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Imperfect Inventory Model for Weibull Deteriorating items with Selling Price Dependent Demand under Reliability Consideration and Fuzzy Environment

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Abstract
In the context of a transforming market, fuzzy inventory system is important for researchers to deal with uncertainty and imprecision. This paper deals with an inventory model with selling price dependent demand and two parameter Weibull deteriorating items with reliability consideration. In this model, shortages are not allowed. Since imperfect production process is considered but it is not re-workable. Due to un-certainty in the market, this inventory model is solved under crisp and fuzzy environment both to evaluate the optimum solutions in different cases. In this model, Centroid Method and Signed Distance Method are used to defuzzify the total cost. Sensitivity analysis is employed to illustrate the example under crisp and fuzziness.
Mathematical Classification: MSC 2010 (90B05, 90B30, 90C70).

Keywords: Imperfect quality, Reliability, Selling price dependent demand, Fuzzy.

1 Introduction:

The classical inventory model assume that all the produced items are of perfect quality but in reality it is not always true because of some inevitable events like machine failure, laziness of workers, and many more. Thus the defective products can either be repaired or sold at reduced cost. At the advancement of time, the manufacturers have a challenge to maintain their performance reliable. The efficiency of production process increases with reliability. Many researcher developed imperfect production model with re-liability.

In most of the inventory models, all the parameters assumed to be fixed but in real life scenario they are uncertain. Fuzzy theory deals with uncertainty of parameters. Fuzzy theory plays an important role in the development of real life inventory models. Researchers have developed a variety of models which are listed as:

Arunadevi E. and Umamaheswari S. [2] proposed an inventory management systems for livestock items and considers both ameliorating and deteriorating products. Bhunia A. and Shaikh A. [3] worked on a model with three parameter weibull deteriorating items. Chiang J., Yao J.S., and Lee H.M. [4] gives a fuzzy inventory model with back-order and defuzzify it with signed distance method. Geetha K. and Reshma S.P. [5] considered a fuzzy model for exponential demand and parabolic deterioration rate using centroid method. Hatibaruah A. and Saha S. [7] studied on ameliorating and deteriorating items with stock and advertisement frequency dependent demand. It is also assumed that retailer invests in preservation technology and suppliers offers a trade credit period. Ichwanto J I et al. [8] proposed a model to determine the value of deterioration with time dependent demand and shortages. Indrajit singha S.K., Samantha P.N., and Mishra U.K. [9] proposed a fuzzy model considering two warehouse problem for single deteriorating items. Kumar S. and Rajput U.S. [11] proposed a model for weibull deteriorating items with time dependent partial backlogging. Kuppulakshmi V. et al. [12] worked on fuzzy economic manufacturing model for imperfect production system with price discount and penalty maintenance cost. Kuraie V.C. et al. [13] developed a model for imperfect production process with selling price dependent demand. Fuzzy set theory also used for uncertain parameters. Maiti A.K. [14] proposed a model to initiate the cloudy fuzzy number in developing a classical EPL model for imperfect production system with demand dependent production rate. Padiyar S.V.S. et al. [15] discussed an integrated inventory model with variable production rate and probabilistic demand pattern under imperfect production process. In addition, they also took inflation and due to uncertainty, they used fuzzy logic in their model and uncertain parameters taken as triangular fuzzy number. Surendra Vikram Singh Padiyar et al. [16] considered a fuzzy inventory model, in which they defined deterioration rate, consumption rate and holding cost as pentagonal fuzzy number and defuzzify the total cost with graded mean representation method. Padiyar S.V.S. et al. [17] developed a fuzzy integrated supply chain model for single producer with multi retailer and multi supplier considering uncertain deterioration and inflation. Pal S. et al. [18] developed an EPQ for weibull deteriorating items with ramp-type demand under the effect of inflation and fuzziness. Palanivel Muthusamy, Venkadesh Murugesan and Vetriselvi Selvaraj [19] give a comprehensive inventory model foe Weibull deterioration items with ramp type demand considering carbon emission reduction and shortages. Vikas Sharma et al. [21] proposed an inventory model for two parameter Weibull deteriorating items, also considered time dependent demand. Singh S.R. and Singh C. [22] considered a two echelon supply chain model to show the prospective of both the manufacturer and the retailer with Weibull deterioration rate under inflationary environment. Vikasdeep Yadav et al. [23] developed a model for demand under fuzzy environment and time varying deterioration rate. Shaikh T.S. et al. [20], Gupta V. et al. [6], Aarya D.D. et al. [1], and Kumar S. and Rajput U.S. [10] worked on fuzzy inventory model for Weibull deteriorating items.

