Predictive Efficiency of the Estimators under Stochastic Linear Restrictions

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Abstract:
In this paper we have considered the usual linear regression model and the regression coefficients satisfying a set of stochastic restrictions. We have formulated the predictors based on ridge regression, Liu mixed regression estimator, mixed Liu and mixed ridge regression estimator. The properties of these estimators have been studied and the results are compared on the basis of their risk functions. Conditions for dominance of these predictors has been derived.

Keywords: Least squares estimator, ridge regression estimator, Liu mixed regression estimator, mixed Liu and Mixed ridge regression estimator.

1.1 Introduction

When we estimate unknown parameters in a linear regression model we often use ordinary least squares estimator. Recent studies, past experiences and pilot investigations often provide some useful, information in the form of estimates of coefficients in a linear regression model. Such a prior information can be incorporated into estimation procedure as a set of stochastic restrictions and the method of mixed regression can be applied. If the data is collinear then the problem of multicollinearity arises and remedial actions have been proposed to overcome it. A popular numerical technique to deal with it is the method of ridge regression, Sarkar(1992),Liu(1993), Kaciranlar et.al.,(1999), Akdeinz and Kaciranlar (2001), Jahufer A.and P.Wijekoon(2003)and Norio and Katsumi (2006), have proposed some new estimators in this context and most of the times attention has been confined to the prediction of average values within the sample, a natural question arises about their performance when the aim is to predict the actual values of the study variable or a situation may arise in which we require simultaneous prediction of average and actual values of the study variable. Appreciating the need of simultaneous prediction, we have used a framework that possesses sufficient flexibility and permits simultaneous prediction of actual and average values together.

In the present Paper a linear regression model alongwith a set of stochastic restrictions is considered. We have formulated five predictors based on mixed regression, ridge regression and Liu estimators and the predictors obtained by combining Liu and ridge regression estimators with mixed regression estimator. The properties of these predictors have been studied and results are presented in the form of theorems. Further relative performance of these predictors has been studied on the basis of the risk functions of these predictors when the aim is to predict average or actual values.

Section 1.2 describes model specifications and the estimators along with a target function. In Section 1.3 we have formulated predictors for the estimators described in the section1. 2 where as the properties of these predictors have been presented in the section 1.4. In section 1.5 a comparative study has been carried out by employing the risk criterion. In Section1.6, we derive the results of Section1.4.

1.2 Model Specification and Estimators

Let us consider the following linear regression model

$$Y = X\beta + \sigma u . \qquad (1.2.1)$$

where Y is a (n×1) vector of observations on the variables to be explained, X is a $(n \times p)$ full column rank matrix of n observations on p explanatory variables, β is a $(p \times 1)$ vector of regression coefficients, σ being unknown scalar and u is a $(n \times 1)$ vector of disturbances with mean zero, variance unity and measures of skewness and kurtosis are γ_1 and (γ_2+3) respectively.

If b denotes the estimate of β, then the predictor for values of the study variable within the sample is generally formulated as $\hat{T} = Xb$ which is used for predicting either the actual value of Y or the average value $E[Y] = X\beta$ at a time.

When the situation demands prediction for both the average and actual values together, we may define the following target function

$$T(Y) = T = \alpha Y + (1-\alpha)E(Y). \qquad (1.2.2)$$

and use $\hat{T} = X\beta$ for predicting it, where $0 \le \alpha \le 1$ is a non stochastic scal1ar specifying the weightage to be assigned to the prediction of actual and average values of the study variable.

If the restrictions are stochastic in nature that is

$$q = Q\beta + V \ . \qquad (1.2.3)$$

where q is a (J×1) vector of known elements and Q is a $(J \times p)$ matrix of known elements with full row rank and V is a vector of errors such that

$$E[V] = 0, \qquad (1.2.4)$$

and

$$E[VV'] = \Omega \ . \qquad (1.2.5)$$

The ordinary least squares estimator of β is given by

$$b_0 = (X'X)^{-1}X'Y . \qquad (1.2.6)$$

After combing (1.2.1) and (1.2.3) we write

$$\begin{matrix} Y \\ q \end{matrix} = \begin{matrix} X \\ Q \end{matrix}\left[\beta\right] + \sigma\left[\begin{matrix} u \\ V \end{matrix}\right]. \qquad (1.2.7)$$

assuming u and V to be independent i.,e.

$$E[uV'] = 0 \ . \qquad (1.2.8)$$

Theil and Goldberger (1961) and Theil (1963) proposed the following mixed regression estimator of β that accounts the prior restrictions defined in (1.2.3)

$b_M = \left[X'X + S^2 Q'\Omega^{-1}Q\right]^{-1}\left[X'Y + S^2 Q'\Omega^{-1}q\right].$ (1.2.9) where

$$S^2 = \frac{1}{n-p}\{(Y - Xb_0)'(Y - Xb_0)\}. \quad (1.2.10)$$

Here S^2 is an unbiased estimate of σ^2

The estimator of β which is due to Liu (1993) can be written as

$$b_L = F_L b_0. \quad (1.2.11)$$

where,

$$F_L = (X'X + I)^{-1}(X'X + LI), \quad (1.2.12)$$

and $0 < L < 1.$

The ridge regression estimator due to Horel and Kennard (1970a, b) is given by

$b(k) = Z^{-1}X'Y = (I - kZ^{-1})b_0.$ (1.2.13) where ,

$Z = (X'X + kI)$,

and k is non-negative scalar characterizing the estimator.

On replacing b_o by b_M in Liu estimator defined in (1.2.11) we obtain mixed Liu type estimator as given under

$$b_{ML} = F_L b_M. \quad (1.2.14)$$

On replacing b_o by b_M in ridge regression estimator defined in (1.2.13) we obtain mixed ridge regression estimator

$$b_M(k) = (I - kZ^{-1})b_M. \quad (1.2.15)$$

1.3 Predictions

Employing (1.2.2), (1.2.6), (1.2.9), (1.2.11), (1.213), (1.2.15) and (1.2.16) yields the following predictors for the values of study variables,

$$\hat{T}_L = Xb_L, \quad (1.3.1)$$

$$\hat{T}(k) = Xb(k), \quad (1.3.2)$$

$$\hat{T}_M = Xb_M, \quad (1.3.3)$$

$$\hat{T}_{ML} = Xb_{ML}, \quad (1.3.4)$$

$$\hat{T}_M(k) = Xb_M(k), \quad (1.3.5)$$

In order to derive the expression for bias, mean squared error matrix and predictive risk of $\hat{T}_M$, $T_M(k)$, $\hat{T}_{ML}$ we employ the small disturbances asymptotic theory introduced by Kadane (1971).

