in the j-direction. This discretization leads to a system of linear algebraic equations characterized by a tridiagonal coefficient matrix. Such a structure arises due to the use of second-order central differences and nearest-neighbour coupling in the spatial domain. The resulting system is efficiently solved using the Thomas algorithm, a specialized direct solver for tridiagonal systems as described by Carnahan et al. [20]. Upon solving, the scalar field C is updated at all grid points along the selected i-level for the new time step.

Once C_{n+1} is obtained, equation (3.3) is similarly discretized and solved to compute the updated temperature field T_{n+1}. This step uses the newly available concentration values, thereby introducing a sequential coupling between scalar transport equations.

Subsequently, the momentum equation (3.2) is evaluated using the updated fields C_{n+1} and T_{n+1} . This yields the intermediate velocity component U_{n+1}, which is computed in the same spatial sweep-wise manner. As a result, the coupled influence of thermal and concentration gradients on the velocity field is incorporated at the current time level.

Finally, the velocity component V_{n+1} is explicitly evaluated from equation (3.1) at each nodal location. Since this equation is treated in an explicit form, no simultaneous system solution is required at this stage.

This entire sequence updating C, then T, followed by U, and finally V is repeated for every i-level across the computational domain. Through this marching procedure, the full field variables C, T, U, and V are systematically advanced to the (n+1)th time level over the entire rectangular grid.

The numerical procedure is extended in the stream wise i-direction in an analogous fashion, ensuring that all internal nodal points across each i-level are systematically updated. Once the solution for all i-levels has been obtained at a given time level n+1, the computation proceeds by advancing to the subsequent time step n+2. This time-marching strategy is repeated iteratively in a sequential manner.

The iteration between successive time levels continues until convergence to a steady-state solution is achieved. The convergence criterion is based on the maximum absolute change in the dependent variables U, V, T, and C between two consecutive time levels. Specifically, the solution is considered steady when the condition $|\varphi_{n+1}(i,j)-\varphi_n(i,j)|<10^{-5}$ is satisfied at every grid point for all variables $\phi\in\{U,V,T,C\}$. This stringent tolerance ensures that temporal variations become negligibly small throughout the computational domain, indicating that the transient effects have decayed and the system has reached a numerically stable steady state.

Numerical Stability via Von Neumann Matrix Approach:

The stability criterion of the finite difference scheme for constant mesh sizes are examined using Von-Neumann technique as explained by Carnahan et al.[20]. The general term of the Fourier expansion for *U, T* and *C* at a time arbitrarily called t = 0, are assumed to be of the form $e^{i\alpha X}e^{i\beta X}$ (here $i=\sqrt{-1}$). At a later time t, these terms will become,

$$U = A(t)e^{i\alpha X}e^{i\beta X}$$
$$T = B(t)e^{i\alpha X}e^{i\beta X}$$
$$C = D(t)e^{i\alpha X}e^{i\beta X} \qquad (4.1)$$

Substituting (4.1) in Equations (3.2) to (3.4); under the assumption that the coefficients U, T and C are constants over any one time step and denoting the values after one time step by A', B' *and* D'. After simplification, we get

$$\frac{(A'-A)}{\Delta t}+\frac{U}{2}\frac{(A'+A)(1-e^{-i\alpha\Delta X})}{\Delta X}+\frac{V}{2}\frac{(A'+A)i\sin\beta\Delta Y}{\Delta Y} \\ =\frac{(B'+B)Gr+(D'+D)Gc-M(A'+A)}{2}+\frac{(A'+A)(\cos\beta\Delta Y-1)}{(\Delta Y)^2} \tag{4.2}$$

$$\frac{(B'-B)}{\Delta t}+\frac{U}{2}\frac{(B'+B)(1-e^{-i\alpha\Delta X})}{\Delta X}+\frac{V}{2}\frac{(B'+B)i\sin\beta\Delta Y}{\Delta Y} \\ =\frac{1}{\Pr}\frac{(B'+B)(\cos\beta\Delta Y-1)}{(\Delta Y)^2}-\frac{R}{2\Pr}(B'+B) \tag{4.3}$$

$$\frac{(D'-D)}{\Delta t}+\frac{U}{2}\frac{(D'+D)(1-e^{-i\alpha\Delta X})}{\Delta X}+\frac{V}{2}\frac{(D'+D)i\sin\beta\Delta Y}{\Delta Y} \\ =\frac{1}{Sc}\frac{(D'+D)(\cos\beta\Delta Y-1)}{(\Delta Y)^2} \tag{4.4}$$