2 Assumptions and Notations:

2.1 Assumptions:

1. Demand rate is the function of marketing parameter i.e.
$D(\alpha, \beta) = (\gamma - k\beta)\alpha^{\eta}$ where γ is an arbitrary constant,
2. Production rate is scalar multiple of demand rate i.e.
$S(t) = \lambda D(t), \lambda > 1$
3. Deterioration rate is two parameter Weibull distribution,
$\theta = abt^{b-1}$
4. Imperfect production is assumed,
5. Shortages are not allowed.

2.2 Notations:

1. R is reliability rate

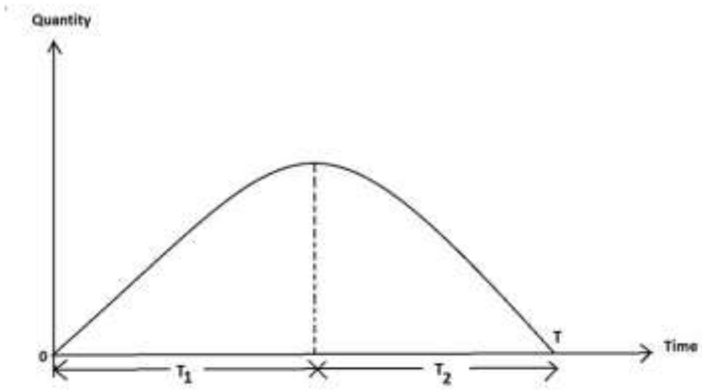

Figure 1: Producer's inventory system

2. S is production rate
3. A is ordering cost per replenishment cycle
4. h is holding cost per unit
5. C_d is deterioration cost per unit
6. η is shape parameter
7. α is selling factor decision variable
8. β is selling price decision variable
9. k is index of price elasticity

3 Mathematical Formulation of Model:

This model is graphically represented by Figure 1.

In this inventory model, time length T is divided into two parts denoted by T_1 and T_2 where T_1 is production period while T_2 is non-production period. The level of inventory decreases due to demand and deterioration and increases due to production in time interval $[0, T_1]$ while in time interval $[T_1, T]$ inventory decreases due to demand and deterioration.

The model can be defined by first order linear differential equations,

$$\frac{dI_1(t)}{dt} = RS - D - \theta I_1(t), \quad 0 \le t \le T_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} = -D - \theta I_2(t), \quad T_1 \le t \le T \qquad (2)$$

With condition $I_1(0) = 0 = I_2(T)$, the solution of equations (1) and (2) are

$$I_1(t) = e^{-at^b} D(R\lambda - 1)[t + \frac{at^{b+1}}{b+1}] \qquad (3)$$

$$I_2(t) = e^{-at^b} D[(T - t) + \frac{a(T^{b+1} - t^{b+1})}{b+1}] \qquad (4)$$

For continuity,

$I_1(T_1) = I_2(T_1)$

$$\Rightarrow T_1 = \frac{1}{R\lambda}[T - \frac{aT^{b+1}}{b+1}] \qquad (5)$$

Now we have discuss all the other inventory related costs

1 Ordering Cost

$$TC_o = A \tag{6}$$

2 Holding Cost

$$TC_h = h[\int_0^{T_1} I_1(t)dt + \int_{T_1}^{T} I_2(t)dt]$$

$$TC_h = h\left[DR\lambda\left(\frac{{T_1}^2}{2} - \frac{ab{T_1}^{b+2}}{(b+1)(b+2)} - \frac{a^2{T_1}^{2(b+1)}}{2(b+1)^2}\right) + D\left(\frac{T^2}{2} + \frac{abT^{b+2}}{(b+1)(b+2)} - \frac{a^2T^{2(b+1)}}{2(b+1)^2} - TT_1 - \frac{aT^{b+1}T_1}{(b+1)} + \frac{a{T_1}^{b+1}T}{(b+1)} + \frac{a^2T^{b+1}{T_1}^{b+1}}{(b+1)^2}\right)\right] \tag{7}$$