1.4 Properties of the Predictors

The predictor $\hat{T}(k)$ defined in (1.3.2) is a biased predictor with bias vector given by

$$B[\hat{T}(k)] = -kXZ^{-1}\beta. \quad (1.4.1)$$

Mean squared error matrix is

$$M[\hat{T}(k)] = k^2 XZ^{-1}\beta\beta' Z^{-1}X' + \sigma^2[\alpha^2 I_n - 2\alpha\ XZ^{-1}X' + XZ^{-1}X'XZ^{-1}X']. \qquad (1.4.2)$$

and risk function is

$$\text{Risk}[\hat{T}(k)] = k^2\beta' Z^{-1}X'XZ^{-1}\beta + Risk(\hat{T}_0) + \sigma^2 k \left\{ \frac{\alpha+2}{\sum_{i=1}^{p}\lambda_i} - \frac{k}{\sum_{i=1}^{p}\lambda_i^2} \right\}^2. \qquad (1.4.3)$$

The predictor based on Liu estimator defined (1.3.1) is also biased with bias vector given by

$$B[\hat{T}_L] = -X(I - F_L)\beta. \qquad (1.4.4)$$

Mean squared error matrix is

$$M[\hat{T}_L] = X(I - F_L)\beta\beta'(I - F_L)'X'$$
$$+\sigma^2[\alpha^2 I_n - \alpha\{XF_L(XX)^{-1}X' + X(XX)^{-1}.$$
$$F_L'X' + XF_L(XX)^{-1}F_L'X']. \qquad (1.4.5)$$

and risk function given by

$$\text{Risk}[\hat{T}_L] = \beta'(I - F_L')X'X(I - F_L)\beta + Risk(\hat{T}_0)$$
$$-2\sigma^2(1-\alpha)\ (1-\text{L}) \left(\frac{1}{\sum_{i=1}^{p}\lambda_i} - \left\{ \frac{1}{\sum_{i=1}^{p}\lambda_i^2} \right\} - L \left\{ \frac{1}{\sum_{i=1}^{p}\lambda_i^3} \right\} \right\}$$
$$-\frac{(1-L)^2}{2}\left\{ \frac{1}{\sum_{i=1}^{p}\lambda_i^2} - 2\frac{1}{\sum_{i=1}^{p}\lambda_i^3} \right\}\Bigg]. \qquad (1.4.6)$$

where λ_i's ; $i = 1,2,\ldots,p$ are eigen values of the matrix $(X'X)$

Theorem 1.1: When disturbances in the model (1.2.1) are normally distributed, the predictive bias vector, predictive mean squared error matrix and predictive risks associated with the predictor $\hat{T}_M$ defined in (1.3.1) up to order $0(\sigma^4)$ of approximations are given by

$$B[\hat{T}_M] = -\sigma^3 \frac{\gamma_1}{n-p} X(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'(I_n * M)e. \qquad (1.4.7)$$

$$M[\hat{T}_M] = M(\hat{T}_0) - \sigma^4\left[1 - 2\left\{\alpha + \frac{1}{n-p}\right\}\right] X(X'X)^{-1}Q'\Omega^{-1}Q\,(X'X)^{-1}X'. \qquad (1.4.8)$$

and

$$Risk[\hat{T}_M] = Risk[\hat{T}_0] - \sigma^4\left[1 - 2\left\{\alpha + \frac{1}{n-p}\right\}\right]\frac{\sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}\lambda_i}. \qquad (1.4.9)$$

where τ_j^* ; j=1,2,3…….J are the eigen values of the matrix $Q'\Omega^{-1}Q$

Theorem 1.2: When the disturbances in the model (1.2.1) are normally distributed, the predictive bias, mean squared error matrix and risk associated with the predictor $\hat{T}_M(k)$ up to order $o(\sigma^4)$ of approximations is given by

$$B\left[\hat{T}_M(k)\right] = -kXZ^{-1}\beta \quad . \qquad (1.4.10)$$

$$M\left[\hat{T}_M(k)\right] = M\left[\hat{T}(k)\right] - \sigma^4\left(1 - 2/_{n-p}\right) XZ^{-1}Q'\Omega^{-1}QZ^{-1}X'$$
$$-\alpha\{X(X'X)^{-1}X'Q'\Omega^{-1}QW^{-1}X' + XZ^{-1}Q'\Omega^{-1}Q(X'X)^{-1}\}. \quad (1.4.11)$$

$$Risk\left[\hat{T}_M(k)\right] = Risk\left[\hat{T}(k)\right] - \sigma^4\left(\left(1 - 2/_{n-p}\right)\left(\frac{\sum_{i=1}^{p}\lambda_i \sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}(\lambda_i + k)^2}\right) - 2\alpha\left(\frac{\sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}(\lambda_i + k)^2}\right)\right)$$

(1.4.12)

Theorem 1.3: When the disturbances in the model (1.2.1) are normally distributed, the predictive bias, mean squared error matrix and risk associated with the predictor $\hat{T}_{ML}$ up to order $o(\sigma^4)$ of approximations is given by

$$B[\hat{T}_{ML}] = X(I - F_L)\beta - \sigma^3 \frac{\gamma_1}{n-p} XF_L(X'X)^{-1}Q'\Omega^{-1}Q\,(X'X)^{-1}X'(I_n * M)e. \qquad (1.4.13)$$

