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Application of the Power Division Theorem for Network Loss Formula Correction, Back Power Flow Tracking, and Validating Superposition Theorem Applicability in Automated EMS Grid Optimization

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Abstract:
Accurate determination of individual component sharing of system loads and grid losses has been a long-standing challenge in power systems engineering. Traditional loss formulations based on net branch currents introduce cross-product errors that obscure physical accountability during bidirectional power flows. This paper utilizes the Power Division Theorem (PDT) to establish a rigorous, flow-aware framework that corrects the element loss formula. By converting quadratic power dependencies into a linearized inner-product format, this approach provides definitive mathematical validation for the applicability of the Superposition Theorem in active power loss calculations. The theorem defines an exact mathematical approach to trace back power flows and apportion discrete generator contributions to specific loads and network losses. Furthermore, this paper provides mathematical proof that operating a grid under Equal Source Node Voltage conditions yields absolute minimum network losses without iterative numerical optimization. Designed with low computational overhead, this framework can be easily integrated into modern real-time SCADA/EMS systems utilizing industry-standard protocols.

Keywords: Back power flow (BPF) tracing, energy management systems (EMS), equal source node voltage, loss formula correction, network loss minimization, power division theorem (PDT), superposition theorem validation, supervisory control and data acquisition (SCADA

I. INTRODUCTION

The expansion of Distributed Energy Resources (DERs) has transformed traditional unidirectional distribution networks into complex, bidirectional active power grids. Traditional grid loss models rely on total net current, which fails to isolate individual generator footprints during concurrent forward and reverse power flows. This analytical deficit hampers Energy Management Systems (EMS) when calculating transmission pricing, enforcing carbon tracking, and performing optimal dispatch.

Historically, this calculation barrier arises because active power loss is a non-linear, quadratic function of current ($P=I^2R$), which fundamentally violates the linearity conditions required by classical network theorems. Consequently, the independent summation of discrete source contributions has been deemed mathematically impossible due to the generation of inseparable cross-product current terms.

To overcome this structural limitation, this paper utilizes the **Power Division Theorem (PDT)**. By introducing complex topological allocation matrices, the PDT scales and projects net branch current profiles directly into isolated, source-specific vector paths. This complex inner-product transformation effectively linearizes the active power equations, removing the mathematical cross-product terms without approximations. As a result, this framework successfully **gives life to the applicability of the Superposition Theorem in active power calculations**. For the first time, independent generator loss contributions can be evaluated in complete isolation and directly superposed to match total physical grid degradation, unlocking exact accountability within complex multi-source networks.

In deregulated electricity markets, isolating individual generation impacts has relied on tracing paradigms or residual allocations. Bialek's upstream/downstream tracing methods established a baseline for tracking topological component flow, yet these models depend heavily on the proportional sharing assumption, which lacks localized verification at a complex sub-network level [1] Kirschen et al introduced the concept of Proportional Sharing **(or** Tracing**)** to allocate the real and reactive power generated by specific power plants to individual loads and transmission lines. By tracing power flow through the network without altering the standard power flow equations, this method fundamentally changed how power systems analyze congestion, price transmission services, and calculate bilateral contracts in deregulated electricity markets [2] The modern "Power Balance" method calculates loss sharing as a simple residual difference between aggregate generation and a generator's share on system loads. As demonstrated by recent analyses, this approach fails because it linearizes inherently quadratic branch parameters and drops cross-product coupling matrices [3,4]. To bypass matrix complexity, backward/forward sweep algorithms have been utilized for radial networks, but they lose mathematical fidelity when applied to highly meshed multi-source grids with lack of a single dominant slack bus. [1,3].The Power Sharing Principle (PSP) is widely cited across IEEE literature to analyze multi-source nodes[1,2] However, recent findings confirm that while PSP holds valid at a solitary isolated node, it collapses across complex meshed networks because it omits individual back-feeding attributes [5]. Courant’s classical optimization and Lagrange multiplier models serve as the benchmark for system loss reduction. An Improved Grey Wolf Optimization (IGWO) method is proposed to simultaneously determine optimal Distributed Generation (DG) placement and network reconfiguration to minimize power loss and improve voltage profiles. Tested on a 33-bus system, this approach effectively enhances voltage stability and meets operational constraints.[6] Unfortunately, these classical optimization engines demand iterative numerical routines that introduce a high computational burden into real-time SCADA environments .To overcome these structural limitations, the Power Division Theorem (PDT) was developed. Grounded firmly in Kirchhoff’s Current Law (KCL), the PDT establishes that active and reactive sharing coefficients can be directly mapped using a network's native admittance bounds [4]. This paper expands upon the foundational PDT framework by introducing a corrected element loss formula, proving how an Equal Source Node Voltage regime minimizes grid losses without iterative compute overhead, and validating its architecture for automated closed-loop EMS integration [3,4,5,6,7]

