A Wavelet Approach for a One-Dimensional Signal

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Abstract
This paper explains a comparative study conducted based on discrete wavelet transform (DWT), discrete wavelet packet transforms (DWPT) and the complex wavelet transforms (CWT) techniques. A mother or basis wavelet is first chosen for five wavelet filter families such as haar, Daubechies (db4), coiflet, symlet and dmey for DWT and WP approach. The signal is then decomposed to a set of scaled and translated versions of the mother wavelet also known as time and frequency parameters. The results show that wavelet filter with multiresolution analysis(MRA) are useful for analysis and synthesis purpose. For Complex Wavelet Transform (CWT) approach we use Haar, Splines, Symlets, Shannon basis function for analysis and synthesis purpose and we verified that SNR values are comparable with CWT also. In terms of signal quality the Haar wavelet has been seen to be the best mother wavelet. This is taken from the analysis of the signal to noise ratio (SNR) value which is around 23dB for normal DWT, around 290 to 310dB for WP and CWT which proves that 1D signal has significant improvement with wavelet transform analysis and synthesis.

Keywords: Discrete wavelet transform (DWT), wavelet packet decomposition (WPD), complex wavelet transform (CWT),

1. INTRODUCTION.

Wavelet Transform (WT) is a powerful mathematical tool for non-stationary signals. Wavelets have the ability to analyze different parts of a signal at different scales. As a consequence, signals with fast oscillations or even discontinuities, in localized regions may be approximated well by a linear combination of relatively few wavelets. In wavelet analysis, a fully scalable modulated window is used for frequency localization [1]–[9]. The window is sliding, and the wavelet transform of a part of the signal is calculated for every position. Then a slightly longer or shorter window is used for each new stage of calculation. Short windows at high frequencies and long windows at low frequencies are used in the wavelet transform. The result of the wavelet transform is a collection of time-scaling representations of the signal with different resolutions. The discrete wavelet transform (DWT) provides a very efficient representation for a broad range of real-world signals. This property has been exploited to develop powerful signal denoising and estimation methods [10] and extremely low-bit-rate compression algorithms [11].The discrete wavelet transform (DWT) is usually implemented using an octave-band tree structure. This is accomplished by dividing each sequence into a component containing its approximate bit-rate compression algorithms [11].The discrete wavelet transform (DWT) is usually implemented using an octave-band tree structure.

This is accomplished by dividing each sequence into a component containing its approximate version (low-frequency part) and a component with the residual details (high-frequency part) and then iterating this procedure at each stage only on the low-pass branch of the tree [12] [13]. The main drawback of the octave-band tree structure is that it does not provide a good approximation of the critical subband decomposition [14]. An alternate means of analysis is sought, so that valuable time-frequency information is not lost. The Wavelet Packet Transform (WPT) is one such time frequency analysis tools. It is a transform that brings the signal into a domain that contains both time and frequency information, (Wicker Hauser, 1991). Thus, analysis of the signal can be done simultaneously in frequency and time. The wavelet packet transform (WPT) offers a great deal of freedom in dealing with different types of transient signals. Indeed the development of the wavelet transform (WT) [15, 16, 17, 18] and wavelet packets [19, 20, 21] has sparked considerable activity in signal representation and in transient and non-stationary signal analysis [22, 23, 24, 25].Wavelet packet decomposition (WPD) (sometimes known as just wavelet packets) is a wavelet transform where the signal is passed through more filters than the DWT. Wavelet packets are the particular linear combination of wavelets. They form bases which retain many of the orthogonality, smoothness, and localization properties of their parent wavelets. The coefficients in linear combinations are computed by a recursive algorithm making each newly computed wavelet packet coefficient, with the result that expansions in wavelet packet bases have low computational complexity. In the DWT, each level is calculated by passing the previous approximation coefficients through high and low pass filters. However, in the WPD, both the detail and approximation coefficients are decomposed and therefore wavelet packet decomposition can provide more precise frequency resolution than wavelet decomposition in time signal analysis and synthesis.