3 Deterioration Cost

$$TC_d = C_d\left[\int_0^{T_1} \theta I_1(t)dt + \int_{T_1}^{T} \theta I_2(t)dt\right]$$

$$TC_d = C_d\left[abDR\lambda\left(\frac{{T_1}^{b+1}}{b+1} - \frac{ab{T_1}^{2b+1}}{(2b+1)(b+1)} - \frac{a^2{T_1}^{3b+1}}{(3b+1)(b+1)}\right) + abD\left(\frac{T^{b+1}}{b(b+1)} + \frac{aT^{2b+1}}{2b(2b+1)} - \frac{a^2T^{3b+1}}{2b(3b+1)} - \frac{a{T_1}^bT^{b+1}}{b(b+1)} - \frac{{T_1}^bT}{b} + \frac{a^2{T_1}^{2b}T^{b+1}}{2b(b+1)} + \frac{a{T_1}^{2b}T}{2b}\right)\right] \tag{8}$$

Therefore the average total cost for the producer is

$$TC = \frac{1}{T}[ordering\ cost + holding\ cost + deterioration\ cost]$$

$$TC = \frac{A}{T} + \frac{h}{T}\left[DR\lambda\left(\frac{{T_1}^2}{2} - \frac{ab{T_1}^{b+2}}{(b+1)(b+2)} - \frac{a^2{T_1}^{2(b+1)}}{2(b+1)^2}\right) + D\left(\frac{T^2}{2} + \frac{abT^{b+2}}{(b+1)(b+2)} - \frac{a^2T^{2(b+1)}}{2(b+1)^2} - TT_1 - \frac{aT^{b+1}T_1}{(b+1)} + \frac{a{T_1}^{b+1}T}{(b+1)} + \frac{a^2T^{b+1}{T_1}^{b+1}}{(b+1)^2}\right)\right] + \frac{C_d}{T}\left[abDR\lambda\left(\frac{{T_1}^{b+1}}{b+1} - \frac{ab{T_1}^{2b+1}}{(2b+1)(b+1)} - \frac{a^2{T_1}^{3b+1}}{(3b+1)(b+1)}\right) + abD\left(\frac{T^{b+1}}{b(b+1)} + \frac{aT^{2b+1}}{2b(2b+1)} - \frac{a^2T^{3b+1}}{2b(3b+1)} - \frac{a{T_1}^bT^{b+1}}{b(b+1)} - \frac{{T_1}^bT}{b} + \frac{a^2{T_1}^{2b}T^{b+1}}{2b(b+1)} + \frac{a{T_1}^{2b}T}{2b}\right)\right] \tag{10}$$

Optimality: To find the optimum solution of model, we have to minimize average total cost. The necessary and sufficient conditions for total cost to be minimum are

$$\frac{\delta TC}{\delta T_1} = 0, \frac{\delta TC}{\delta T} = 0, \frac{\delta^2 TC}{\delta {T_1}^2} > 0\ and\ \frac{\delta^2 TC}{\delta {T_1}^2}\frac{\delta^2 TC}{\delta T^2} - \left(\frac{\delta^2 TC}{\delta T_1 \delta T}\right)^2 > 0$$

We can find the value of $T_1$and T with the help of given partial derivatives respectively,

$$\frac{\delta TC}{\delta T_1} = 0$$

$$\Rightarrow \frac{h}{T}\left[DR\lambda\left(T_1 - \frac{ab{T_1}^{b+1}}{(b+1)} - \frac{a^2{T_1}^{2b+1}}{b+1}\right) + D\left(-T - \frac{aT^{b+1}}{b+1} + a{T_1}^bT + \frac{a^2T^{b+1}{T_1}^b}{b+1}\right)\right] + \frac{C_d}{T}\left[abDR\lambda\left({T_1}^b - \frac{ab{T_1}^{2b}}{(b+1)} - \frac{a^2{T_1}^{3b}}{(b+1)}\right) + abD\left(-\frac{a{T_1}^{b-1}T^{b+1}}{(b+1)} - {T_1}^{b-1}T + \frac{a^2{T_1}^{2b-1}T^{b+1}}{(b+1)} + a{T_1}^{2b-1}T\right)\right] = 0 \tag{10}$$