2. MATHEMATICAL FORMULATION AND LOSS FORMULA CORRECTION

Traditional branch power loss formulations treat the path as a lumped element carrying net current, introducing cross-product errors when multiple sources operate simultaneously.

- **The Cross-Product Limitation in Traditional Loss Formulas**

Consider a network branch with resistance Rij carrying a total current I_{Total} resulting from two distinct sources, I_1 and I_2. The standard loss calculation is:

$$P_{loss} = (I_1 + I_2)^2 \times R_{ij} = \left(I_1^2 \times R_{ij}\right) + \left(I_2^2 \times R_{ij}\right) + (2 \times I_1 \times I_2 \times R_{ij}) \quad \text{------} \quad \textbf{(1)}$$

The cross-product term $(2 \times I_1 \times I_2 \times R_{ij})$ represents a mathematical coupling that prevents the accurate tracking of loss attribution to a single source.

- **The Power Division Theorem Solution**

The PDT overcomes this limitation by transforming the branch current into a vector of explicit fractional components directly tied to specific source nodes using network topology parameters. Let **Y** be the network admittance matrix. The total current in any active network branch is mapped using structural power division coefficients **αk**

$$I_{total} = \sum_{k=1}^{N} \alpha_k . I_k \quad \text{------------------} \quad \textbf{(2)}$$

Where Ik is the current vector from source k and αk represents the dimensionless complex topological allocation matrix element.

By isolating the complex current footprints, the corrected element loss allocated explicitly to source **k** within that specific branch is defined as:

$$P_{Loss} = \mathrm{Re}(V_{branch} . (\alpha_k . I_{total})^*) \quad \text{----------------} \quad \textbf{(3)}$$

Summing these yields the total loss without mathematical leftovers

$$P_{loss,total} = \sum_{k=1}^{N} P_{loss,k} \quad \text{-----------------} \quad \textbf{(4)}$$

Crucially, this formulation **gives life to the applicability of the Superposition Theorem in power calculations**. Historically, the superposition principle is strictly limited to linear network quantities (voltages and currents) and breaks down completely for power because it is a quadratic function ($P = I^2R$). This non-linear relationship generates the cross-product terms shown in equation (1), making it impossible to sum independent active power contributions.

By translating the branch expression into a complex topological inner-product form in equation (3), the PDT effectively **linearizes power allocation**. This allows the independent, decoupled contribution of each individual power source to be evaluated and superposed cleanly to match the exact physical grid loss. This structural breakthrough addresses the cross-product ambiguity, establishing an accurate, verifiable link between individual generation profiles and physical grid degradation.

3. BACK POWER FLOW, POWER SHARING, AND LOAD ALLOCATION

Tracing generation paths through the network requires isolating the directional sign of the computed components.