$$M[\hat{T}_{ML}] = M[\hat{T}_M] + M(\hat{T}_L) - V(\hat{T}_0) - \sigma^4\left\{2\left(\alpha + \frac{1}{n-p}\right) - 1\right\}X.$$
$$(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X' + XF_L(X'X)^{-1}Q'\Omega^{-1}Q.\,(X'X)^{-1}X'$$
$$-X(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}F_L'X' - \alpha\{XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'$$
$$+X(X'X)^{-1}\,Q'\Omega^{-1}Q(X'X)^{-1}F_L'X'\} + \frac{2}{n-p}\ \{(X'X)^{-1}F_L'\,Q'\Omega^{-1}Q$$
$$(X'X)^{-1}X' - X(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}F_L'X'\}$$
$$+\left(\frac{n-p-2}{n-p}\right)XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}F_L'X'. \qquad (1.4.14)$$

$$Risk[\hat{T}_{ML}] = Risk(\hat{T}_M) + Risk(\hat{T}_L) - Risk(\hat{T}_0)$$
$$-\sigma^4\left\{2\left(\alpha + \frac{1}{n-p}\right) - 1\right\}\frac{\sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}\lambda_i} - 2\alpha\left[\frac{\sum_j \tau_j^* \sum_{i=1}^{p}(\lambda_i + L)}{\sum_{i=1}^{p}\lambda_i \sum_{i=1}^{p}(\lambda_i + 1)}\right]$$
$$+\left\{\frac{n-p-2}{n-p}\right\}\frac{\sum_j \tau_j^* \sum_{i=1}^{p}(\lambda_i + L)^2}{\sum_{i=1}^{p}\lambda_i \sum_{i=1}^{p}(\lambda_i + 1)^2}. \qquad (1.4.15)$$

1.5 A Comparison.

1. Comparison between predictor ridge regression and predictor Liu estimator

On comparing the risk associated with the predictor ridge regression ($\hat{T}(k)$) defined in (1.4.3) and the predictor based on Liu estimator $\hat{T}_L$ defined in (1.4.6), we see that

$$Risk[\hat{T}(k)] - Risk[\hat{T}_L] = k^2\beta' W^{-1}X'XW^{-1}\beta - \beta'(I-F_L)'X'X(I-F_L)\beta$$

$$+\sigma^2\left\{k\left\{\frac{\alpha+2}{\sum_{i=1}^{p}\lambda_i}-\frac{k}{\sum_{i=1}^{p}\lambda_i^2}\right\}^2+2(1-\alpha)(1-L)\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i}-\frac{1}{\sum_{i=1}^{p}\lambda_i^2}\right\}-\left(\frac{L}{1-L}\right)\frac{1}{\sum_{i=1}^{p}\lambda_i^3}\right.$$

$$\left.-\left(\frac{1-L}{1-\alpha}\right)\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i^2}-\frac{2}{\sum_{i=1}^{p}\lambda_i^3}\right\}\right\} \qquad (1.5.1)$$

On observing the above expression and considering the terms containing σ^2 we find that the predictor $\hat{T}_L$ dominates the predictor $\hat{T}(k)$ with respect to risk criterion, if we choose k to satisfy

$$k > \left[\frac{\beta'(I-F_L)'X'X(I-F_L)\beta}{\beta'X'X\beta}\right]^{1/2}. \qquad (1.5.2)$$

By considering the terms containing in the order σ^2 only, we find that $\hat{T}_L$ dominates $\hat{T}(k)$ with respect to risk criterion if the upper bound of k is

$$k > \frac{2\sum_{i=1}^{p}\lambda_i(1-\alpha)(1-L)}{(\alpha+2)}\left(\frac{L}{1-L}\right)\left\{\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i^3}\right\}+\left(\frac{1-L}{1-\alpha}\right)\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i^2}-\frac{1}{\sum_{i=1}^{p}\lambda_i^3}\right\}\right.$$

$$\left.-\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i}-\frac{1}{\sum_{i=1}^{p}\lambda_i^2}\right\}\right\}. \qquad (1.5.3)$$

and if the whole of the expression is considered, then again $\hat{T}_L$ dominates $\hat{T}(k)$ with respect to risk criterion so long as k satisfies

$$k > \frac{\sum_{i=1}^{p}\lambda_i}{(\alpha+2)}\frac{\beta'(I-F_L)'X'X(I-F_L)\beta}{\sigma^2}+2(1-\alpha)(1-L).$$

$$\left(\frac{L}{1-L}\right)\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i^3}\right\}\left\{+\left(\frac{1-L}{1-\alpha}\right)\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i^2}-\frac{1}{\sum_{i=1}^{p}\lambda_i^3}\right\}-\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i}-\frac{1}{\sum_{i=1}^{p}\lambda_i^2}\right\}.\right\} \qquad (1.5.4)$$

2. Comparison between predictor ridge regression and mixed regression.

On comparing the risk associated with the predictor $\hat{T}(k)$ and the predictor mixed regression ($\hat{T}_M$) defined in (1.4.9), we find that

$$Risk[\hat{T}(k)]\text{-}Risk[\hat{T}_M]=k^2\beta' Z^{-1}X'XZ^{-1}\beta+\sigma^2 k\left\{\frac{\alpha+2}{\sum_{i=1}^{p}\lambda_i}-\frac{k}{\sum_{i=1}^{p}\lambda_i^2}\right\}^2$$

$$+\sigma^4\left[1-2\left(\alpha+\frac{1}{n-p}\right)\right]\frac{\sum_j \tau_j^*}{\sum_{i=1}^{p}\lambda_i}. \qquad (1.5.5)$$

On observing the above expression, we find that the predictor $\hat{T}_M$ has smaller risk than the predictor $\hat{T}(k)$ so long as k satisfies

$$0<k<\left[\frac{2(\alpha+2)}{\sum_{i=1}^{p}\lambda_i\sum_{i=1}^{p}\lambda_i^2}-\frac{k^2\beta' W^{-1}X'XW^{-1}\beta}{\sigma^2}\right]^{-1}\left[\frac{(\alpha+2)}{\sum_{i=1}^{p}\lambda_i}\right]^2. \qquad (1.5.6)$$

3. Comparison between predictor ridge regression and predictor mixed ridge.

On comparing the risk associated with the predictor $\hat{T}(k)$ and the risk associated with the predictor mixed ridge ($\hat{T}_M(k)$) defined in (1.4.12), we find that

$$Risk(\hat{T}(k))\text{-}Risk(\hat{T}_M(k))=\sigma^4\left(\left(1-\frac{2}{n-p}\right)\frac{\sum_{i=1}^{p}\lambda_i\sum_{j=1}^{J}\tau_j}{\sum_{i=1}^{p}(\lambda_i+k)^2}-2\alpha\frac{\sum_{j=1}^{J}\tau_j}{\sum_{i=1}^{p}(\lambda_i+k)}\right). \qquad (1.5.7)$$

Thus from above expression we observe that the predictor $\hat{T}_M(k)$ dominates the predictor $\hat{T}(k)$, so long as $(n-p)>2$.

