

# When Does Ontology Buy Phenomenology? A Critical Assessment of Quantum-Form Black Holes as Dark Matter, the Universality Bottleneck, and the Graviton Condensate Program

Shivashish Borah

Independent Researcher, Bengaluru, India

## Abstract

We assess the proposition that dark matter consists of black holes in a “quantum form” and ask under what conditions such an ontological reinterpretation produces phenomenology distinguishable from standard  $\Lambda$ CDM or the well-studied primordial black hole (PBH) dark matter program. We articulate a universality theorem: cosmology is an infrared (IR) science, and most ultraviolet (UV) reinterpretations of the dark sector flow to identical IR effective theories under the coarse-graining intrinsic to cosmological observation. We identify the condition under which this universality fails: the UV physics must generate at least one operator that is relevant in the renormalization-group sense, or that enters a population-level evolution equation integrating over cosmic time. We analyze the memory-burden mechanism of Dvali and collaborators as the canonical surviving example. We then analyze the Dvali–Gomez graviton condensate framework as the leading candidate for a UV-motivated dark matter program in which a single microscopic parameter — the graviton occupation number  $N$  — could in principle control multiple independent observables. We derive the algebraic consequences in the compact-object (critical) regime and identify what we argue is the central obstruction to extending the framework to galactic halos: halos are subcritical condensates by a factor of order  $10^8$  in linear size, and the existence of a non-trivial subcritical fixed point in the condensate variational problem is an open question. We propose a falsifiability criterion — the two-observable, one-parameter principle — under which a quantum-gravity-inspired dark matter model becomes scientifically vulnerable, and we list concrete open problems whose resolution would determine whether the program graduates from ontological interpretation to predictive theory.

## 1 Introduction

The hypothesis that dark matter consists of black holes is among the oldest non-particle proposals in cosmology (Hawking1971?; Chapline1975?). In its modern primordial black hole (PBH) form (CarrHawking1974?; Carr2016?; CarrKuhnel2020?), the proposal is well-defined, fully gravitational, and constrained from multiple directions: microlensing surveys (Niikura2019?; Niikura2024?), cosmic microwave background (CMB) accretion bounds (AliHaimoud2017?), gravitational wave merger rates (Sasaki2018?), and Hawking-evaporation gamma-ray backgrounds (Carr2010?). The constraints have

closed most of the available mass window, leaving asteroid-mass PBHs ( $10^{17}$ – $10^{22}$  g) as the principal surviving regime in which PBHs may constitute 100% of the dark matter (CarrKuhnel2020?).

A distinct but often conflated proposal asserts that dark matter consists of black holes in a “quantum form.” The phrase is suggestive but, as we shall argue, requires unpacking before it constitutes a physical theory. Three readings are common:

1. Black holes as primordial relics behaving as cold gravitating point masses (the standard PBH-DM hypothesis).
2. Black holes whose Hawking evaporation is modified by quantum-gravity effects, opening sub-asteroid mass windows — most prominently the memory-burden mechanism of Dvali and collaborators (Dvali2018?; Dvali2020?; Alexandre2024?).
3. Black holes understood as localized excitations of quantum spacetime — e.g., graviton condensates (DvaliGomez2011?; DvaliGomez2013?), holographic constructs (Maldacena1999?), or ER=EPR configurations (MaldacenaSusskind2013?) — whose collective behavior could in principle constitute the dark sector.

Reading (i) is well-defined but offers no new physics beyond the constraints already established. Reading (iii) is conceptually rich but has rarely been developed into a quantitative dark matter program. Reading (ii) sits between them: it inherits the gravitational simplicity of PBH-DM while exploiting a UV correction that survives into IR phenomenology. In this paper we ask: *what general principle determines which of these readings produces new predictions, and which collapses to standard PBH-DM or  $\Lambda$ CDM?* We argue that the relevant principle is a form of renormalization-group universality. Cosmological observables — the CMB acoustic peaks, the matter power spectrum  $P(k)$ , weak lensing observables, large-scale structure formation — are sensitive only to coarse-grained features of the dark sector: its stress-energy tensor, its sound speed, and possibly its self-interaction cross-section. Two microscopically distinct dark matter models that produce identical coarse-grained stress-energy will produce identical cosmological observables. This is the same universality that underlies the success of  $\Lambda$ CDM as a phenomenological model: it does not need to know what dark matter *is* in order to fit the data, because the data does not depend on what dark matter is, only on how it gravitates.

The implication is that ontological reinterpretation is, by itself, never enough. A quantum-gravity-inspired dark matter program acquires scientific content only when it modifies a quantity to which the IR coarse-graining is sensitive. We will make this precise in Section 2.

For orientation, Table 1 summarizes the principal dark matter candidates discussed in this paper, the channel by which each escapes the universality bottleneck, and their current observational status.

*Summary of dark matter candidates by microscopic parameter and route by which their UV physics survives cosmological coarse-graining (escape routes E1–E4 defined in Section 2). The graviton condensate program is the central object of analysis in this paper.*

Candidate	Microscopic parameter	Escape route	Status
$\Lambda$ CDM (collisionless)	none (effective)	—	Reference model.
WIMPs	$m_\chi$ , $\sigma_{\text{ann}}$	E1 (cross-section), E4 (annihilation rate)	Constrained by direct detection (LZ, XENONnT) and indirect detection.
Axion / ultra-light scalars (FDM)	$m_\phi$ (and $f_a$ )	E3 (de Broglie wavelength)	Mass window narrowed by Lyman- $\alpha$ and substructure.

Candidate	Microscopic parameter	Escape route	Status
Standard PBH-DM	$dN/dM$ (mass function)	E4 (granularity)	Asteroid-mass window $10^{17} - 10^{22}$ g remains open.
Memory-burdened PBHs	$M_i, k$ (burden exponent)	E2 (modified rate equation)	New mass window $10^9 - 10^{15}$ g opened; constrained by gamma-ray observations.
Self-interacting DM	$\sigma/m$	E1 (cross-section)	Constrained by Bullet Cluster, cluster cores.
Graviton condensate (compact-object regime)	$N \sim (M/M_p)^2$	E2 + E3 (rate + coherence)	Compact-object predictions exist; cosmological extension open.
Graviton condensate (halo regime, hypothetical)	$N$ (subcritical)	E3 (coherence length)	Subcritical fixed point unknown; no derived $P(k; N)$ .