The wavelet packet transform has more important benefits than the discrete wavelet transforms. Wavelet packet functions comprise a rich family of building block functions. Wavelet packet functions are still localized in time but offer more flexibility than wavelets in representing different types of signals. In particular, wavelet packets are better at representing signals that exhibit oscillatory or periodic behaviour. The wavelet packets allow more flexibility in adapting the basis to the frequency contents of a signal and it is easy to develop a fast wavelet packet transform. The power of wavelet packet lies in the fact that we have much more freedom in deciding which basis function is to be used to represent the given function. It can be computed very fast, it demands only $O(M \log M)$ time, where M is the number of data points which is important in particular in real time applications. It also has compact support in time as well as in frequency domain and adapts its support locally to the signal which is important in time varying signals. With wavelet packets we have a much finer resolution of the signal and a greater variety of options for decomposing it. Though standard DWT is a powerful tool for analysis and processing of many real-world signals and images, it suffers from three major disadvantages, (1) Shift- sensitivity, (2) Poor directionality, and (3) Lack of phase information. These disadvantages severely restrict its scope for certain signal and image processing applications. Other extensions of standard DWT such as Wavelet Packet Transform (WPT) reduce only the first disadvantage of shift- sensitivity but with the cost of very high redundancy and involved computation. Recent research suggests the possibility of reducing two or more above-mentioned disadvantages using different forms of Complex Wavelet Transforms (CWT) [26,27,28,29] with only limited (and controllable) redundancy and moderate computational complexity. Complex discrete wavelet transform (CDWT) is a form of discrete wavelet transform, which generates complex coefficients by using a dual tree of wavelet filters to obtain their real and imaginary parts. What makes the complex wavelet basis exceptionally useful is that it provides a high degree of shift-invariance and better directionality compared to the real DWT. The initial motivation behind the development of CWT was to avail explicitly both magnitude and phase information.

The paper is organized as follows. In Section 2 brief background information on Wavelet theory is discussed. In section 3 the present work is explained. The results are given in Section 4 and Section 5 gives the conclusion.

2. REVIEW OF WAVELET THEORY

2.1 Discrete Wavelet Transform

From the signal processing point of view, the discrete wavelet transform is a *filter bank* algorithm iterated on the low-pass output. A filter bank is a pair of lowpass and highpass filters followed by down sampling by two. The lowpass filtering produces an *approximation* of the signal, which is expressed by the *scaling* coefficients, while the highpass filtering reveals the *details* (i.e., the differences between two successive approximations) that are expressed by the *wavelet* coefficients. At the reconstruction, the scaling and the wavelet coefficients are first up sampled (by introducing a zero between each two samples) and then filtered with a lowpass and a highpass filter, respectively, followed by summation of the filtered outputs. The DWT is also related to a multiresolution framework. The idea is to represent a signal as a series of approximations (lowpass version) to the signal and details (high-pass version) at different resolutions. To start with, the input signal is assigned a resolution 1. Then at each stage (resolution 2^{-j}, $j = 1,2\ldots$) the signal is low-pass filtered to give an approximation signal and high-pass filtered to give a detail signal, which represents the information lost when going from a higher resolution to a lower resolution. Then the wavelet representation is the set of detail coefficients at all resolutions and approximation coefficients at the lowest resolution.

2.2 Multiresolution analysis

An orthogonal wavelet decomposition of a signal $x(t)\,\varepsilon\, L^2(R)$ to coefficients $\{W_j^{\,k}(x)\}_{k,j\varepsilon Z^2}$ such that

$$W_j^{\,k}(x) \cong \left\langle x(t), 2^{-j/2}\,\psi\left(2^{-j}t-k\right)\right\rangle$$

$$\cong 2^{-j/2}\int_{-\infty}^{\infty} x(t)\,\psi^*\left(2^{-j}t-k\right)dt \qquad (1)$$

where the function ψ is usually referred to as a mother wavelet and $*$ stands for the complex conjugation. The orthonormal wavelet basis $\left\{2^{-j/2}\,\psi\left(2^{-j}t-k\right),\ (k,j)\in Z^2\right\}$ may be built from a multiresolution analysis of $L^2(R)$. In this case, the approximation of the signal at resolution 2^{-j} can be described by the coefficients

$$A_j^{\,k}(x) \cong \left\langle x(t), 2^{-j/2}\,\phi\left(2^{-j}t-k\right)\right\rangle,\ k\in Z \qquad (2)$$

where the function ϕ is the scaling function. The mother wavelet and the scaling function then satisfy the so called two- scale equations:

$$2^{-1/2}\,\phi\left(\frac{t}{2}-k\right) = \sum_{l=-\infty}^{\infty} h_{l-2k}\,\phi(t-l) \qquad (3)$$

$$2^{-\frac{1}{2}}\psi\left(\frac{t}{2}-k\right)=\sum_{l=-\infty}^{\infty} g_{l-2k}\psi(t-l) \qquad (4)$$