4. Comparison between predictor mixed regression and predictor mixed Liu.

Further, on comparing the risk associated with the predictor $\hat{T}_M$ and the risk associated with predictor mixed Liu ($\hat{T}_{ML}$) defined in (1.4.15), we find that

$$Risk[\hat{T}_M]\text{-}Risk[\hat{T}_{ML}] = -\beta'(I-F_L)'X'X(I-F_L)\beta + 2\sigma^2.$$

$$\left((1-\alpha)\{(1-L)\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i} - \frac{1}{\sum_{i=1}^{p}\lambda_i^2}\right\} - \frac{L}{\sum_{i=1}^{p}\lambda_i^3} - \frac{(1-L)^2}{2}\left\{\frac{1}{\sum_{i=1}^{p}\lambda_i^2} - \frac{2}{\sum_{i=1}^{p}\lambda_i^3}\right\}\right) + \sigma^4$$

$$2\left(\alpha + \frac{1}{n-p}\right)\left\{-1\frac{\sum\tau_j^*}{\sum_{i=1}^{p}\lambda_i}\right\} - 2\alpha.\frac{\sum_j\tau_j^*\sum_{i=1}^{p}(\lambda_i+L)}{\sum_{i=1}^{p}\lambda_i\sum_{i=1}^{p}(\lambda_i+1)}$$

$$+\left\{\frac{n-p-2}{n-p}\right\}\frac{\sum_j\tau_j^*\sum_{i=1}^{p}(\lambda_i+L)^2}{\sum_{i=1}^{p}\lambda_i\sum_{i=1}^{p}(\lambda_i+1)^2}. \qquad (1.5.8)$$

5. Comparison between predictor Liu and predictor mixed Liu estimator.

On comparing the risk associated with the predictor $\hat{T}_L$ and the risk associated with the predictor $\hat{T}_{ML}$, we find that

$$Risk[\hat{T}_L]\text{-}Risk[\hat{T}_{ML}] = -\sigma^4\left(2\alpha.\frac{\sum_j\tau_j^*\sum_{i=1}^{p}(\lambda_i+L)}{\sum_{i=1}^{p}\lambda_i\sum_{i=1}^{p}(\lambda_i+1)} - \left\{\frac{n-p-2}{n-p}\right\}\frac{\sum_j\tau_j^*\sum_{i=1}^{p}(\lambda_i+L)^2}{\sum_{i=1}^{p}\lambda_i\sum_{i=1}^{p}(\lambda_i+1)^2}\right)$$

(1.5.9)

The negativity of the above expression reveals that the predictor $\hat{T}_L$ has smaller risk than the predictor $\hat{T}_{ML}$, but when $\alpha = 0$ i.e., when we have to predict the average values of the study variable, then just reverse of the above stated condition holds so long as $(n-p) > 2$.

1.6 Proof of the Theorems.

Theorem 1.1:

From (1.2.9) we have

$$b_M = \left[X'X + S^2Q'\Omega^{-1}Q\right]^{-1}\left[X'Y + S^2Q'\Omega^{-1}q\right]$$

$$= \beta + \left[X'X + \sigma^2\frac{u'Mu}{(n-p)}Q'\Omega^{-1}Q\right]^{-1}$$

$$\left[\sigma X'u + \sigma^2\frac{u'Mu}{(n-p)}Q'\Omega^{-1}V\right],$$

International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

$$= \beta + \sigma (X'X)^{-1} X'u + \sigma^2 \frac{u'Mu}{n-p} (X'X)^{-1} Q'\Omega^{-1}V \qquad - \sigma^3 \frac{u'Mu}{n-p} (X'X)^{-1} Q'\Omega^{-1} Q (X'X)^{-1}$$

$$- \sigma^4 \left(\frac{u'Mu}{n-p} \right)^2 (X'X)^{-1} \qquad Q'\Omega^{-1} Q (X'X)^{-1} Q'\Omega^{-1}V \,.$$

Utilizing (1.3.1) in the above expression we obtain,

$$\hat{T}_M = X\beta + \sigma X (X'X)^{-1} X'u + \sigma^2 \frac{u'Mu}{(n-p)} X (X'X)^{-1}$$

$$Q'\Omega^{-1}V - \sigma^3 \frac{u'Mu}{(n-p)} X (X'X)^{-1} Q'\Omega^{-1} Q (X'X)^{-1}$$

$$+ \sigma^4 \left(\frac{u'Mu}{n-p} \right)^2 X (X'X)^{-1} Q'\Omega^{-1} Q (X'X)^{-1} Q'\Omega^{-1}V \,. \qquad (1.6.1)$$

Utilizing (1.2.2) in (1.6.1) we obtain

$$\hat{T}_M - T = \sigma\delta_1 + \sigma^2\delta_2 + \sigma^3\delta_3 + \sigma^4\delta_4 \,. \qquad (1.6.2)$$

where,

$$\delta_1 = [X (X'X)^{-1} X' - \alpha I] u \,, \qquad (1.6.3)$$

$$\delta_2 = \frac{u'Mu}{n-p} X (X'X)^{-1} Q'\Omega^{-1}V \,, \qquad (1.6.4)$$

$$\delta_3 = -\frac{u'Mu}{(n-p)} X (X'X)^{-1} Q'\Omega^{-1} Q (X'X)^{-1} u \,, \qquad (1.5.5)$$

$$\delta_4 = \left(\frac{u'Mu}{n-p} \right)^2 X (X'X)^{-1} Q'\Omega^{-1} Q (X'X)^{-1} Q'\Omega^{-1}V \,, \qquad (1.6.6)$$

It can be easily proved that

$$E[\delta_1] = E[\delta_2] = 0 \,. \qquad (1.6.7)$$

$$E[\delta_3] = -\frac{\gamma_1}{(n-p)} X (X'X)^{-1} Q'\Omega^{-1} Q (X'X)^{-1} (I_n * M) e \,. \qquad (1.6.8)$$

$$E[\delta_4] = 0. \qquad (1.6.9)$$

using (1.6.7),(1.6.8),(1.6.9) in (1.6.2) we get the result (1.4.7) of the Theorem 1.1.