The structure of the paper is as follows. Section 2 formalizes the universality argument in effective field theory language and states the central criterion under which UV ontology can survive into IR phenomenology. Section 3 introduces a three-level hierarchy (ontology, effective theory, observables) that organizes the discussion. Section 4 analyzes the memory-burden mechanism as the canonical surviving example: a UV correction whose effect on a population-level evolution equation amplifies into a measurable IR signature. Section 5 reviews the Dvali–Gomez graviton condensate framework and derives the critical scaling relations  $R \sim \sqrt{N} \ell_p$  and  $M \sim \sqrt{N} M_p$  for compact-object configurations. Section 6 addresses the extension of the condensate picture to galactic halos and identifies the subcriticality problem as the central obstruction. Section 7 discusses the route from a condensate microphysics to a candidate matter power spectrum  $P(k; N)$ , drawing analogies with fuzzy dark matter. Section 8 states the two-observable, one-parameter falsifiability criterion. Section 9 lists open problems. Section 10 concludes.

## 2 The Universality Problem

### 2.1 Cosmology as an infrared science

The observable consequences of dark matter physics enter cosmological data primarily through the dark sector contribution to the stress-energy tensor:

$$T^{\mu\nu}(\text{DM})(x) = \rho_{\text{DM}}(x) u^\mu u^\nu + P_{\text{DM}}(x) (g^{\mu\nu} + u^\mu u^\nu) + \pi^{\mu\nu}(x), \quad \text{\label{eq:stressenergy}}$$

where  $\rho_{\text{DM}}$  is the energy density,  $P_{\text{DM}}$  the isotropic pressure,  $u^\mu$  the four-velocity, and  $\pi_{\mu\nu}$  the anisotropic stress. The Einstein equations and the linearized perturbation equations then determine the evolution of cosmological observables in terms of these quantities together with the sound speed  $c_s^2 \equiv \delta P / \delta \rho$  and the viscosity  $\mathcal{c}_{\text{vis}}^2$  encoding the anisotropic-stress evolution (Hu1998?; MaBertschinger1995?).

A key observation is that the mapping from microphysics to  $(\rho, P, \pi_{\mu\nu}, c_s)$  is many-to-one. Two ultraviolet completions  $A$  and  $B$  of the dark sector that produce identical coarse-grained  $(\rho, P, \pi_{\mu\nu}, c_s)$

will produce identical Einstein-equation solutions, identical linear perturbation evolution, and therefore identical CMB acoustic peaks, identical matter power spectra at the perturbative level, and identical lensing observables.

Cold dark matter (CDM), in this framework, is defined by

$$\begin{equation} P_{\text{CDM}} \approx 0, \quad \pi^{\text{CDM}}_{\mu\nu} \approx 0, \quad c^2_{s,\text{CDM}} \approx 0. \quad \text{[eq:cdmdef]} \end{equation}$$

Any dark matter candidate that satisfies [eq:cdmdef] to within current observational precision is observationally indistinguishable from CDM at the level of linear cosmological perturbations. The only routes to phenomenological distinction are:

1. Non-zero pressure or anisotropic stress at some scale, e.g. from finite de Broglie wavelength (fuzzy dark matter (Hu2000?; Hui2017?)).
2. Non-zero self-interaction cross-section, modifying nonlinear structure (self-interacting dark matter (SpergelSteinhardt2000?; TulinYu2018?)).
3. Granularity of the dark matter distribution on small scales, modifying microlensing and dynamical friction observables (PBHs, massive compact halo objects).
4. Late-time deviations from collisionless behavior — decays, annihilations, or evaporation — modifying the abundance over cosmic time.

These are the four routes by which UV physics can survive into IR cosmological observables. We will argue below that route (d) is the one exploited by memory-burden PBHs, and that any quantum-gravity-inspired dark matter program must in practice find a way into one of these four channels.

### 2.2 Effective field theory formalism

Consider an effective action for the dark sector, expanded around the  $\Lambda$ CDM limit:

$$\begin{equation} S_{\text{eff}}[g_{\mu\nu}, \chi] = S_{\Lambda\text{CDM}}[g_{\mu\nu}, \chi] + \sum_i \frac{c_i}{\Lambda^{d_i-4}} \int d^4x \sqrt{-g} \mathcal{O}^{(d_i)}_i(g, \chi), \quad \text{[eq:effaction]} \end{equation}$$

where  $\chi$  schematically denotes dark-sector degrees of freedom,  $\Lambda$  is the UV cutoff (typically the Planck mass  $M_P$  for quantum-gravity-induced operators), and  $\mathcal{O}_i^{(d_i)}$  are local operators of mass dimension  $d_i$ . Under renormalization-group flow from  $\Lambda$  to an IR scale  $\mu$ , the coefficients run schematically as

$$c_i(\mu) \sim c_i(\Lambda) \left(\frac{\mu}{\Lambda}\right)^{d_i-4},$$

modulo logarithmic running and anomalous dimensions. The classification of operators is then:

- $d_i > 4$  (irrelevant):  $c_i(\mu) \rightarrow 0$  as  $\mu/\Lambda \rightarrow 0$ . The effect is Planck-suppressed at cosmological scales.
- $d_i = 4$  (marginal): logarithmic running only; the effect is approximately scale-invariant.
- $d_i < 4$  (relevant):  $c_i(\mu)$  grows in the IR, potentially producing large effects at cosmological scales.

Most quantum-gravity-induced operators are irrelevant in this classification: they arise from integrating out Planck-scale physics and are suppressed by powers of  $\mu/M_P$ . For cosmological scales  $\mu \sim H_0 \sim 10^{-33}$  eV and  $M_P \sim 10^{19}$  GeV, the suppression factor is of order  $(10^{-60})^{d_i-4}$  — catastrophic for any  $d_i > 4$ .

This yields the universality theorem in its simplest form:

**Theorem 1** (Universality Theorem). *Let A and B be two UV completions of the dark sector that differ only by operators of mass dimension  $d > 4$  in the effective action [eq:effaction]. Then, at cosmological scales  $\mu \ll \Lambda$ , the linear cosmological observables computed in A and B differ by quantities of order  $(\mu/\Lambda)^{d-4}$ , which are unobservable for any  $\Lambda \gtrsim 10^{15}$  GeV and  $d \geq 5$ .*

*Remark.* The theorem is not strictly mathematical — it depends on identifying the relevant scaling dimension and assumes the absence of fine-tuned cancellations. But it captures the essential reason why most “different UV story, same IR physics” proposals collapse: the corrections they generate are dimensionally suppressed to invisibility.