Where $\{h_k\}_{k\in Z}$ and $\{g_k\}_{k\in Z}$ are respectively the impulse response of lowpass and highpass paraunitary Quadrature mirror filters (QMF) [18]. If we denote the vector spaces

$V_j \cong span\{\phi(2^{-j}t-k), k\in Z\}$ and $O_j \cong span\{\psi(2^{-j}t-k), k\in Z\}$, it results from equations (3) and (4) that $V_{j+1}=V_j \overset{\perp}{\oplus} O_j^{\;2}$. We then find that for every $j_m \in Z$,

$\left\{2^{-j/2}\psi(2^{-j}t-k), k\in Z, j\le j_m\right\} \cup \left\{2^{-j_m/2}\phi(2^{-j_m}t-k), k\in Z\right\}$ is an orthonormal basis of $L^2(R)$.

The interest in the QMF filters lies in the efficient computation of the orthogonal wavelet decomposition via a two channel filter bank structure. The decomposition which is useful in emphasizing the local features of a signal, presents however a limitation, namely its non variance in time (or space). This implies that the wavelet coefficients of $T_\tau[x(t)]\cong x(t-\tau), \tau\in R$ are generally not delayed versions of $\{W_j^{\,k}(x)\}_{k\in Z}$

3. WAVELET PACKET DECOMPOSITION

The wavelet packet decomposition [30][31][32] is an extension of the wavelet representation which allows the best matched analysis to a signal. To define wavelet packets, we introduce functions of $L^2(R)$, $W_m(t), m\in N$ such that

$$\int_{-\infty}^{\infty} W_0(t)dt = 1 \qquad (5)$$

and for all $k\in Z$,

$$2^{-\frac{1}{2}}W_{2m}\left(\frac{t}{2}-k\right)=\sum_{l=-\infty}^{\infty} h_{l-2k}W_m(t-k) \qquad (6)$$

$$2^{-\frac{1}{2}}W_{2m+1}\left(\frac{t}{2}-k\right)=\sum_{l=-\infty}^{\infty} g_{l-2k}W_m(t-k) \qquad (7)$$

Where $\{h_k\}_{k\in Z}$ and $\{g_k\}_{k\in Z}$ are the previously defined impulse responses of the QMF filters. If for every $j\in Z$, we define the vector space $\Omega_{j,m}\cong span\{W_m(2^{-j}t-k), k\in Z\}$ then it can be shown that $\Omega_{j,m}=\Omega_{j+1,2m}\overset{\perp}{\oplus}\Omega_{j+1,2m+1}$. As a result, if we denote by P a partition of R^+ into intervals $I_{j,m}=[2^{-j}m,....,2^{-j}(m+1)], j\in Z \; and \; m\in\{0,1,....2^j-1\}$, then

$$L^2(R)=\overset{\perp}{\bigoplus_{(j,m)/\, I_{j,m}\in P}}\Omega_{j,m} \qquad (8)$$

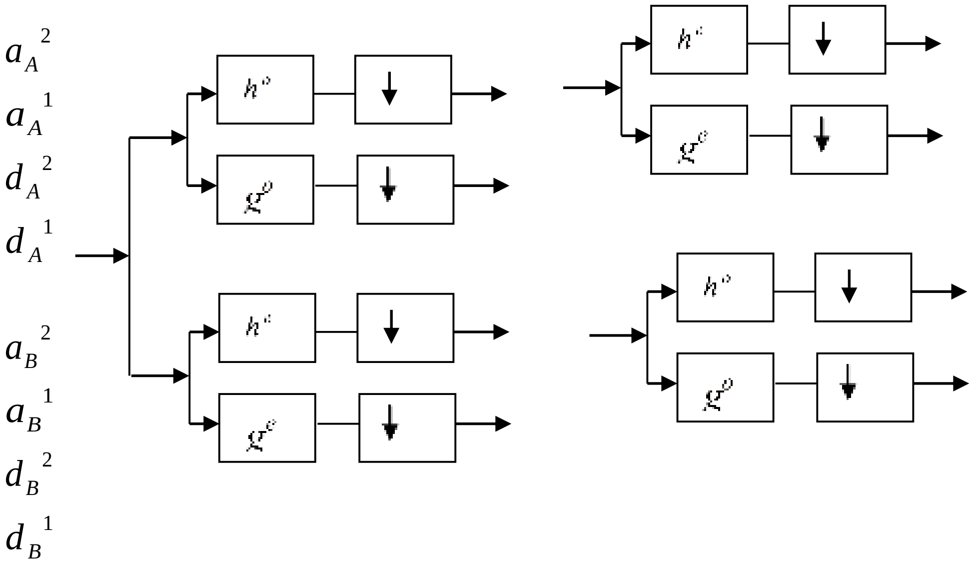
In an equivalent way, $\left\{2^{-j/2}W_m(2^{-j}t-k), k\in Z, (j,m)/\, I_{j,m}\in P\right\}$ is an orthonormal basis of $L^2(R)$. Such a basis is called a wavelet packet. The coefficients resulting from the decomposition of a signal $x(t)$ in this basis are