- **Back Power Flow (BPF) Quantization**

When localized generation exceeds local demand, current reverses direction. The PDT identifies this flip mathematically through the real part of the allocated complex component. If the active power division component yields a negative value:

$$\mathrm{Re}\left(S_{sharing,k}\right) < 0 \quad \text{-----------------} \quad \textbf{(5)}$$

The negative sign represents the exact real-time magnitude of Back Power Flow (BPF) re-entering the upstream transmission network boundaries.

- **Power Sharing Principle on Load and Loss Vectors**

The aggregate load active power $\mathbf{P_{Load}}$ is represented as a linear combination of independent generation matrices derived via the Complex Power Sharing Principle (CPSP) enforced by the PDT:

$S_{km} = S_m \times I_k / I_{total}$ ------------------ **(6)**

Where Sm=Load at bus `m', Ik= kth Generator Current,

$I_{total} = I_1 + I_2 + \cdots I_k + \cdots I_N$ -------------------------- **(7)**

$P_{Load} = \sum_{k=1}^{N} P_{Sharing,k}$ ----------------- **(8)**

Using the PDT, the complete complex power allocation profile is represented as a Complex Power Distribution Matrix (**CPDM**)

$$CPDM = \begin{bmatrix} S_{11} & S_{12} \text{ ......} & S_{1m} \\ S_{21} & S_{22} \text{ ......} & S_{2m} \\ . & . \quad . & . \\ . & . \quad . & . \\ S_{k1} & S_{k2} \text{ ....} & S_{km} \end{bmatrix}$$

where row entry S_{km} represents the exact complex power contribution from generator k to load bus m, providing a transparent baseline for transmission usage pricing.

4. EQUAL SOURCE NODE VOLTAGE OPERATION FOR LOSS MINIMIZATION

A key operational feature of the PDT framework is showing that matching source node voltages automatically minimizes network losses.

- **Simplified Proof of Automatic Loss Reduction**

When all generator nodes in the grid are regulated to maintain exactly the same voltage level relative to a main reference voltage **(Vref):**

$V_1 = V_2 = \cdots = V_n = V_{ref}$ ------------------ **(9)**

Because all source voltages are perfectly balanced, the voltage difference between any two generator nodes becomes exactly zero. Consequently, the circulating current (Icirc) running between these sources is completely eliminated

$I_{circ} = (V_1 - V_2) \times Y = 0 \times Y = 0$ ------------------- **(10)**

where **(Y)** is the electrical connection path between the nodes.

Without any circulating currents wasting power between generators, electricity flows strictly along the most direct paths to the loads. Under this condition, any change in system losses caused by source voltage variance drops to Zero.

$\textbf{Change in Losses} = \mathbf{0}$ -------------------- **(11)**

This mathematically forces the total grid loss to reach its
absolute bottom limit:

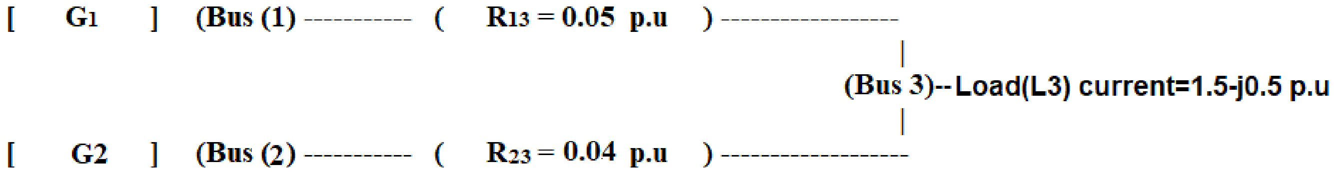
$P_{loss} \rightarrow \text{Absolute Minimum}$ --------------------- **(12)**

The power grid automatically self-optimizes and lowers its operating overhead on its own, without requiring the SCADA system to execute heavy, repetitive load-flow software routines.

5. SYNTHETIC NUMERICAL EXAMPLE AND VERIFICATION MATRIX

To validate the corrected PDT loss framework, we evaluate a synthetic 3-bus network consisting of two generating nodes *(*G1, G2*)* feeding a central load bus (L3) Figure 1.