### 2.3 Escape routes from universality

The theorem does not assert that all UV physics is invisible at the IR; it asserts that only certain classes of UV physics survive. The escape routes are:

#### 2.3.0.1 (E1) *Relevant or marginal operators.*

If the UV physics generates an operator of dimension  $d \leq 4$ , the RG flow does not kill it. Examples include modified gravity couplings entering at dimension four (e.g., scalar-tensor terms (Clifton2012?)) and mass terms for new fields, which are relevant by power counting.

#### 2.3.0.2 (E2) *Integration over cosmic time.*

Even an exponentially small modification of a rate equation, integrated over the age of the universe, can produce a large effect. If the UV physics modifies a decay rate  $\Gamma$  such that

$$\frac{dN}{dt} = -\Gamma(t) N,$$

then a small change in  $\Gamma$  can produce an exponentially different surviving population  $N(t_0)$  at cosmic age  $t_0$ . This is the mechanism by which memory-burden corrections to Hawking evaporation survive into observable PBH-DM phenomenology, even though the underlying quantum gravity correction is Planck-suppressed.

#### 2.3.0.3 (E3) *Resonant or coherent enhancement.*

If the dark sector physics admits a macroscopically coherent state — a Bose–Einstein condensate, a quantum-pressure-supported soliton, or an axion field — the coherence length sets a new physical scale that can fall in the cosmologically relevant range, even if the underlying coupling is gravitational. Fuzzy dark matter exploits this route via the de Broglie wavelength  $\lambda_{\text{dB}} = \hbar/(mv)$ , which for  $m \sim 10^{-22}$  eV is of order kpc.

#### 2.3.0.4 (E4) *Discreteness signatures.*

The granularity of dark matter, irrelevant for fluid-level observables, becomes observable through microlensing, halo substructure, and stochastic gravitational wave backgrounds. PBH-DM exploits this route.

**Criterion 1** (Survival Criterion). *A quantum-gravity-inspired dark matter proposal produces observable consequences distinct from  $\Lambda$ CDM if and only if its UV physics enters one or more of the escape routes (E1)–(E4): generating a relevant operator, modifying a population-level rate equation, producing a macroscopically coherent state with a kpc-or-larger coherence length, or producing observable discreteness.*

We will use Criterion 1 to assess the graviton condensate program in Section 5 and Section 6.

## 2.4 The Bullet Cluster as a Level 2 constraint

A useful illustration of how cosmology constrains Level 2 directly — and bypasses Level 1 entirely — is the Bullet Cluster (Clowe2006?). Two galaxy clusters that recently collided show a clear spatial offset between the X-ray-emitting baryonic gas (concentrated in the central region, decelerated by collisional pressure) and the gravitational lensing signal (concentrated with the galaxies, indicating dark matter

passed through nearly unperturbed). This rules out dark matter models with self-interaction cross-section per unit mass exceeding approximately

$$\frac{\sigma}{m} \lesssim 1 \sim \frac{\text{cm}^2}{\text{g}} \quad \text{\label{eq:bulletcluster}}$$

at relative velocities of order  $10^3$  km/s. The constraint applies to any dark matter candidate, regardless of its microscopic identity.

For our purposes, the Bullet Cluster illustrates two structural features of cosmological inference:

1. It constrains a Level 2 quantity ( $\sigma/m$ ) directly, with no reference to Level 1.
2. It is satisfied trivially by any Level 1 candidate that does not produce a Level 2 self-interaction — including all gravitational-only proposals (PBHs, graviton condensates, fuzzy DM at the cosmological scale, and so on).

The constraint therefore tells us very little about the microscopic identity of dark matter beyond the negative statement that it cannot have substantial non-gravitational self-interactions. The same coarse-graining that makes [eq:bulletcluster] a powerful Level 2 constraint is exactly what makes it useless as a discriminator among gravitational candidates.

### 3 The Three-Level Hierarchy

It is useful to organize the discussion into three levels, corresponding to the distinct mathematical objects involved.

#### 3.0.0.1 Level 1 — *Ontology*.

The microscopic description of dark matter. Examples: a WIMP field of mass  $m_\chi$  with specified interactions; an axion-like scalar with a potential  $V(\phi)$ ; a population of primordial black holes with a mass function  $dN/dM$ ; a graviton condensate with occupation number  $N$ ; an entanglement structure in quantum spacetime.

#### 3.0.0.2 Level 2 — *Effective Theory*.

The coarse-grained description in terms of fluid variables:  $\rho(x, t)$ ,  $P(x, t)$ ,  $\pi_{\mu\nu}(x, t)$ ,  $c_s^2(k, t)$ , plus any non-gravitational interaction cross-sections  $\sigma(v)$ . This is the level at which the Einstein equations and the linear perturbation equations are written.

#### 3.0.0.3 Level 3 — *Observables*.

The quantities directly accessed by experiment: the CMB anisotropy spectrum  $C_\ell^{TT,TE,EE}$ , the matter power spectrum  $P(k)$ , weak lensing convergence statistics, large-scale structure correlation functions, gravitational-wave signals  $h(t)$ , and so on.

The flow of information is:

$$\text{Microphysics} \rightarrow (\underbrace{\rho, P, \pi_{\mu\nu}, c_s, \sigma}_{\text{Level 1}}) \rightarrow (\underbrace{C_\ell, P(k), \kappa, h}_{\text{Level 3}}).$$

*Observation.* Most ontological proposals fail because they specify Level 1 without ever producing a derivation of the Level 2 quantities that differs from the  $\Lambda$ CDM defaults. The model is then observationally indistinguishable from  $\Lambda$ CDM at Level 3, regardless of how exotic the Level 1 story may be.

The criterion for a proposal to escape this fate is that the map Level 1  $\rightarrow$  Level 2 must produce something non-trivial: a non-zero pressure, a non-zero anisotropic stress, a non-zero coherence length, a

non-zero cross-section, or a non-zero decay rate. The Universality Theorem above can be restated in this language: irrelevant operators at Level 1 produce zero modification at Level 2 in the IR limit.