$$C^k_{\;j,m}(x)\cong\left\langle x(t), 2^{-j/2}W_m(2^{-j}t-k)\right\rangle \qquad (9)$$

By varying the partition P, different choices of wavelet packets are possible[33]-[36]. While a fixed and dyadic partitioning of time frequency domain is imposed in the case of the wavelet transform, the idea of wavelet packets is to introduce more flexibility making this partitioning adaptive to spectral content of the signal.

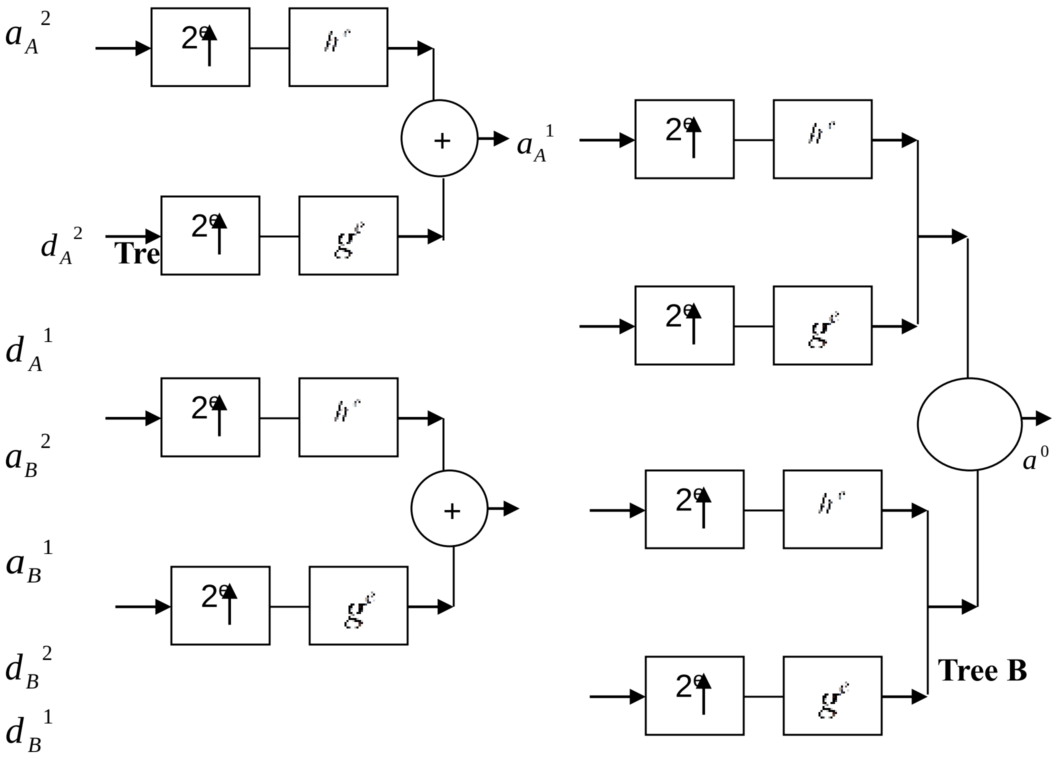
4. The 1-D Dual-Tree Complex Wavelet Transform