$$\frac{\delta TC}{\delta T} = 0$$

$$\Rightarrow -\frac{A}{T^2} + h\left[\frac{-DR\lambda}{T^2}\left(\frac{{T_1}^2}{2} - \frac{ab{T_1}^{b+2}}{(b+1)(b+2)} - \frac{a^2{T_1}^{2(b+1)}}{2(b+1)^2}\right) + D\left(\frac{1}{2} + \frac{abT^b}{(b+2)} - \frac{a^2T^{2b}(2b+1)}{2(b+1)^2} - \frac{abT^{b-1}T_1}{(b+1)} + \frac{a^2bT^{b-1}{T_1}^{b+1}}{(b+1)^2}\right)\right] + C_d\left[\frac{-abDR\lambda}{T^2}\left(\frac{{T_1}^{b+1}}{b+1} - \frac{ab{T_1}^{2b+1}}{(2b+1)(b+1)} - \frac{a^2{T_1}^{3b+1}}{(3b+1)(b+1)}\right) + abD\left(\frac{T^{b-1}}{(b+1)} + \frac{aT^{2b-1}}{(2b+1)} -\right.\right.$$

$$\frac{a^2T^{3b-1}}{(3b+1)} - \frac{aT_1{}^bT^{b-1}}{(b+1)} + \frac{a^2T_1{}^{2b}T^{b-1}}{2(b+1)}\Big)\Big] = 0 \tag{11}$$

With the help of numerical example we can easily find the value of T_1 and T and also prove the convexity of total cost. We can see the convexity of total cost in different environment with the help of figure 2, figure 3 and figure 4.

4 Solution Procedure:

4.1 Crisp environment:

Let $h = 6, R = 0.8, \lambda = 2, a = 0.1, b = 1, C_d = 4, A = 17, \gamma = 15, k = 2,$
$\beta = 1.56, \alpha = 0.5, \eta = 0.002$
Then we find optimum values $T_1 = 0.6938, T = 1.0965$ and total Cost
$TC = 30.9049$

4.2 Fuzzy environment:

Due to uncertainty in parameters, it is not easy to show the exact variation in parameters of model, so we consider the following parameters in fuzzy sense
Let $\tilde{h} = (3, 4.5, 6), \tilde{A} = (10, 14.5, 20), \tilde{C}_d = (1.5, 2, 3.5)$
Using Centroid method, the total cost is $TC = 24.9144$
Using Signed distance method, the total cost is $TC = 24.8222$