#### 4 Memory Burden: The Canonical Surviving Example

The cleanest example of a quantum-gravity correction that survives universality via route (E2) is the memory-burden mechanism for black hole evaporation (Dvali2018?; Dvali2020?; Alexandre2024?). We review the argument because it serves as the prototype for the kind of derivation that any condensate-based dark matter program must produce.

##### 4.1 Semi-classical evaporation and the Hawking lifetime

A Schwarzschild black hole of mass  $M$  has Hawking temperature

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \simeq \frac{M_p^2}{8\pi M} \quad (c = \hbar = k_B = 1).$$

The semi-classical mass-loss rate is

$$\frac{dM}{dt} = -\frac{\alpha}{G^2 M^2},$$

with  $\alpha$  an  $\mathcal{O}(10^{-3})$  greybody-dependent constant. Integrating yields the standard Hawking lifetime

$$\tau_H(M) = \frac{G^2 M^3}{3\alpha} \simeq \frac{M^3}{M_p^4}.$$

Setting  $\tau_H(M_*) = t_0 \approx 10^{17}$  s yields the standard PBH evaporation cutoff  $M_* \approx 5 \times 10^{14}$  g; below this mass, semi-classical PBHs cannot survive to the present epoch and so cannot constitute a non-negligible component of dark matter.

##### 4.2 The memory-burden correction

The semi-classical analysis assumes that the black hole behaves as a thermal source independent of its prior emission history. Dvali’s argument is that this assumption fails for any system carrying a large quantum information capacity. The Bekenstein–Hawking entropy

$$S_{BH} = \frac{A}{4G\hbar} = 4\pi \frac{M^2}{M_p^2}$$

counts an exponentially large number of internal states  $\mathcal{N} = e^{S_{BH}}$ , encoding information about the initial state of collapse. As the black hole evaporates and  $M$  decreases, the information content of the remaining  $M$  does not decrease proportionally: a parametrically large amount of information remains stored in degrees of freedom that must eventually be radiated away.

The argument of (Dvali2018?) is that this stored information “backreacts” on the emission rate. Specifically, once the black hole has lost a fraction of its mass — typically estimated as  $\sim 1/2$  (Alexandre2024?) — the decay channels saturate and the further emission rate is suppressed by a factor of order  $\mathcal{N}^{-k}$  for some positive  $k$ . The modified lifetime becomes

$$\tau_{MB}(M_i) \sim \tau_H(M_i) \left(\frac{M_i}{M_p}\right)^{2k},$$

where  $M_i$  is the initial mass. For  $k \geq 1$  this enhancement can be enormous: a  $10^9$  g black hole has  $\tau_H \sim 10^{-12}$  s but  $\tau_{MB} \gtrsim 10^{17}$  s, allowing such ultra-light PBHs to survive to the present.

##### 4.3 Cosmological implication

The memory-burden correction [eq:mblifetime] opens a new PBH-DM mass window  $M \sim 10^9\text{--}10^{15}$  g, well below the semi-classical cutoff  $M_* \approx 5 \times 10^{14}$  g. PBHs in this window emit a characteristic late-

time gamma-ray background as they enter the second, exponentially suppressed phase of evaporation. Recent analyses have begun to use Fermi-LAT, EGRET, and INTEGRAL data to constrain or detect this signature (Thoss2024b?; Thoss2024a?).

#### 4.4 Why this works: explicit amplification estimate

The memory-burden mechanism survives universality not because it modifies a relevant operator (it does not), but because it modifies a population-level rate equation. The integral

$$N_{\text{PBH}}(t) = N_{\text{PBH}}(0) \exp\left(-\int_0^t \frac{dt'}{\tau(M(t'))}\right) \quad \text{\label{eq:survivingpopulation}}$$

is exponentially sensitive to the lifetime  $\tau$ , so even a Planck-suppressed correction to the underlying microphysics can produce an exponentially different surviving population. This is the canonical illustration of escape route (E2) above.

To make the amplification explicit, consider a fractional correction  $\epsilon$  to the evaporation rate:

$$\frac{dM}{dt} = -\frac{1}{G^2 M^2} (1 + \epsilon), \quad |\epsilon| \ll 1.$$

The corrected lifetime is  $\tau(M_i)(1 + \epsilon)^{-1}$ , and the fractional change in the surviving fraction at the present epoch is

$$\frac{\Delta N_{\text{PBH}}}{N_{\text{PBH}}} \sim \epsilon \cdot \frac{t_0}{\tau(M_i)}. \quad \text{\label{eq:fractionalchange}}$$

For PBHs near the semi-classical cutoff  $M_i \sim M_*$ , the prefactor  $t_0/\tau$  is of order unity, and the effect is modest. But for  $M_i$  a few orders of magnitude below  $M_*$  with the memory-burden suppression switched on, the exponent of the surviving fraction changes by a factor of order  $(M_*/M_i)^3$ . For  $M_i \sim 10^9$  g, this is a factor of  $\sim 10^{15}$ , corresponding to a difference of order  $10^{15}$  in the surviving abundance — the difference between “zero” and “unity” as a fraction of the cosmological dark matter density.

This is the mechanism by which a microscopic quantum-gravity correction — even one with a small dimensionless coefficient — produces a macroscopically  $\mathcal{O}(1)$  modification of the cosmological PBH population. The key ingredient is that the underlying observable (the surviving population) sits inside an exponential of the lifetime, so the relevant figure of merit is the exponent, not the prefactor. Few cosmological observables share this structure, which is why memory burden is unusual among quantum-gravity-motivated dark matter mechanisms.

*Observation.* The memory-burden mechanism is the principal published example of a quantum-gravity-motivated correction that converts an ontological reinterpretation of black holes into a measurable cosmological signature. Any condensate-based dark matter program should be benchmarked against it.

## 5 The Graviton Condensate Framework

The deepest of the three readings of “black holes in a quantum form” identified in Section 1 is the proposal that black holes are macroscopic bound states of soft gravitons — a graviton Bose–Einstein condensate (DvaliGomez2011?; DvaliGomez2013?; DvaliGomez2014?). We review the framework and derive its principal scaling relations, with an eye toward the dark matter extension to be addressed in Section 6.