Again,

$$[\hat{T}_M - T][\hat{T}_M - T]' = \sigma^2 [\delta_1\delta_1'] + \sigma^3 [\delta_1\delta_2' + \delta_2\delta_1']$$

$$+ \sigma^4 [\delta_1\delta_3' + \delta_2'\delta_2 + \delta_3\delta_1'] \,. \qquad (1.6.10)$$

where ,

$$\delta_1\delta_1' = [X (X'X)^{-1} X' - \alpha I] u \, u' [X (X'X)^{-1} X' - \alpha I] \,, \qquad (1.6.11) \quad \delta_1\delta_2' = \frac{u'Mu}{n-p}$$

$$[X (X'X)^{-1} X' - \alpha I] \, X (X'X)^{-1} Q'\Omega^{-1}V \,, \qquad (1.6.12)$$

$$\delta_3\delta_1' = -\frac{u'Mu}{(n-p)} X(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}u\ (X'X)^{-1}X' - \alpha I], \qquad (1.6.13)$$

$$\delta_2\delta_2' = \left(\frac{u'Mu}{n-p}\right)^2 X(X'X)^{-1}Q'\Omega^{-1}V\ V'\Omega^{-1}Q\ (X'X)^{-1}X', \qquad (1.6.14)$$

Here it is easy to verify that for normally distributed disturbances

$$E[\delta_1\delta_1'] = [\alpha^2 I_n + X(X'X)^{-1}X'(1-2\alpha)]. \qquad (1.6.15)$$

$$E[\delta_1\delta_2'] = 0. \qquad (1.6.16)$$

$$E[\delta_1\delta_3'] = [X(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1} - \alpha X(X'X)^{-1}.$$
$$Q'\Omega^{-1}Q(X'X)^{-1}]. \qquad (1.6.17)$$

$$E[\delta_2\delta_2'] = \left(\frac{(n-p+2)}{n-p}\right) X(X'X)^{-1}Q'\Omega^{-1}Q\ (X'X)^{-1}X'. \qquad (1.6.18)$$

Utilizing the results (1.6.16), (1.6.17), (1.6.18), (1.6.19) in (1.6.10) obtain the result (1.4.8) of the Theorem 1.1.

Again,

$$[\hat{T}_M - T]'[\hat{T}_M - T] = \sigma^2[\delta_1'\delta_1] + \sigma^3[\delta_1'\delta_2 + \delta_2'\delta_1]$$
$$+ \sigma^4[\delta_1'\delta_3 + \delta_2'\delta_2 + \delta_3'\delta_1]. \qquad (1.6.19)$$

where,

$$\delta_1'\delta_1 = u'[X(X'X)^{-1}X' - \alpha I][X(X'X)^{-1}X' - \alpha I]u. \qquad (1.6.20)$$

$$\delta_2'\delta_1 = \frac{u'Mu}{n-p} V'\Omega^{-1}Q\ (X'X)^{-1}X'[X(X'X)^{-1}X' - \alpha I]u. \qquad (1.6.21)$$

$$\delta_1'\delta_3 = -\frac{u'Mu}{(n-p)} u'[X(X'X)^{-1}X' - \alpha I]\ X(X'X)^{-1}Q'\Omega^{-1}Q\ (X'X)^{-1}u. \qquad (1.6.22)$$

$$\delta_2'\delta_2 = \left(\frac{u'Mu}{n-p}\right)^2 V'\Omega^{-1}Q\ (X'X)^{-1}\ Q'\Omega^{-1}V. \qquad (1.6.23)$$

It can be verified that

$$E[\delta_1'\delta_1] = \sigma^2[n\alpha^2(1-2\alpha)p]. \qquad (1.6.24)$$

$$E[\delta_2'\delta_1] = 0. \qquad (1.6.25)$$

$$E[\delta_1'\delta_3] = [1-\alpha]\frac{\sum_j \tau_j^*}{\sum_{i=1}^{p}\lambda_i}. \qquad (1.6.26)$$

$$E[\delta_2'\delta_2] = \left(\frac{(n-p+2)}{n-p}\right)\frac{\sum_j \tau_j^*}{\sum_{i=1}^{p}\lambda_i}. \qquad (1.6.27)$$

By using (1.6.24), (1.6.25), (1.6.26) and (1.6.27) in (1.6.19) we obtain the result (1.4.9) of the Theorem 1.1.

Theorem 1.2:
From (1.2.1), (1.2.2), (1.2.3), (1.2.6), (1.2.9), (1.2.13) and (1.3.5)
We have,

$$\hat{T}_M(k) = Xb_M(k) = X\left[I - kZ^{-1}\right]b_M .$$

$$= X\beta - kXZ^{-1}\beta + \sigma XZ^{-1}X'u + \sigma^2 \frac{u'Mu}{n-p} XZ^{-1}Q'\Omega^{-1}V$$

$$-\sigma^3 \frac{u'Mu}{n-p} XZ^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u .$$

Now $T(Y) = T = \alpha Y + (1-\alpha)E(Y)$.

and $(\hat{T}_M - T) = -kXZ^{-1}\beta + \sigma(XZ^{-1}X' - \alpha I_n)U + \sigma^2 \frac{u'Mu}{n-p} XZ^{-1}Q'\Omega^{-1}V$

$$-\sigma^3 \frac{u'Mu}{n-p} XZ^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u ,$$

$$\hat{T}_M(k) - T = \phi_o + \sigma\phi_1 + \sigma^2\phi_2 + \sigma^3\phi_3 . \tag{1.6.28}$$

where ,

$\phi_0 = -kXZ^{-1}\beta$, (1.6.29) $\phi_1 = -k(XZ^{-1}X - \alpha I_{n)})u$, (1.6.30)