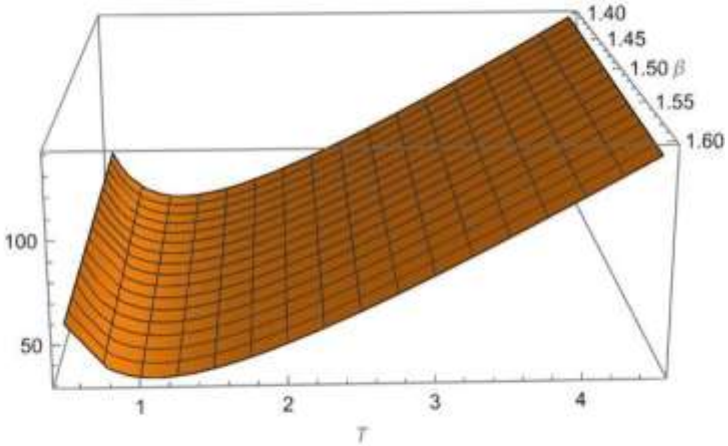

Figure 2: Convex graph of Crisp Total Cost

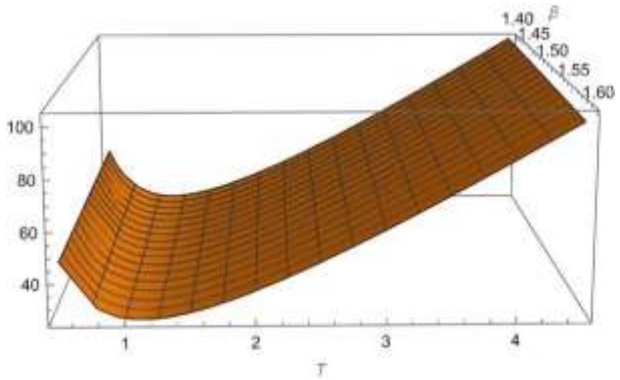

Figure 3: Convex graph of fuzzy Total Cost with Centroid method

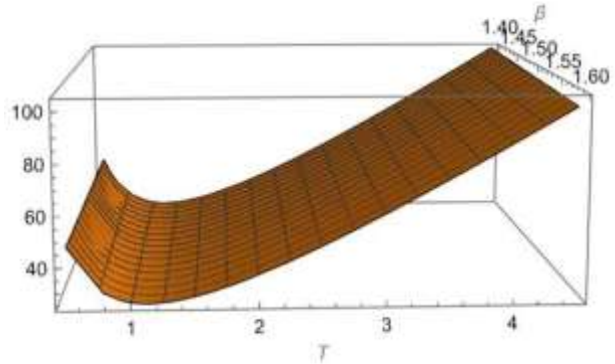

Figure 4: Convex graph of fuzzy Total Cost with Signed Distance method

5 Sensitivity Analysis:

Parameters	Changes	Crisp Model	Centroid Method	Signed Distance Method
R	0.80	28.0535	22.8848	22.7894
	0.85	26.9459	22.0387	21.9415
	0.90	25.9672	21.2909	21.1921
	0.95	25.0963	20.6253	20.5250
a	0.10	28.0535	22.8848	22.7894
	0.15	28.6120	22.9404	22.9352
	0.20	29.0941	23.2152	23.1108
	0.25	29.5205	23.2320	23.1263
b	1	28.0535	22.8848	22.7894
	2	29.0065	23.5782	23.4803
	3	29.4187	23.8916	23.7943
	4	29.6201	24.0532	23.9571

This section of the paper illustrates the sensitivity analysis of parameters. Given table provides the impact of various parameters on total cost in different methods.

As we see from figure 5, that total cost of model decreases in all methods as we increases the reliability factor. Reliability is the most important factor for the customers as well as producers. If we increase the reliability of the product then deterioration rate reduces due to which total cost decreases.

Similarly, if we see figure 6 and 7, it is evident that total cost of model in-creases in all methods if we increases the deterioration parameters. If we see the table of sensitivity analysis it is evident that signed distance method is the most appropriate among all the methods used in this model.