### 5.1 The N-portrait of a Schwarzschild black hole

A Schwarzschild black hole of mass  $M$  has Schwarzschild radius

$$R_S = \frac{2GM}{c^2} = \frac{2M}{M_p^2} \quad (c = \hbar = 1).$$

The Dvali–Gomez proposal is that the black hole is a self-bound state of  $N$  gravitons of characteristic wavelength  $\lambda \sim R_S$ . Each graviton has energy

$$\epsilon \sim \frac{\hbar c}{\lambda} \sim \frac{1}{R_S} = \frac{M_P^2}{2M}$$

The total mass is then  $M = N\epsilon$ , yielding

$$N \sim \frac{M^2}{M_P^2}$$

Equivalently, the Schwarzschild radius can be written as

$$R_S = \sqrt{N} \ell_P,$$

where  $\ell_P = 1/M_P$  is the Planck length. The black hole is, in this picture, a  $\sqrt{N}$ -magnification of the Planck scale by the bosonic enhancement of the condensate.

### 5.2 Self-consistency and quantum criticality

The gravitational coupling among the constituent gravitons is, by dimensional analysis,

$$\alpha_g \sim G\epsilon^2 \sim \frac{1}{M^2/M_P^2} = \frac{1}{N}.$$

The product  $\alpha_g N \sim 1$  identifies the condensate as sitting at the marginal stability point — the maximally packed bound state. This is the quantum criticality condition of (DvaliGomez2013?): the condensate is precisely at the threshold of collapse, balanced by the bosonic enhancement of the attractive coupling.

Three predictions follow from this critical structure:

1. The mass–radius relation  $R \propto M$ , equivalent to the Schwarzschild relation  $R_S \propto M$ , is reproduced.
2. The Bekenstein–Hawking entropy emerges as the multiplicity of states of the marginal condensate:  $S_{BH} \sim N$ , matching  $A/(4G\hbar) = (R_S/\ell_P)^2 \sim N$ .
3. The Hawking radiation rate is reproduced as the rate of quantum depletion of the condensate, with characteristic depletion time  $\tau \sim N^{3/2}/M_P \sim M^3/M_P^4$ .

### 5.3 Memory burden in the condensate language

In the condensate picture, the memory-burden mechanism arises naturally: the  $N$  gravitons in the condensate carry  $\mathcal{O}(N)$  bits of quantum information, and the rate at which this information can be released back into Hawking radiation is bounded by unitarity. Once the condensate has shed roughly half of its constituents, the residual information “burdens” the further depletion, suppressing the rate. This provides a microscopic mechanism for the lifetime correction [eq:mlifetime].

The condensate framework therefore unifies two of the three readings of “quantum-form black holes” into a single picture: ontological reinterpretation (Reading iii) and modified evaporation (Reading ii) emerge from the same  $N$ -portrait.

### 5.4 Numerical scale

For benchmarking, we record the values of  $N$  implied by [eq:nportrait] at various physical mass scales:

$$\begin{array}{l} \begin{array}{l} \text{Planck-mass BH} \\ \text{asteroid-mass PBH} \\ \text{Solar-mass BH} \\ \text{Supermassive BH} \end{array} \quad \& \quad \begin{array}{l} 1 \\ 10^9 \\ 10^{37} \\ 10^{41} \end{array} \\ \text{g (memory-burden lower edge)} & \& \approx 10^{28} \\ \text{g (asteroid-mass PBH)} & \& \approx 10^{44} \\ \text{Solar-mass BH} & \& 10^{76} \\ \text{Galaxy mass } (10^9, M_\odot) & \& 10^{94} \\ \text{Galaxy mass } (10^{12}, M_\odot) & \& 10^{100} \end{array}$$

(All values approximate;  $M_P \approx 2.2 \times 10^{-5} \text{ g} \approx 1.2 \times 10^{19} \text{ GeV}$ .) The condensate framework therefore has  $N$  ranging over more than a hundred orders of magnitude across physically relevant regimes, all con-

trolled by a single scaling relation.

## 6 Extending the Condensate to Galactic Halos: The Subcriticality Problem

The question of whether the graviton condensate framework can be extended to galactic dark matter halos is, in our view, the central technical question of this entire program. We argue that it is more subtle than has sometimes been suggested in the literature, and that the answer is currently unknown.

### 6.1 Naive extension: equating $L_{\text{halo}}$ to $\sqrt{N} \ell_p$

A natural starting point is to ask: for what value of  $N$  would the condensate length scale  $L = \sqrt{N} \ell_p$  coincide with a galactic halo radius  $R_{\text{halo}} \sim 1 \sim 10^{19} \text{ m}$ ?

$$L = \sqrt{N} \ell_p \approx 1 \sim 10^{19} \text{ m}. \quad \text{[eq:naiveextension]}$$

With  $\ell_p \approx 1.6 \times 10^{-35} \text{ m}$ , this gives

$$N_{\text{halo}} \sim \left( \frac{1 \sim 10^{19}}{1.6 \times 10^{-35}} \right)^2 \approx 4 \times 10^{108}. \quad \text{[eq:nhalo]}$$

### 6.2 Comparison with the critical condensate prediction

The Dvali–Gomez critical relation [eq:nportrait], however, predicts  $N$  from the mass of the bound state. For a galaxy of mass  $M_{\text{halo}} \sim 10^{12} M_{\odot} \approx 2 \times 10^{42} \text{ kg} \approx 10^{50} M_p$ , the critical  $N$  would be

$$N_{\text{crit}}(M_{\text{halo}}) \sim \left( \frac{M_{\text{halo}}}{M_p} \right)^2 \sim 10^{100}. \quad \text{[eq:ncrit]}$$

The two values — [eq:nhalo] from the radius and [eq:ncrit] from the mass — differ by approximately eight orders of magnitude. This is not a small mismatch.

### 6.3 Interpretation: halos are subcritical

The discrepancy has a clean physical interpretation: *a galactic halo is not at quantum criticality*. If a halo of mass  $10^{12} M_{\odot}$  were a critical Dvali–Gomez condensate, it would have radius  $\sqrt{N_{\text{crit}}} \ell_p \sim 10^{50} \ell_p \sim 10^{15} \text{ m}$  — approximately a few AU, i.e. the Schwarzschild radius for that mass. It would be a galaxy-mass black hole, not a galaxy.