Equations (4.2) to (4.4) can be rewritten as,

$$(1+E)A' = (1-E)A+\frac{Gr}{2}(B'+B)\Delta t+\frac{Gc}{2}(D'+D)\Delta t \tag{4.5}$$

$$(1+F)B' = (1-F)B \tag{4.6}$$

$$(1+G)D' = (1-G)D \tag{4.7}$$

Where,

$$E=\frac{U}{2}\frac{\Delta t}{\Delta X}(1-e^{(-i\alpha\Delta X)})+\frac{V}{2}\frac{\Delta t}{\Delta Y}i\sin(\beta\Delta Y)-(\cos\beta\Delta Y-1)\frac{\Delta t}{(\Delta Y)^2}+\frac{M\Delta t}{2}$$

$$F=\frac{U}{2}\frac{\Delta t}{\Delta X}(1-e^{(-i\alpha\Delta X)})+\frac{V}{2}\frac{\Delta t}{\Delta Y}i\sin\beta\Delta Y-\frac{(\cos\beta\Delta Y-1)}{\Pr}\frac{\Delta t}{(\Delta Y)^2}+\frac{R\Delta t}{2\Pr}$$

$$G=\frac{U}{2}\frac{\Delta t}{\Delta X}(1-e^{(-i\alpha\Delta X)})+\frac{V}{2}\frac{\Delta t}{\Delta Y}i\sin\beta\Delta Y-\frac{(\cos\beta\Delta Y-1)}{Sc}\frac{\Delta t}{(\Delta Y)^2}$$

After eliminating B' and D' in Equation (4.5) using Equations (4.6) and (4.7),the resultant equation is given by,

$$(1+E)A' = (1-E)A + B\frac{Gr\Delta t}{(1+F)} + D\frac{Gc\Delta t}{(1+G)} \tag{4.8}$$

Equations (4.6) to (4.8) can be written as follows:

$$A' = \left(\frac{1-E}{1+E}\right)A + \left(\frac{Gr\,\Delta t}{(1+E)(1+F)}\right)B + \left(\frac{Gc\,\Delta t}{(1+E)(1+G)}\right)D$$

$$B' = \left(\frac{1-F}{1+F}\right)B$$

$$D' = \left(\frac{1-G}{1+G}\right)D$$

In matrix form

$$\begin{bmatrix} A' \\ B' \\ D' \end{bmatrix} = \begin{bmatrix} \frac{1-E}{1+E} & Q_1 & Q_2 \\ 0 & \frac{1-F}{1+F} & 0 \\ 0 & 0 & \frac{1-G}{1+G} \end{bmatrix} \begin{bmatrix} A \\ B \\ D \end{bmatrix}$$

Where, $Q_1 = \frac{Gr\,\Delta t}{(1+E)(1+F)}$ and $Q_2 = \frac{Gc\,\Delta t}{(1+E)(1+G)}$

From the above system of equations amplification matrix is given by

$$\begin{bmatrix} \frac{1-E}{1+E} & Q_1 & Q_2 \\ 0 & \frac{1-F}{1+F} & 0 \\ 0 & 0 & \frac{1-G}{1+G} \end{bmatrix} \tag{4.9}$$

Now, for stability of the finite difference scheme, the modulus of each eigen value of the amplification matrix does not exceed unity. Since the matrix equation (4.9) is triangular, the eigen values are its diagonal elements. The eigen values of the amplification matrix are, $(1-E)/(1+E)$, $(1-F)/(1+F)$ and $(1-G)/(1+G)$. Assuming that, U is everywhere non-negative and V is everywhere non-positive, we get

$$E = 2a\sin^2\left(\frac{\alpha\Delta X}{2}\right) + 2c\sin^2\left(\frac{\beta\Delta Y}{2}\right) + \frac{M\Delta t}{2} + i\left(a\sin\alpha\Delta X - b\sin\beta\Delta Y\right)$$

Where $a = \frac{U}{2}\frac{\Delta t}{\Delta X}$, $b = \frac{|V|}{2}\frac{\Delta t}{\Delta Y}$, $c = \frac{\Delta t}{(\Delta Y)^2}$

Since the real part of E is greater than or equal to zero, $|(1-E)/(1+E)| \leq 1$ always. Similarly, $|(1-F)/(1+F)| \leq 1$ and $|(1-G)/(1+G)| \leq 1$.

Hence the finite difference scheme is unconditionally stable. The local truncation error is $O(\Delta t^2 + \Delta Y^2 + \Delta X)$ and it tends to zero as Δt , ΔX and ΔY tend to zero. Hence the scheme is compatible. Stability and compatibility ensures convergence.

Conclusion:

In conclusion, the present study confirms that the proposed implicit finite difference scheme provides a stable and dependable numerical framework for analyzing unsteady magnetohydrodynamic radiative flow with mass diffusion. The Von Neumann matrix stability analysis, supported by the examination of amplification matrix eigenvalues, demonstrates unconditional stability with bounded growth for all discretization choices. Moreover, the scheme maintains its stability characteristics even under significant variations in radiation, magnetic field strength, and diffusion parameters. These findings establish the robustness and efficiency of the method, making it highly suitable for accurately simulating complex coupled MHD heat and mass transfer phenomena in engineering and physical applications.

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