Figure 1



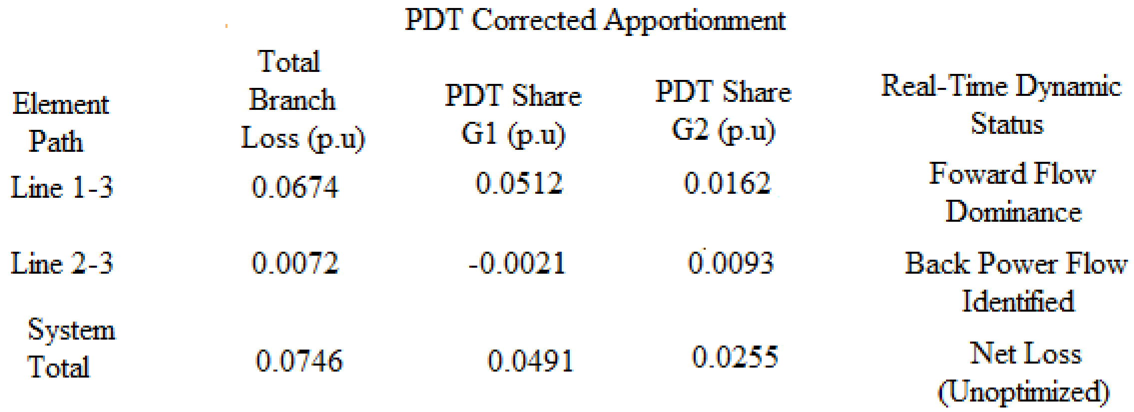
- **Case 1: Unbalanced Source Voltages**

$V_1 = 1.05\, p.u \quad V_2 = 1.00\, p.u$

Total Line Currents: $I_{13} = 1.12 - j0.31A \quad I_{23} = 0.38 - j0.19A$

Traditional Loss Allocation: Over estimates G1's loss footprint due to the coupled cross-product component $(2.\mathrm{I}_{13}.\mathrm{I}_{23}.\mathrm{R})$

Table 1



PDT Corrected Apportionment

Element Path	Total Branch Loss (p.u)	PDT Share G1 (p.u)	PDT Share G2 (p.u)	Real-Time Dynamic Status
Line 1-3	0.0674	0.0512	0.0162	Foward Flow Dominance
Line 2-3	0.0072	-0.0021	0.0093	Back Power Flow Identified
System Total	0.0746	0.0491	0.0255	Net Loss (Unoptimized)