The factor of  $\sim 10^8$  by which the halo radius exceeds the Schwarzschild radius is precisely the factor by which the halo is subcritical: it is far from gravitational collapse, supported (in the standard CDM picture) by the kinetic motion of its constituent particles.

**Proposition 1** (Subcriticality Problem). *A galactic dark matter halo cannot be described by the critical Dvali–Gomez condensate ansatz  $R = \sqrt{N} \ell_p$ ,  $M = \sqrt{N} M_p$  with  $\alpha_g N \sim 1$ . A subcritical extension — in which  $\alpha_g N \ll 1$  and the condensate is supported by a stabilizing mechanism other than quantum criticality — is required.*

### 6.4 Possible resolutions

There are at least three logically distinct routes by which the subcriticality problem might be resolved, each corresponding to a distinct physical model:

#### 6.4.0.1 (R1) Halo as a non-gravitational dark condensate.

The condensate constituents are not gravitons but some hypothetical dark-sector boson with self-coupling weaker than gravity. The condensate is stabilized by the dark coupling, with gravity acting only as a background. This effectively reduces the program to a self-interacting bosonic dark matter model, of which fuzzy dark matter (Hu2000?; Hui2017?) is the most studied example. The connection to the

original “black hole in a quantum form” ontology is then weak; the model has decoupled from its compact-object origin.

**6.4.0.2 (R2) Halo as a coherent superposition of subcritical graviton states.**

The condensate is genuinely gravitational, but the gravitons are arranged in a coherent superposition that is far from the maximally packed critical configuration. The stabilization is provided by the macroscopic motion (a virial balance between graviton kinetic content and self-gravity), with a quantum pressure analogous to the Madelung pressure of fuzzy dark matter providing the small-scale cutoff. Whether such a state is consistent with the variational principle of the underlying condensate theory is, to our knowledge, an open question.

**6.4.0.3 (R3) Halo as a composite of microscopic critical condensates.**

The dark matter halo consists of many small critical condensates — e.g., asteroid-mass PBHs each at quantum criticality — with their collective dynamics producing the halo. This reduces to PBH-DM with no new condensate physics at the halo scale; the “quantum form” enters only at the compact-object scale via memory burden and similar corrections.

Of these, (R3) is best understood (it is essentially PBH-DM plus memory burden), (R1) is best constrained (it reduces to fuzzy DM and inherits its constraints), and (R2) is the most novel but also the least developed. We believe (R2) is where the genuinely new physics, if any, would emerge from the condensate program, but the requisite calculations have not been performed.

**6.5 The species-bound complication**

A separate concern with naive identifications of  $N$  at the halo scale is the species bound of Dvali (Dvali2010?):

$$M_*^2 \sim \frac{M_P^2}{N_{\text{species}}}, \quad \text{\label{eq:speciesbound}}$$

where  $M_*$  is the effective gravitational cutoff in the presence of  $N_{\text{species}}$  light particle species. If  $N$  in the condensate picture is identified with a species count, the species bound implies  $M_* \sim M_P/\sqrt{N}$ . For  $N \sim 10^{108}$ , this gives  $M_* \sim 10^{-100}$  GeV, far below any consistent IR scale. The species interpretation of  $N$  is therefore not viable for halo-scale physics.

By contrast, the graviton occupation number interpretation does not directly invoke the species bound:  $N$  counts gravitons in a single condensate state, not distinct particle species. This interpretation is consistent with halo-scale values of  $N$  in principle, but the subcriticality problem (Proposition 1) still applies.

**6.6 Status**

We summarize the state of the halo-extension program:

- The critical Dvali–Gomez condensate is well-defined and quantitatively predictive in the compact-object regime, where the scaling  $N \sim (M/M_P)^2$  is self-consistent.
- Halo-mass configurations are subcritical by a factor of  $\sim 10^8$  in size and cannot be described by the critical ansatz.
- No published derivation produces a subcritical condensate solution corresponding to a galactic halo from the underlying variational problem.
- The species interpretation of  $N$  is ruled out for halo scales by [eq:speciesbound]. The graviton occupation interpretation remains viable in principle.

The honest summary is that the extension to halos is currently an aspiration, not a derivation.

## 7 Toward $P(k; N)$ : Cosmological Phenomenology

If the subcriticality problem is resolved — e.g., by a successful version of resolution (R2) above — the next step is to derive the matter power spectrum  $P(k; N)$  as a function of the microscopic parameter  $N$ , and to compare it with  $\Lambda$ CDM and current cosmological data. We sketch what such a derivation would look like, what observable scales it would target, and where the principal difficulties lie.

### 7.1 The fuzzy dark matter template

The closest existing analog is fuzzy dark matter (FDM) (**Hu2000?**; **Hui2017?**): an ultralight scalar field  $\phi$  of mass  $m$  with de Broglie wavelength

$$\lambda_{\text{dB}} = \frac{\hbar v}{m} \sim 1 \text{ kpc} \left( \frac{10^{-22} \text{ eV}}{m} \right) \left( \frac{100 \text{ km/s}}{v} \right). \quad \text{\label{eq:debroglie}}$$

The Madelung formulation of the Schrödinger–Poisson system yields an effective fluid with a quantum pressure

$$P_Q = -\frac{\hbar^2}{2m^2} \rho \nabla^2 \left( \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right),$$

which suppresses linear power below the Jeans wavenumber

$$k_J = \left( \frac{16\pi G \rho a^2 m^2}{\hbar^2} \right)^{1/4}.$$

For  $m \sim 10^{-22}$  eV,  $k_J \sim \text{Mpc}^{-1}$ , suppressing structure on galactic scales while leaving the CMB acoustic peaks unaffected.

### 7.2 Adapting the template to a condensate

For a graviton condensate of occupation number  $N$ , the analog of the de Broglie wavelength is the condensate coherence length  $\xi(N)$ . Within the (hypothetical) subcritical resolution (R2), one expects schematically

$$\xi(N) \sim N^\alpha \ell_p,$$

for some exponent  $\alpha$  determined by the variational problem. Matching  $\xi(N) \sim \text{kpc}$  would then fix  $N$ , at which point the analog of [eq:quantumpressure] determines the small-scale suppression of  $P(k)$ .