It is well-known that the real biorthogonal wavelet transform can provide PR and no redundancy, but it is lack of shift variant. Then Kingsbury (Julian Magarey1998;N. Kingsbury, 1998a ; Nick Kingsbury,1998b; Serkan Hatipoglu, 1999) has developed a dual-tree algorithm with a real biorthogonal transform, and an approximate shift invariance can be obtained by doubling the sampling rate at each scale, which is achieved by computing two parallel subsampled wavelet trees respectively. For one dimension signal, we can compute two parallel wavelet trees. There is one sample offset delay between two trees at level 1, which is achieved by doubling all the sample rates. The shift invariance is perfect at level 1, since the two trees are fully decimated. To get uniform intervals between two trees beyond level 1, there have to be half a sample delay. The term will be satisfied using odd-length and even-length filters alternatively from level to level in each tree. Because we use the decimated form of a real discrete wavelet transform beyond level 1, the shift invariance is approximate. The transform algorithm is described as following. Its process is illustrated by figure 1 and figure2.



Level 1 **Level 2**

Fig 1. The one-dimensional dual tree complex wavelet transfo



Level 2 **Level 1**

Fig 2. One-dimensional dual tree inverse complex wavelet transforms

1D Complex Wavelet Transform:

- At level 1, there is one sample offset between the trees.

$$\left(a_A^{\,1}\right)_n = \left(a^0 * h^0\right)_{2n} \qquad \left(d_A^{\,1}\right)_n = \left(a^0 * g^0\right)_{2n}$$
$$\left(a_B^{\,1}\right)_n = \left(a^0 * h^0\right)_{2n+1} \qquad \left(d_B^{\,1}\right)_n = \left(a^0 * g^0\right)_{2n+1} \qquad (10)$$

- Beyond level 1, there must be half a sample difference between the trees.

$$\left(a_A^{\,j+1}\right)_n = \left(a_A^{\,j} * h^e\right)_{2n} \qquad \left(d_A^{\,j+1}\right)_n = \left(a_A^{\,j} * g^e\right)_{2n}$$
$$\left(a_B^{\,j+1}\right)_n = \left(a_B^{\,j} * h^0\right)_{2n+1} \qquad \left(d_B^{\,j+1}\right)_n = \left(a_B^{\,j} * g^0\right)_{2n+1} \qquad (11)$$

1D Inverse Complex Wavelet Transform:

- Level j (j > 0)

$$\left(a_A^{\,j}\right)_n = \left(\breve{a}_A^{\,j+1} * \tilde{h}^e\right)_n + \left(\breve{d}_A^{\,j+1} * \tilde{g}^e\right)_n \qquad (12)$$
$$\left(a_B^{\,j}\right)_n = \left(\hat{a}_B^{\,j+1} * \tilde{h}^o\right)_n + \left(\hat{d}_B^{\,j+1} * \tilde{g}^o\right)_n$$

- At j = 0

$$a_n^{\,0} = \frac{1}{2}\left(\left(\breve{a}'_A * \tilde{h}^o\right)_n + \left(\breve{d}_A' * \tilde{g}^o\right)_n + \left(\hat{a}_B * \tilde{h}^o\right)_n + \left(\hat{d}_B' * \tilde{g}^o\right)_n\right) \qquad (13)$$

Where $\begin{matrix} x_n = x_p & if\ n = 2p \\ 0 & if\ n = 2p+1 \end{matrix}$, $\begin{matrix} \hat{x}_n = x_p & if\ n = 2p+1 \\ 0 & if\ n = 2p \end{matrix}$

5. PROPOSED WORK

5.1. The block diagram shown in figure (3) gives the actual implementation of the method proposed in this paper.

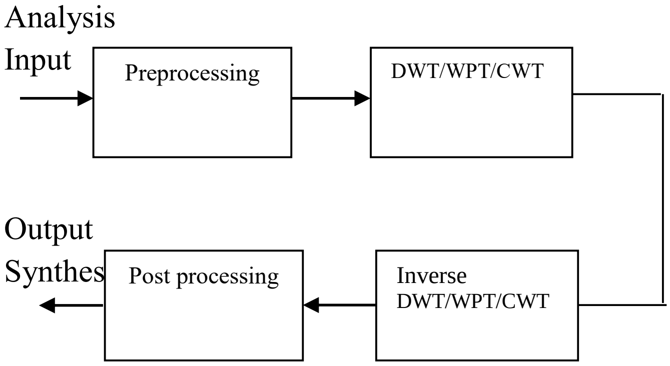


Fig.3 Block diagram of time domain signal Using DWT/ WPT/CWT

Input time domain signal:

The system described above was simulated on a computer with floating point numbering system. The input time domain wave is pre- emphasized, low pass filtered with sampling frequency 8 K to 44.1 KHz range and 1 – 10 sec duration samples. The time domain is digitized to 16 bits.