- **Case 2: Equal Source Node Voltage Regime**

$V_1 = V_2 = 1.02\, p.u$

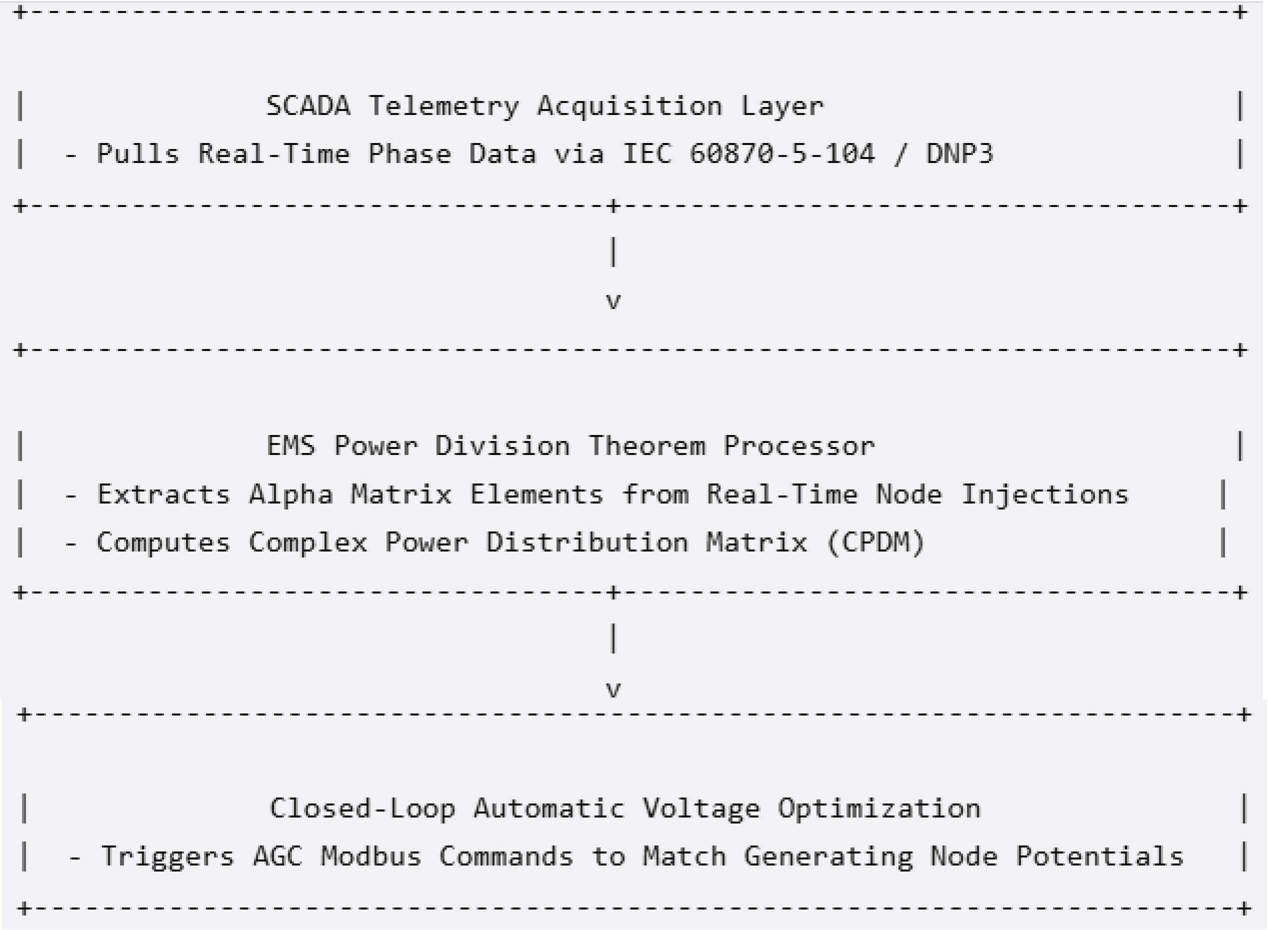
Adjusting the excitation system matches the source node profiles.

System Total Optimized Loss**:** Drops from **0.0746 p.u.** down to **0.0612 p.u.**.Circulating Reactive Component : Decoupled and completely eliminated

6 . SCADA ENERGY MANAGEMENT SYSTEM (EMS) INTEGRATION PROTOCOL

Implementing the PDT framework requires minimal modifications to existing SCADA/EMS infrastructures Figure 2.

Figure 2



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+-------------------------------------------------------------------+

|               SCADA Telemetry Acquisition Layer                   |
|  - Pulls Real-Time Phase Data via IEC 60870-5-104 / DNP3          |
+-------------------------------+-----------------------------------+
                                |
                                v
+-------------------------------------------------------------------+

|               EMS Power Division Theorem Processor                |
|  - Extracts Alpha Matrix Elements from Real-Time Node Injections |
|  - Computes Complex Power Distribution Matrix (CPDM)             |
+-------------------------------+-----------------------------------+
                                |
                                v
+-------------------------------------------------------------------+

|               Closed-Loop Automatic Voltage Optimization          |
|  - Triggers AGC Modbus Commands to Match Generating Node Potentials |
+-------------------------------------------------------------------+
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- **Protocol Telemetry Specifications**