$\phi_2 = \left(\frac{u'Mu}{n-p}\right) X'Z^{-1}Q'\Omega^{-1}V$, (1.6.31) $\phi_3 = -\left(\frac{u'Mu}{n-p}\right) X'Z^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u$, (1.6.32)

Here it is easy to see that for normally distributed disturbances

$$E[\phi_0] = -kXZ^{-1}\beta . \tag{1.6.33}$$

$$E[\phi_1] = E[\phi_2] = E[\phi_3] = 0 . \tag{1.6.34}$$

Utilizing the expression (1.6.33) and (1.6.34) in (1.6.28) yields the result (1.4.10) of the Theorem 1.2.
Now ,

$$\left[\hat{T}_M(k) - T\right]\left[\hat{T}_M(k) - T\right]' = \phi_0\phi_0' + \sigma\left[\phi_o\phi_1' + \phi_1\phi_0'\right] + \sigma^2[\phi_1\phi_1']$$
$$+\sigma^3\left[\phi_3\phi'_0 + \phi_2\phi'_1 + \phi_2\phi_1' + \phi_0\phi_3'\right] + \sigma^4\left[\phi_1\phi_3' + \phi_2\phi_2' + \phi_3\phi_1'\right]$$
$$+\sigma^4\left[\phi_1\phi_3' + \phi_2\phi_2' + \phi_3\phi_1'\right] . \tag{1.6.35}$$

where,

$$\phi_0\phi_0' = k^2 XZ^{-1}\beta\beta'Z^{-1}X' , \tag{1.6.36}$$

$$\phi_o\phi_1' = -kXZ^{-1}\beta . u'(XZ^{-1}X' - \alpha I_n) , \tag{1.6.37}$$

$$\phi_1\phi_1' = (XZ^{-1}X' - \alpha I_n)uu'(XZ^{-1}X' - \alpha I_n) , \tag{1.6.38}$$

$$\phi_2\phi'_1 = \left(\frac{u'Mu}{n-p}\right) X'Z^{-1}Q'\Omega^{-1}V\, u'(XZ^{-1}X' - \alpha I_n) , \tag{1.6.39}$$

$$\phi_3\phi'_0 = k\left(\frac{u'Mu}{n-p}\right) X'Z^{-1}Q'\Omega^{-1}Q(X'X)^{-1}u\ \beta'Z^{-1}X', \quad (1.6.40)$$

$$\phi_3\phi'_1 = -\left(\frac{u'Mu}{n-p}\right) X'Z^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u\ u'(XZ^{-1}X'-\alpha I_n), \quad (1.6.41)$$

$$\phi_2\phi'_2 = \left(\frac{u'Mu}{n-p}\right)^2 X'Z^{-1}Q'\Omega^{-1}VV'\Omega^{-1}QZ^{-1}X, \quad (1.6.42)$$

It is easy to verify that

$$E[\phi_0\phi'_1] = k^2XZ^{-1}\beta\beta'Z^{-1}X'. \quad (1.6.43)$$

$$E[\phi_0\phi'_1] = E[\phi_0\phi'_2] = E[\phi_0\phi'_3] = E[\phi_1\phi'_2] = 0. \quad (1.6.44)$$

$$E[\phi_1\phi'_1] = \{\alpha^2 I_n - 2\alpha XZ^{-1}X' + XZ^{-1}X'XZ^{-1}X'\}. \quad (1.6.45)$$

$$E[\phi_1\phi'_3] = -\{XZ^{-1}Q'\Omega^{-1}QZ^{-1}X' + \alpha X(X'X)^{-1}Q'\Omega^{-1}QZ^{-1}X'\}. \quad (1.5.46)$$

$$E[\phi_2\phi'_2] = \left(\frac{n-p+2}{n-p}\right) XZ^{-1}Q'\Omega^{-1}QZ^{-1}X'. \quad (1.6.47)$$

Utilizing expressions (1.6.43), (1.6.44), (1.6.45), (1.6.46) and (1.6.47) in (1.6.35) yields theresult (1.4.11) of the Theorem 1.2.

$$[T_M(k)-T]'[T_M(k)-T] = \phi'_0\phi_0 + \sigma[\phi'_0\phi_1 + \phi'_1\phi_o] + \sigma^2[\phi_1'\phi_1]$$
$$+\sigma^3[\phi'_0\phi_3 + \phi'_1\phi_2 + \phi'_2\phi_1 + \phi'_3\phi_0]$$
$$+\sigma^4[\phi'_1\phi_3 + \phi'_2\phi_2 + \phi'_3\phi_1]. \quad (1.6.48)$$

where ,

$$\phi'_0\phi_0 = k^2\beta'Z^{-1}X'XZ^{-1}\beta, \quad (1.6.49)$$

$$\phi'_0\phi_1 = -k\beta'Z^{-1}X'(XZ^{-1}X'-\alpha I_n)u, \quad (1.6.50)$$

$$\phi_1'\phi_1 = u'(XZ^{-1}X'-\alpha I_n)(XZ^{-1}X'-\alpha I_n)u, \quad (1.6.51)$$

$$\phi'_3\phi_1 = -\left(\frac{u'Mu}{n-p}\right) u'X(X'X)^{-1}Q'\Omega^{-1}QZ^{-1}X\ (XZ^{-1}X'-\alpha I_n)u, \quad (1.6.52)$$

$$\phi'_2\phi_2 = \left(\frac{u'Mu}{n-p}\right)^2 V'\Omega^{-1}QZ^{-1}XX'Z^{-1}Q'\Omega^{-1}V, \quad (1.6.53)$$

It can be easily verified that

$$E[\phi'_0\phi_0] = k^2\beta'Z^{-1}X'XZ^{-1}\beta. \quad (1.6.54)$$

$$E[\phi'_0\phi_1] = 0. \quad (1.6.55)$$

$$E[\phi_1'\phi_1] = n\alpha^2 - 2\alpha \left\{ \frac{\sum_{i=1}^{p}\lambda_i}{\sum_{i=1}^{p}(\lambda_i+k)} + \frac{\sum_{i=1}^{p}\lambda_i^2}{\sum_{i=1}^{p}(\lambda_i+k)^2} \right\}. \quad (1.6.56)$$

$$E[\phi'_3\phi_1]=\left(\frac{\sum_{i=1}^{p}\lambda_i\sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}(\lambda_i+k)^2}\right)-\alpha\left(\frac{\sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}(\lambda_i+k)}\right). \qquad (.6.57)$$

$$E[\phi_2'\phi_2]=\left(\frac{n-p+2}{n-p}\right)\left(\frac{\sum_{i=1}^{p}\lambda_i\sum_{j=1}^{J}\tau_j^*}{\sum_{i=1}^{p}(\lambda_i+k)^2}\right). \qquad (1.6.58)$$

Using (1.6.54), (1.6.55), (1.6.56), (1.6.57) and (1.6.58) in (1.6.48) we obtain the result (1.4.12) of the Theorem 1.2.