Preprocessing:

In this block unwanted noise is removed when the signal gets recorded with the help of various digital filter such as Low pass filter (LPF) with 3.4KHz, Notch filter to remove the line frequency effect i.e. 50Hz.

Wavelet transforms:

In this block we applied the wavelet filter coefficients as low and high band and filter the input signal to enhance the band energies.

Inverse Wavelet transforms:

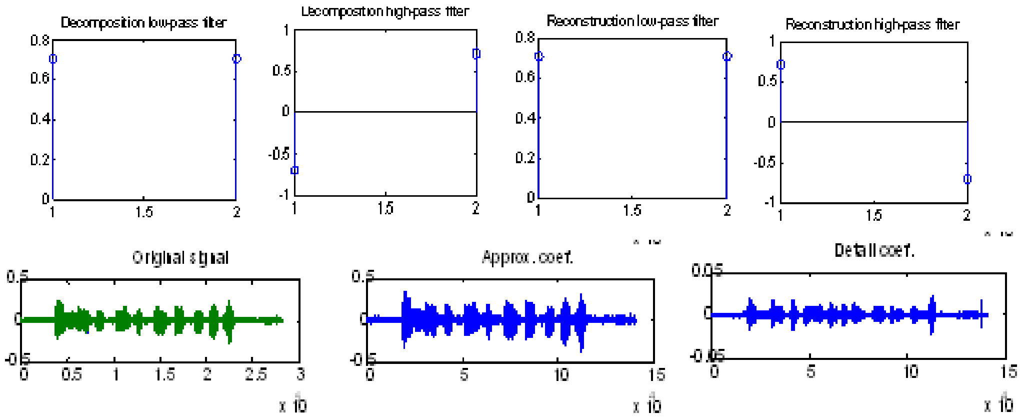
Analysis of interpolated features is reordered with respect to the 2 band system to extract the original contents from the feature vector for input signal synthesis purpose.

Post Processing:

Final tuning is done in the post processing block.

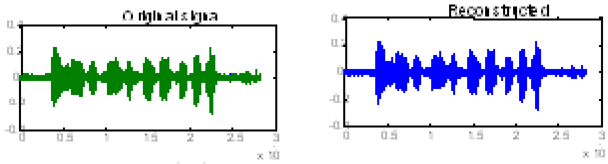
5.2. Implementation:

Following example shows how the block diagram wise analysis and synthesis carried out.



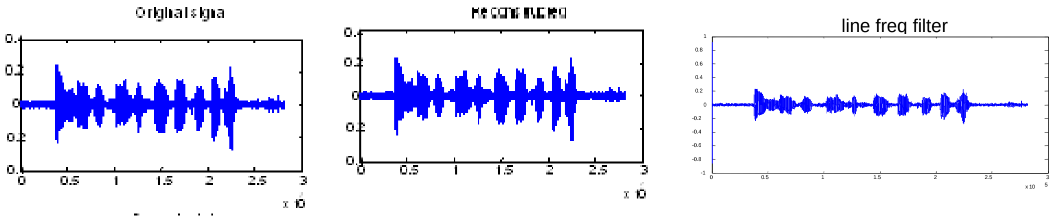
Haar mother wavelet.

Analysis and Synthesis using DWT for sample T7.



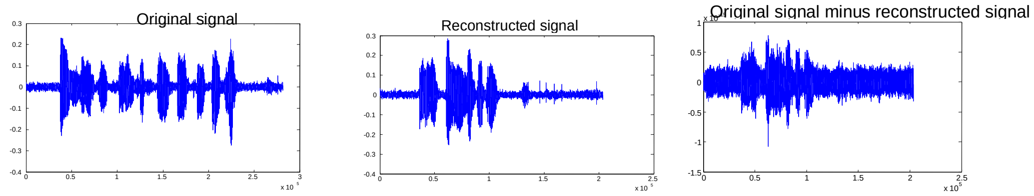
Wavelet Filtering

Haar mother wavelet. **Analysis and Synthesis using Wavelet packet Analysis for sample T7**

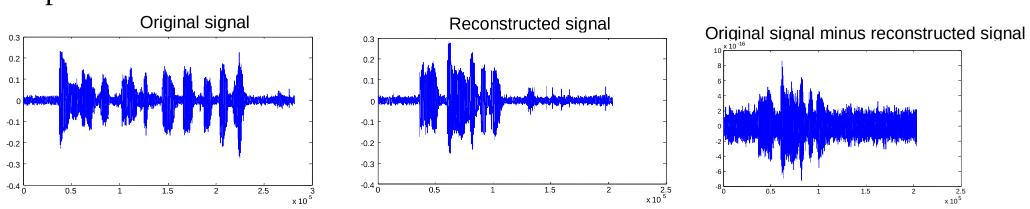


Analysis and Synthesis using Complex wavelet transform for sample T7.