The principal difficulty is that  $\alpha$  is not currently known from first principles in the halo regime. In the critical regime  $\alpha = 1/2$ , but as we have argued in Section 6, this regime does not apply to halos. The subcritical exponent could differ substantially, and without it the link between  $N$  and the cosmological suppression scale is not established.

### 7.3 Where current cosmology constraints would bite

If a derivation of  $\xi(N)$  in the subcritical regime were achieved, the resulting model would be subject to the same constraints that have narrowed the parameter space of fuzzy dark matter:

#### 7.3.0.1 Lyman- $\alpha$ forest.

The Lyman- $\alpha$  flux power spectrum at  $z \sim 2\text{--}5$  probes the matter power spectrum down to  $k \sim 5\text{--}10 \text{ h} \text{ Mpc}^{-1}$ . Recent reanalyses (**RogersPeiris2021?**; **Irsic2024?**) push the lower bound on the FDM mass to  $m \gtrsim 2 \times 10^{-20}$  eV at 95% CL, well above the value  $\sim 10^{-22}$  eV that was originally invoked to address the small-scale crisis. Any condensate model that mimics FDM-like suppression on  $\sim \text{kpc}$  scales will face the same constraint.

### 7.3.0.2 $S_8$ tension.

The amplitude of matter clustering at  $z = 0$  on  $\sim 8 h^{-1}$  Mpc scales is parameterized by  $S_8 = \sigma_8 \sqrt{\Omega_m}/0.3$ . Weak-lensing measurements (DES, KiDS, HSC) prefer values approximately  $2-3\sigma$  below the Planck CMB prediction (**DES2022?**; **KiDS2021?**). A condensate model that suppresses small-scale power could relieve this tension — but the same suppression risks conflicting with the Lyman- $\alpha$  bound above.

### 7.3.0.3 Substructure observables.

The abundance of Milky Way satellite galaxies and stellar streams in the Galactic halo constrain the dark matter power spectrum at  $k \sim 10-100 h \text{ Mpc}^{-1}$ . Recent analyses based on the Dark Energy Survey (**Nadler2021?**) place tight constraints on warm and fuzzy dark matter models.

### 7.4 The narrow window

For a derived condensate coherence length  $\xi(N)$  to relieve the  $S_8$  tension without violating Lyman- $\alpha$  constraints, the suppression must operate in a narrow scale window. This is the same difficulty that has confronted FDM, warm dark matter, and self-interacting dark matter for the past decade. A condensate model does not, in itself, evade the constraint; it must derive the suppression scale from microphysics and demonstrate that it falls in the allowed window.

## 8 Falsifiability: The Two-Observable, One-Parameter Principle

### 8.1 Statement of the criterion

Successful theories in physics are typically overconstrained: a single underlying parameter controls multiple independent observables, so that fitting one observable predicts another. We codify this as a criterion for assessing dark matter proposals:

**Criterion 2** (Two-Observable, One-Parameter). *A proposed dark matter framework attains the status of a falsifiable physical theory — as distinct from an ontological interpretation — when there exists a microscopic parameter  $\theta$  such that:*

1.  $\theta$  controls at least one cosmological observable  $\mathcal{O}_{\text{cosmo}}(\theta)$  (e.g., the matter power spectrum suppression scale, the  $S_8$  amplitude, the CMB damping tail);
2.  $\theta$  simultaneously controls at least one independent strong-field or compact-object observable  $\mathcal{O}_{\text{strong}}(\theta)$  (e.g., a modified Hawking lifetime, a quasi-normal-mode frequency shift, a ringdown spectrum deviation);
3. The functional dependences  $\mathcal{O}_{\text{cosmo}}(\theta)$  and  $\mathcal{O}_{\text{strong}}(\theta)$  are derived from microphysics, not fitted to data.

### 8.2 Historical precedent

This criterion has historical precedent. General relativity (**Einstein1915?**) produced two independent post-Newtonian predictions — the perihelion precession of Mercury and the deflection of light by the Sun — from a single dimensionless parameter (the post-Newtonian  $\gamma$ ). The Higgs mechanism predicted the gauge boson masses and the existence of a scalar resonance from a single vacuum expectation value (**Higgs1964?**; **Weinberg1967?**). Inflation linked the flatness problem, the horizon problem, and the spectrum of primordial perturbations to a single inflaton potential (**Guth1981?**; **LiddleLyth2000?**). In each case, fitting one observable produced a non-trivial prediction for the other.

### 8.3 Application to the condensate program

For the graviton condensate framework with microscopic parameter  $N$ , Criterion 2 would require:

$$N \rightarrow \{\xi(N), \tau(N), \delta\omega_{\text{QNM}}(N), \dots\}$$

where the cosmological observable might be the small-scale power spectrum suppression  $\xi(N)$ , and the strong-field observables might include the memory-burden-modified lifetime  $\tau(N)$  and ringdown frequency shifts  $\delta\omega_{\text{QNM}}(N)$ . The criterion is satisfied if:

- The same value of  $N$  predicts a cosmologically relevant suppression scale and a phenomenologically relevant compact-object signature;
- The functional dependences are derived from the variational principle of the condensate, not fitted;
- At least two of the predictions can in principle be measured.

The compact-object side is partially developed: the memory-burden corrections to  $\tau(N)$  are explicit in the literature, and quasi-normal-mode deviations from quantum-hair effects have been computed in related frameworks (CalmetKuipers2022?). The cosmological side, as discussed in Section 6, remains aspirational.

#### 8.4 Why this is the right filter

The criterion is more demanding than “does the model fit the data” for two reasons. First, any model with sufficient parameters can fit any finite data set; the question is whether the model commits itself to specific predictions before the data is consulted. Second, the criterion separates the ontological component of a proposal — which may be philosophically attractive but has no scientific content on its own — from the phenomenological component, which is what experiment can test. A theory that satisfies the criterion has placed itself in a position of risk: the predictions can be wrong, and if they are wrong, the theory is wrong. That risk is the source of scientific value.

### 9 Open Problems

We close with a list of concrete open problems whose resolution would determine whether the graviton condensate framework, or a related “quantum-form black hole” program, can graduate to a falsifiable dark matter theory. The list is not exhaustive but identifies what we view as the most consequential bottlenecks.