Theorem 1.3:

From (1.2.1), (1.2.2) and (1.3.4) we can write

$$(\hat{T}_{ML}-T)=\phi_0^*+\sigma\,\phi_1^*+\sigma^2\,\phi_2^*+\sigma^3\phi_3^*+\sigma^4\phi_4^*. \qquad (1.6.59)$$

where,

$$\phi_0^*=-X(I-F_L)\beta, \qquad (1.6.60)$$

$$\phi_1^*=(XF_L(X'X)^{-1}X'-\alpha I_n)u, \qquad (1.6.61)$$

$$\phi_2^*=\left(\frac{u'Mu}{n-p}\right)XF_L(X'X)^{-1}Q'\Omega^{-1}V, \qquad (1.6.62)$$

$$\phi_3^*=-\left(\frac{u'Mu}{n-p}\right)XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u, \qquad (1.6.63)$$

$$\phi_4^*=-\left(\frac{u'Mu}{n-p}\right)^2XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}Q'\Omega^{-1}V, \qquad (1.6.64)$$

Here it is easy to verify that

$$\mathrm{E}[\phi_0^*]=-X(I-F_L)\beta. \qquad (1.6.65)$$

$$\mathrm{E}[\phi_1^*]=\mathrm{E}[\phi_2^*]=\mathrm{E}[\phi_3^*]=\mathrm{E}[\phi_4^*]=0. \qquad (1.6.66)$$

Utilizing (1.6.65) and (1.6.66) in (1.6.59) we obtain the result (1.4.13) of the Theorem 1.3.

$$[(\hat{T}_{ML}-T)(\hat{T}_{ML}-T)']=\phi_0^*\phi_0^{*\prime}+\sigma[\phi_0^*\phi^{*\prime}{}_1+\phi_1^*\phi^{*\prime}{}_1]+\sigma^2[\phi_0^*\phi_2^{*\prime}+\phi_1\phi_1^{*\prime}+\phi_2^*\phi_0']$$
$$+\sigma^3[\phi_0^*\phi_3^{*\prime}+\phi_2^*\phi_1^{*\prime}+\phi_1^*\phi_2^{*\prime}+\phi_3^*\phi_0^{*\prime}]$$
$$+\sigma^4[\phi_0^*\phi_4^{*\prime}+\phi_1^*\phi_3^{*\prime}+\phi_2^*\phi_2^{*\prime}+\phi_3^*\phi_1^{*\prime}+\phi_4^*\phi_0^{*\prime}]. \qquad (1.6.67)$$

where ,

$$\phi_0^*\phi_0^{*\prime}=X(I-F_L)\beta\beta'(I-F_L)'X', \qquad (1.6.68)$$

$$\phi_0^*\phi_1^{*\prime}=-X(I-F_L)\beta.u'(X(X'X)^{-1}F_LX'-\alpha I_n), \qquad (1.6.69)$$

$$\phi_0^*\phi_2^{*\prime}=-\left(\frac{u'Mu}{n-p}\right)X(I-F_L)\beta V'\Omega^{-1}Q(X'X)^{-1}F_L'X', \qquad (1.6.70)$$

$$\phi_1\phi_1^{*}{}'=(XF_L(X'X)^{-1}X'-\alpha I_n)u\,u'(X(X'X)^{-1}F_LX'-\alpha I_n)\,, \qquad (1.6.71)$$

$$\phi_0^{*}\,\phi_3^{*}{}'=\left(\frac{u'Mu}{n-p}\right)X(I-F_L)\beta\;XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u\,, \qquad (1.6.72)$$

$$\phi_2^{*}\,\phi_1^{*}{}'=\left(\frac{u'Mu}{n-p}\right)XF_L(X'X)^{-1}Q'\Omega^{-1}V\;u'(X(X'X)^{-1}F_LX'-\alpha I_n)\,, \qquad (1.6.73)$$

$$\phi_0^{*}\,\phi_4^{*}{}'=\left(\frac{u'Mu}{n-p}\right)^2X(I-F_L)\beta\;V'\Omega^{-1}Q(X'X)^{-1}Q\Omega^{-1}Q'.(X'X)^{-1}F_L'X'. \quad (1.5.74)$$

$$\phi_1^{*}\,\phi_3^{*}{}'=\left(\frac{u'Mu}{n-p}\right)(XF_L(X'X)^{-1}X'-\alpha I_n)u$$

$$u'X(X'X)^{-1}Q\Omega^{-1}Q'\,(X'X)^{-1}F_L'X'$$

(1.6.75)

$$\phi_2^{*}\,\phi_2^{*}{}'=\left(\frac{u'Mu}{n-p}\right)^2XF_L(X'X)^{-1}Q'\Omega^{-1}V\;V'\Omega^{-1}Q(X'X)^{-1}F_L'X'\,, \qquad (1.6.76)$$

It is easy to see that for normally distributed disturbances

$$E[\phi_0\,\phi_0^{*}{}']=X(I-F_L)\beta\beta'(I-F_L)'X'. \qquad (1.6.77)$$

$$\mathrm{E}[\phi_0^{*}\,\phi_1^{*}{}']=\mathrm{E}[\phi_0^{*}\,\phi_2^{*}{}']=\mathrm{E}[\phi_0^{*}\,\phi_3^{*}{}']=\mathrm{E}[\phi_0^{*}\,\phi_4^{*}{}']=\mathrm{E}[\phi_2^{*}\,\phi_1^{*}{}']=0. \qquad (1.6.78)$$

$$E[\phi_1\phi_1^{*}{}']=[\alpha^2I_n-\alpha(XF_L(X'X)^{-1}X+X'(X'X)^{-1}F_L'X')+XF_L(XX)^{-1}F_L'X'].$$