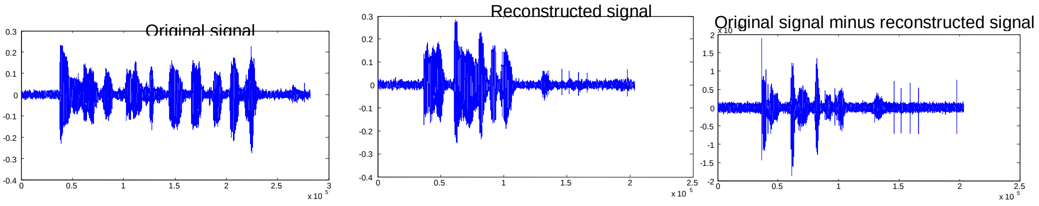
For Haar mother wavelet



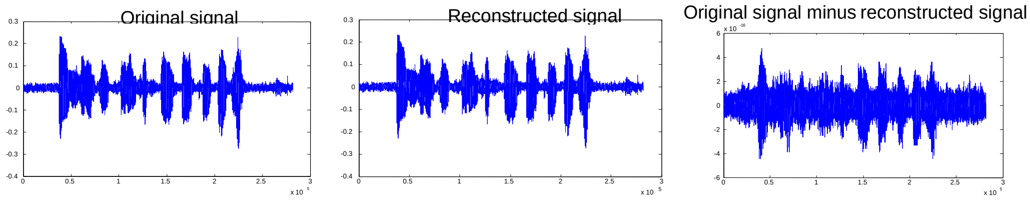
For Splines



For Symlets



For Shannon



5.3 Testing Setup

Matlab programs were written to implement the structure shown in figure (3). Time domain signals in a *.wav format sampled at 8 KHz were used for all simulations. Results for different setups are given in the next section. Several examples of time domain signals of different sampling frequencies, with five wavelet filter families are given below.

5.4 Performance Evaluation

Performance evaluation tests can be done by subjective quality measures and objective quality measures. Objective measures provide a measure that can be easily implemented and reliably reproduced. Objective measures are based on mathematical comparison of the original and processed time domain signals. The majority of objective quality measures quantify time domain quality of the signal in terms of a numerical distance measure. The signal to noise ratio is the most widely used method to measure time domain signal quality. It is calculated as the ratio of the signal to noise power in decibels.

$$SNR = 10\log_{10}\left(\frac{\sum_n s^2(n)}{\sum_n [s(n)-\hat{s}(n)]^2}\right) \quad (14)$$

Where $s(n)$ is the clean time domain signal and $\hat{s}(n)$ is the processed time domain signal.

6. RESULTS

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	22.52	9.24	21.26	22.53	4.78
T2	22.41	8.92	20.94	22.52	4.41
T3	20.99	8.68	20.52	21.01	5.15
T4	23.24	10.03	22.02	23.23	5.66
T5	21.85	8.45	20.48	21.86	3.94
T6	22.63	9.33	21.33	22.63	4.78
T7	22.52	9.48	21.46	22.52	5.08
T8	21.30	9.04	20.54	21.41	5.03
T9	23.15	9.58	21.65	23.15	4.89
T10	21.58	9.06	20.57	21.62	4.68

Table 1. SNR calculation for various mother wavelets using DWT.

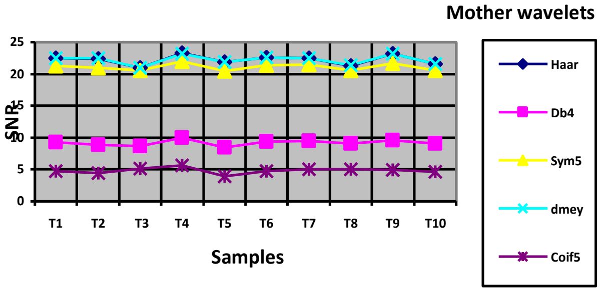


Fig 4. Graphical presentation of SNR and mother wavelets

From Table1 we observe that the Haar mother wavelet give good SNR values i.e. 20-24dB for DWT analysis purpose.