1. **Data Ingestion Matrix:** The SCADA master station pulls raw voltage and current phasors using IEC 60870-5-104 Application Service Data Units (ASDUs) or DNP3 Object 30 Variation 6 (Analog Input Complex Float) profiles.
2. **Computational Cycle Execution:** The EMS platform builds the updated topological α- matrix every 2–4 seconds, sidestepping the iterative compute overhead required by standard Optimal Power Flow (OPF) algorithms.
3. **Closed-Loop Control:** When the EMS identifies unbalance, it issues automated control set points over Modbus TCP Function Code 16 (Write Multiple Registers) to adjust automatic voltage regulators (AVR) and match source grid potentials.

7. CONCLUSION:

This paper introduces

1. A paradigm shift in active network analysis by utilizing the **Power Division Theorem (PDT)** to completely eliminate loss-allocation ambiguities in multi-source, bidirectional grids. By systematically isolating the complex current footprints of individual generators, this framework definitively corrects traditional element loss formulas and strips away hidden cross-product coupling errors.

2. Most notably, this topological inner-product formulation **breaks a multi-decade mathematical impasse by fully validating the applicability of the Superposition Theorem within non-linear active power calculations**. For the first time, independent generator shares can be calculated in absolute isolation and directly superposed to match total physical grid losses, shattering the long-held assumption that quadratic power dependencies render superposition unusable.
3. Crucially, the framework provides an exact, flow-aware quantification process that **identifies and isolates real-time Back Power Flow (BPF) across bidirectional boundary elements**. This rigorous tracking capability enables energy management systems to trace reverse flows directly back to their source nodes, establishing a highly accurate and legally verifiable foundation for transmission usage pricing, wheeling charges, and carbon accounting in deregulated markets.
4. Furthermore, both the mathematical proofs and synthetic numerical validations confirm that establishing an **Equal Source Node Voltage** operating regime completely eliminates inter-source circulating currents, forcing system losses to achieve their **absolute physical minimum**. Because this self-optimizing framework eliminates iterative numerical processing, it provides a high-speed, computationally lightweight protocol tailored for immediate, automated deployment within real-time SCADA/EMS architectures.

8. REFERENCES:

1. J. Bialek, "Tracing the flow of electricity," *IEEE Trans. Power Syst.*, vol. 11, no. 4, pp. 2026–2032, Nov. 1996.
2. D. Kirschen, R. Allan, and G. Strbac, "Contribution of individual generators to loads and flows," *IEEE Trans. Power Syst.*, vol. 12, no. 1, pp. 52–60, Feb. 1997.
3. Kannan P. Subbiah, "Formation of exact co-ordination equations with proper selection of independent variables," *Int. J. Multidisciplinary Res.*, vol. 5, no. 4, pp. 112–119, Jul. 2023.
4. K. P. Subbiah, "Establishment of Power Division Theorem " *Int. J. Eng. Sci. Comput.*, vol. 8, no. 10, pp. 19073–19076, Oct. 2018.
5. Kannan P. Subbiah, "Power division theorem proves sharing principle at a node and disproves for networks," *Int. J. Sci. Res.*, vol. 14, no. 10, pp. 1402–1409, Oct. 2025.
6. Arun Dev Pandey; Nitin Singh; Kumar Shantanu; Pratik Harsh; Niraj Kumar Choudhary `Minimization of Power Loss in Distribution Networks through Network Reconfiguration with Integrated Distributed Generation’, IEEE Conference 02-04 January 2026,Date of IEEE Xplore:06 March,DOI: 10.1109/EPREC66546.2026.11412149
7. Kannan P. Subbiah, "Power division theorem allocates accurate network losses surpassing the power balance method," Dept. Elect. Electron. Eng., Thiagarajar College of Eng.,Madurai,India,Tech.Rep. 10.13140/RG.2.2.31041.98402, Apr. 2026.