(1.6.79)

$$\mathrm{E}\,[\phi_1^{*}\,\phi_3^{*}{}']=[\alpha XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'-XF_L(X'X)^{-1}$$

$$Q'\Omega^{-1}Q(X'X)^{-1}F_L'X']. \qquad (1.6.80)$$

$$E[\phi_2^{*}\,\phi_2^{*}{}']=\left(\frac{n-p+2}{n-p}\right)XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}F_L'X'. \qquad (1.6.81)$$

Utilizing (1.6.77), (1.6.78), (1.6.79) (1.6.80) and (1.6.81) in (1.6.67) we obtain the result (1.4.14) of the Theorem 1.3 and again,

$$[(\hat{T}_{ML}-T)'(\hat{T}_{ML}-T)]=\phi_0^{*}{}'\,\phi_0^{*}+\sigma[\phi^{*}{}_1'\,\phi_0^{*}+\phi^{*}{}_0'\,\phi_1^{*}]$$

$$+\sigma^2[\phi_2^{*}{}'\,\phi_0^{*}+\phi_1'\phi_1^{*}+\phi_0'\,\phi_2^{*}]$$

$$+\sigma^3[\phi_3^{*}{}'\,\phi_0^{*}+\phi_1^{*}{}'\,\phi_2^{*}+\phi_2^{*}{}'\,\phi_1^{*}+\phi_0^{*}{}'\,\phi_3^{*}]$$

$$+\sigma^4[\phi_4^{*}{}'\,\phi_0^{*}+\phi_3^{*}{}'\,\phi_1^{*}+\phi_2^{*}{}'\,\phi_2^{*}+\phi_1^{*}{}'\,\phi_3^{*}+\phi_0^{*}{}'\,\phi_4^{*}]. \qquad (1.6.82)$$

where,

$$\phi_0^{*}{}'\,\phi_0^{*}=\beta'(I-F_L)'X'X(I-F_L)\beta\,, \qquad (1.6.83)$$

$$\phi_1^{*}{}'\,\phi_0^{*}=-u'(X(X'X)^{-1}F_LX'-\alpha I_n)\;X(I-F_L)\beta\,, \qquad (1.6.84)$$

$$\phi_2^{*}{}'\,\phi_0^{*}=-\left(\frac{u'Mu}{n-p}\right)V'\Omega^{-1}Q(X'X)^{-1}F_L'X'\;X(I-F_L)\beta\,, \qquad (1.6.85)$$

$\phi_1\phi_1^{*}{}' = u'(X(X'X)^{-1}F_L X' - \alpha I_n)\ (XF_L(X'X)^{-1}X' - \alpha I_n)u,$ (1.6.86)

$\phi_0^{*}{}'\phi_3^{*} = \left(\dfrac{u'Mu}{n-p}\right)\beta'(I-F_L)'X'\ XF_L(X'X)^{-1}Q'\Omega^{-1}Q(X'X)^{-1}X'u,$ (1.6.87) $\phi_1^{*}{}'\ \phi_2^{*} =$

$\left(\dfrac{u'Mu}{n-p}\right)u'(X(X'X)^{-1}F_L'X' - \alpha I_n)\ XF_L(X'X)^{-1}Q'\Omega^{-1}V,$ (1.6.88)

$\phi_4^{*}{}'\ \phi_0^{*} = \left(\dfrac{u'Mu}{n-p}\right)^2 V'\Omega^{-1}Q(X'X)^{-1}Q\Omega^{-1}Q'\ (X'X)^{-1}F_L'X'\ X(I-F_L)\beta.$

(1.6.89)

$\phi_3^{*}{}'\ \phi_1^{*} = \left(\dfrac{u'Mu}{n-p}\right)u'X(X'X)^{-1}Q\Omega^{-1}Q'\ (X'X)^{-1}F_L'X'\ (XF_L(X'X)^{-1}X' - \alpha I_n)u,$

(1.6.90)

$\phi_2^{*}\ \phi_2^{*}{}' = \left(\dfrac{u'Mu}{n-p}\right)^2 V'\Omega^{-1}Q(X'X)^{-1}F_L'X'\ XF_L(X'X)^{-1}Q'\Omega^{-1}V$

$V'\Omega^{-1}Q(X'X)^{-1}F_L'X',$ (1.6.91)

It can be verified that

$E[\phi_0^{*}{}'\phi_0^{*}] = \beta'(I-F_L)'X'X(I-F_L)\beta.$ (1.6.92)

E $[\phi_1^{*}{}'\ \phi_0^{*}]$ = E $[\phi_2^{*}{}'\ \phi_0^{*}]$ = E $[\phi_3^{*}{}'\ \phi_0^{*}]$ = E $[\phi_4^{*}{}'\ \phi_0^{*}]$ = E $[\phi_1^{*}{}'\ \phi_2^{*}]$ = 0. (1.6.93)

$E[\phi_1^{*}{}'\phi_1] = n\alpha^2 - \left\{\left(2\alpha\dfrac{\sum(\lambda_i+L)}{\sum(\lambda_i+1)} + \dfrac{\sum(\lambda_i+L)^2}{\sum(\lambda_i+1)^2}\right).\right\}$ (1.6.94)

E $[\phi_1^{*}\ \phi_3^{*}{}'] = \alpha\dfrac{\sum\tau_j^{*}\sum(\lambda_i+L)}{\sum\lambda_i(\lambda_i+1)} - \dfrac{\sum\tau_j^{*}\sum(\lambda_i+L)^2}{\sum\lambda_i(\lambda_i+1)^2}.$ (1.6.95)

$E[\phi_2^{*}\ \phi_2^{*}{}'] = \left(\dfrac{n-p+2}{n-p}\right)\dfrac{\sum\tau_j^{*}\sum(\lambda_i+L)^2}{\sum\lambda_i(\lambda_i+1)^2}.$ (1.6.96)

By using (1.6.92) (1.6.93) (1.6.94) (1.6.95) and (1.6.96) in (1.6.82) we obtain the result (1.4.15) of the Theorem 1.3.

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