Sample	Haar	Db4	Sym5	dmey	Coif5
T1	310.41	232.64	247.31	121.37	165.81
T2	310.41	232.64	247.31	121.34	165.82
T3	310.40	232.64	247.31	121.36	165.81
T4	310.41	232.64	247.31	121.39	165.81
T5	310.38	232.64	247.31	121.36	165.81
T6	310.40	232.64	247.31	121.42	165.81
T7	310.40	232.64	247.31	121.33	165.82
T8	310.39	232.64	247.31	121.42	165.81
T9	310.40	232.65	247.31	121.25	165.82
T10	310.41	232.64	247.31	121.34	165.82

Table 2. SNR calculation for various mother wavelets using wavelet packet analysis

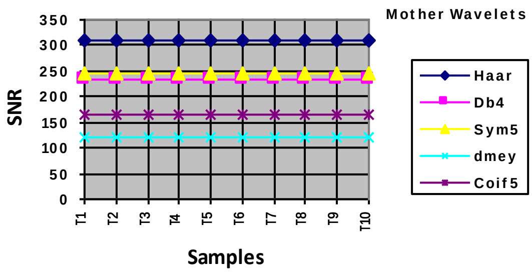


Fig 5. Graphical presentation of SNR and mother wavelets

From Table 2 we observed that the Haar mother wavelet give good SNR values i.e. 310dB for Wavelet packet analysis purpose.

Sample	Haar	Splines	Symlets	Shannon
T1	293.82	294.20	249.72	297.74
T2	284.54	287.25	248.82	288.38

T3	**294.74**	**295.30**	**249.40**	**296.56**
T4	**290.08**	**291.05**	**251.41**	**292.60**
T5	**293.61**	**295.11**	**249.35**	**297.33**
T6	**294.48**	**295.16**	**250.33**	**296.72**
T7	**292.04**	**292.95**	**250.82**	**294.77**
T8	**296.95**	**297.24**	**250.82**	**298.99**
T9	**292.12**	**292.89**	**251.18**	**294.85**
T10	**290.59**	**291.75**	**250.97**	**293.23**

Table 3. SNR calculation using Complex wavelet transform

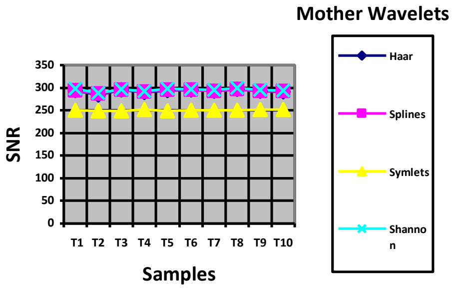


Fig 6. Graphical presentation of SNR and mother wavelets

From Table 3 we observe that the Shannon mother wavelet gives good SNR values i.e. 293dB for complex wavelet transform.

7. CONCLUSION

A comparative study for analysis and synthesis of time signals using different wavelet filtering techniques is presented. From this study we could understand and experience the effectiveness of wavelet transform techniques in time signal analysis and synthesis. The performance of wavelet packet is appreciable while comparing with the discrete wavelet transform decomposition technique since wavelet packet analysis can provide a more precise frequency resolution than the wavelet analysis. It also has compact support in time as well as in frequency domain and adapts its support locally to the signal which is important in time varying signals. With wavelet packets we have a greater variety of options for decomposing the signal. The complex wavelet transform are useful since it provides a high degree of shift-invariance and better directionality compared to the real DWT. Moreover, CWT can avail explicitly both magnitude and phase information. The method presented is used for time as well as frequency analysis of time varying signals. From the results we conclude that the wavelet filtering find applications in the time domain analysis and synthesis era. In terms of signal quality, Haar wavelet has been seen to be the best mother wavelet. This is taken from the analysis of the signal to noise ratio (SNR) value which is around 23dB for normal DWT, around 290 to 310dB for WP and CWT which proves that 1D signal has significant improvement with wavelet transform analysis and synthesis. The system has been tested with various sampling frequencies for time domain samples which give satisfactory output. Hence we conclude that the system will behave stable with wavelet filter and can be used for time signal analysis and synthesis purpose.

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