#### 9.0.0.1 Problem 1: Subcritical condensate solutions.

*Status: open.*

Does the variational problem underlying the Dvali–Gomez condensate admit a non-trivial subcritical solution corresponding to a galactic-halo configuration? In the critical regime  $\alpha_g N \sim 1$ , the maximal-packing condition fixes the radius–mass relation. In the subcritical regime, an additional stabilizing mechanism is required. Candidate mechanisms include macroscopic angular momentum, finite-temperature pressure of the constituent gravitons, and Madelung-type quantum pressure analogous to fuzzy dark matter. None has been carried through to a calculation of halo profiles.

#### 9.0.0.2 Problem 2: Derivation of $\xi(N)$ in the subcritical regime.

*Status: open, conditional on Problem 1.*

If a subcritical condensate solution exists, what is the dependence of the coherence length  $\xi$  on the occupation number  $N$ ? In the critical regime,  $\xi = \sqrt{N} \ell_p$ . In the subcritical regime, the scaling exponent must be derived from the variational problem and may differ substantially. This is the parameter that controls whether the resulting cosmology can be made consistent with both the  $S_8$  tension and the Lyman- $\alpha$  forest.

**9.0.0.3 Problem 3: Linear perturbation theory of a graviton condensate.****Status: open.**

The full relativistic linear perturbation theory of a graviton condensate coupled to baryons and radiation has not been formulated. The standard Boltzmann-hierarchy machinery for cold dark matter and fuzzy dark matter rests on a known stress-energy tensor and an equation of state. The analog for a graviton condensate requires the condensate's effective stress-energy in the cosmological background as input. Producing a transfer function  $T(k; N)$  that can be compared with Planck and DES is the principal computational task.

**9.0.0.4 Problem 4: Initial conditions from inflation.****Status: open.**

For the model to be predictive, the initial amplitude and spectrum of dark-sector perturbations must be specified, not fitted. A complete theory would derive the condensate initial conditions from the inflationary epoch, in the same way that the inflaton's vacuum fluctuations source the standard adiabatic CDM perturbations. The mechanism by which a graviton condensate would form during or after inflation is not currently known.

**9.0.0.5 Problem 5: Consistency with binary merger gravitational-wave observations.****Status: partially constrained.**

If a non-trivial fraction of dark matter consists of compact condensate objects (PBH-like at small mass, or larger condensate "balls"), the predicted merger rate and waveform must be consistent with LIGO/Virgo/KAGRA observations (**GWTC3?**). The ringdown spectrum of a condensate object is expected to differ from that of a Kerr black hole at the level of quantum-hair corrections (**CalmetKuipers2022?**); detecting or constraining these deviations is a focus of ongoing analysis.

**9.0.0.6 Problem 6: The species-cutoff puzzle for very large  $N$ .****Status: open conceptually.**

If the condensate  $N$  at galactic scales is genuinely  $\sim 10^{100}$ , the species-bound interpretation [eq:speciesbound] is inconsistent with the gravitational sector remaining weakly coupled at sub-Planck scales. While the graviton-occupation interpretation evades the immediate inconsistency, the relation between the two interpretations of  $N$  across regimes (compact-object vs. halo) has not been clarified.

**9.0.0.7 Problem 7: Distinguishing the condensate proposal from PBH-DM in the granularity sector.****Status: open.**

If the halo-extension fails and the condensate program reduces to "PBH-DM plus memory burden," the resulting phenomenology is granular: microlensing, halo substructure, and stochastic gravitational-wave-background signatures distinguish it from smooth dark matter. The condensate ontology adds nothing to the granular signatures over standard PBH-DM, but it may modify the mass function  $dN/dM$  at formation. Deriving the condensate-predicted formation mass function and comparing it with the PBH-formation mass function is an unsolved problem.

**10 Discussion and Conclusion**

The proposition that "dark matter is black holes in a quantum form" is intuitive, ontologically attractive, and connected to several active research programs in quantum gravity. Whether it is also a physical theory depends on whether the ontology can be made to produce observables distinguishable from standard  $\Lambda$ CDM or the well-developed primordial black hole program.

We have argued that this question reduces to a renormalization-group universality argument: most ultraviolet reinterpretations of dark matter flow to identical infrared effective theories, and the cosmological data lives entirely at the infrared end. The escape routes from this universality are limited and well-understood: a relevant operator, a modified rate equation integrated over cosmic time, a macroscopic coherence length, or an observable granularity. Memory-burden corrections to Hawking evaporation (**Dvali2018?**; **Dvali2020?**) exploit the second route and are, as far as we are aware, the only published example of a quantum-gravity-motivated dark matter mechanism that converts ontological reinterpretation into a measurable cosmological signature.

The graviton condensate framework of Dvali and Gomez (**DvaliGomez2011?**; **DvaliGomez2013?**) offers, in principle, a more ambitious unification: black holes and dark matter halos as different phases of a single condensate description, controlled by a single microscopic parameter  $N$ . We have argued that the principal obstruction to realizing this unification is the subcriticality problem: galactic halos lie eight orders of magnitude (in linear size) away from the critical condensate ansatz  $R = \sqrt{N} \ell_p$ , and no subcritical extension has been derived from the variational problem. Without such a derivation, the framework is well-defined for compact objects (where it is essentially a reformulation of the standard Schwarzschild geometry plus a specific calculation of Hawking radiation as condensate depletion) and aspirational for galaxies.

The criterion we have proposed for assessing the program — the two-observable, one-parameter principle (Criterion 2) — is intended to identify when ontological work begins to yield phenomenological dividends. The criterion is demanding: it requires that the same  $N$  control both a cosmological and a strong-field observable, with both functional dependences derived rather than fitted. The compact-object side is partially developed; the cosmological side is not. The honest assessment is that the program currently has the structure of a candidate theory but lacks the calculations that would render it falsifiable.

We close with a methodological remark. The conversation that motivated this paper followed a pattern that may be useful to record. The initial proposal — “dark matter is black holes in a quantum form” — was tested against successively sharper criteria: Does it differ from PBH-DM? Does the ontology survive cosmological coarse-graining? Does it predict something independent? Does one parameter control multiple observables? At each step, the proposal had to give up some of its initial generality to acquire any scientific content. By the end of the analysis, what remained was not a new theory of dark matter, but a precise diagnosis of what such a theory would have to deliver to count as one. We hope that this diagnosis is useful even if — as we expect — the specific condensate route ultimately requires substantial new physics to succeed.

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