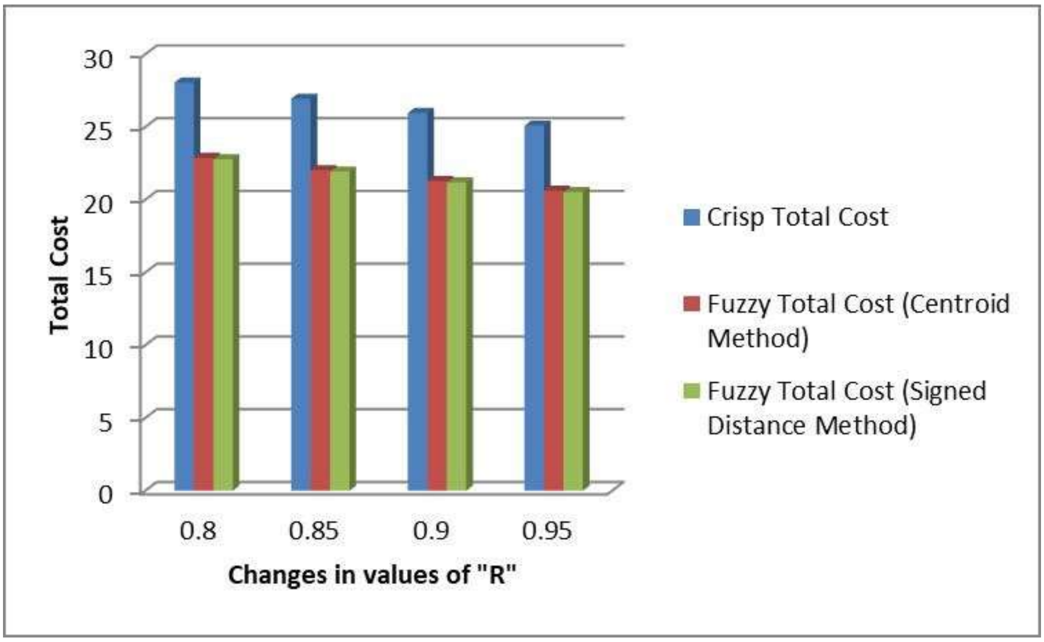

Figure 5: Variation in total cost w.r.t. reliability

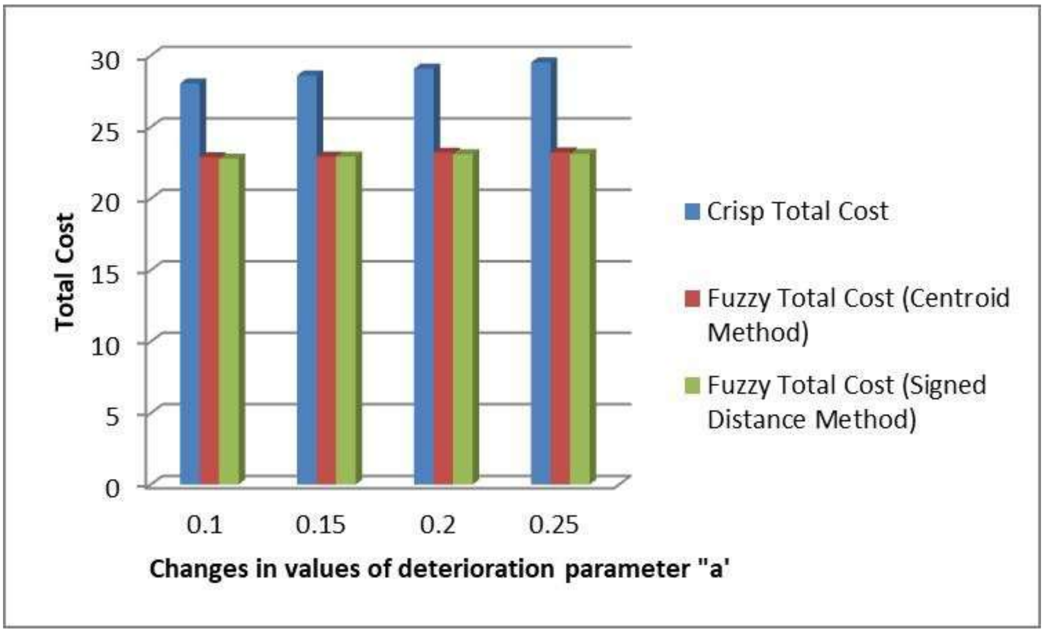

Figure 6: Variation in total cost w.r.t. "a"

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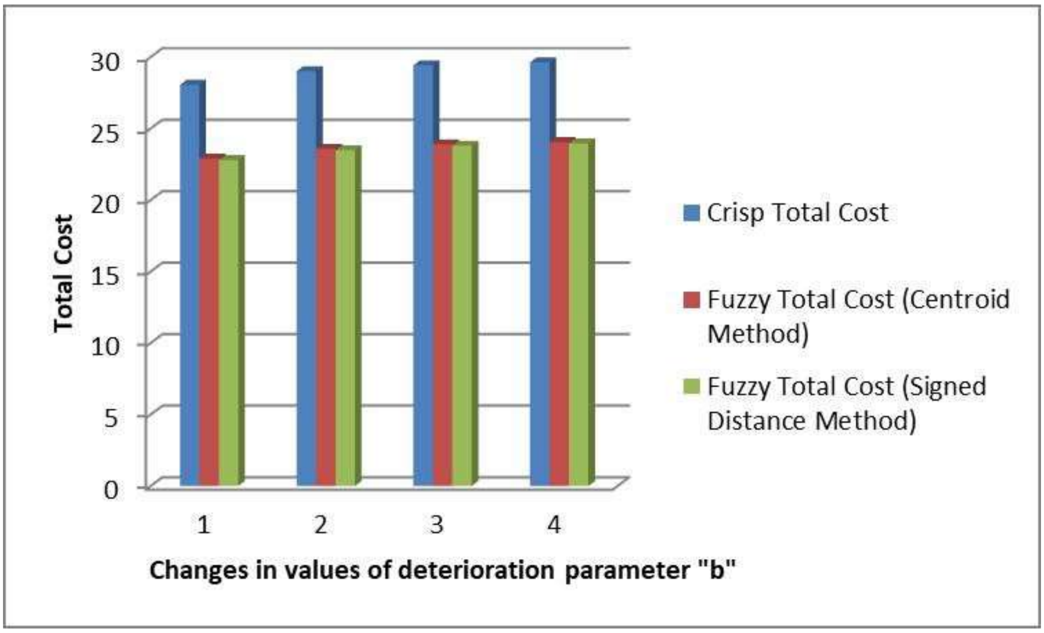

Figure 7: Variation in total cost w.r.t. "b"

6 Conclusion:

In this proposed model, imperfect production is considered. This paper also considers reliability factor because reliability of the product is key factor for producers to retain the customers for long time. Also, we all know that marketing of the products is used to attract consumers therefore demand is the function of selling price as well as marketing factor. We also consider two parameter Weibull deterioration rate. This paper gives a total cost minimization model in crisp as well as fuzzy environment. Numerical analysis shows that total cost is affected by reliability parameter and deterioration parameters. If we increase the reliability parameter we see that total cost decreases. From sensitivity analysis it is also evident that signed distance method is the most appropriate method among all the methods used in this model to obtain minimum total cost.

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