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Performance Evaluation of Redundant Systems with Faulty Switches

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Abstract:
In order to evaluate complicated systems, reliability analysis is essential, particularly those incorporating redundant components and imperfect switching mechanisms. This study examines the reliability of standby redundant systems with imperfect switching devices by contrasting Bayesian and classical statistical methods. Using real-world failure data and simulated datasets, the Classical approach focuses on point estimation and confidence intervals, while the Bayesian method integrates prior knowledge with observed data to provide probabilistic insights. Results demonstrate the strengths and limitations of both methodologies in capturing system behavior under uncertainty. This comparison highlights the flexibility of Bayesian analysis in handling sparse data and incorporating prior expertise. Using Monte Carlo simulations alongside Classical and Bayesian approaches to model the reliability of such systems under various operational conditions. Monte Carlo methods allow for robust exploration of uncertainty and variability in system performance, providing detailed insights into failure probabilities and reliability metrics. The findings offer valuable insights for decision-making in system design and maintenance planning.

Keywords: Reliability analysis, Bayesian and Classical analysis, Standby redundant systems, Imperfect switching devices, Failure data, System design, System performance, Maintenance strategies

Introduction

Complex equipment in many real-world scenarios is made up of strategic parts, the failure of which could cause disastrous consequences. The failure of strategic units should be minimised in order to handle such difficult circumstances. Critical unit failure can be reduced by introducing appropriate redundancy during the equipment design phase. The standby redundancy strategy, which offers theoretically far higher reliability than parallel redundancy, must be employed in the design stage if the failure detection and switching over device are very dependable. In a standby arrangement, the same backup unit is activated manually or by a switch in the event that the primary unit in use fails. For instance, in the event that an aeroplane's radio breaks down, the aircraft can still fly thanks to a backup communication system. So far, quite a good number of studies have been carried out by many authors.

Iqbal [5] suggests a number of empirical investigations to improve system reliability. Pan et al. [11], Jain [6], Sharma et al. [13], and Singh et al. [15] provided the imperfect coverage models with different status and trends. An unreliable machine repair system with warm standby switching failure and retrial behaviour is the subject of a study by Wang et al. [19]. Jyh-Bin et al. [4] investigate the reliability metric of repairable systems with standby switching failures and reboot delay. Amari et al. [1, 2], Wang and Chen [18], and Ke and Liu [8] have also made significant contributions. Their work has taken detection, incomplete coverage, and reboot into account.

The failure and repair of system units have been modelled by the majority of the previously cited literature using certain probability distributions with defined characteristics. However, these characteristics are frequently unknown or ambiguous in real-world applications. In these situations, determining these parameters requires the use of appropriate estimating methodologies in addition to the assessment of critical reliability metrics including Mean Time to System Failure (MTSF), system availability, and overall profitability. Ghosh et. al. [3], Kishan et.al. [10], Patawa et.al. [12] , Thiagarajan et. al. [16] & Sharma et.al. [14] have applied the Bayesian approach to integrate previous knowledge into the analysis, enabling the investigation of repairable systems' behaviour under different hypotheses.

In this study examines at a complex system that is made up of two interrelated subsystems, A and B, which are arranged in series, as shown in Figure 1(a). Two identical units in standby redundancy make up Subsystem A, which also includes an imperfect switching device (S) that is in charge of turning on the backup unit as necessary. In contrast, Subsystem B consists of N non-identical components that work in parallel. Subsystem B as a whole enters a condition of lower efficiency if any of its units malfunction. Subsystem B is notable for having a 2-out-ofN:F arrangement.

The transition-state diagram for the system is shown in figure 1(b). The failure rates of other components are assumed to follow an exponential distribution, whereas repair times, with the exception of the switching device, are assumed to follow a general time distribution. The mathematical model, which was developed using the additional variable technique, is solved using the Laplace transform. To improve the model's usefulness, a particular case study and the system's ergodic behaviour have also been examined.

Random elements are taken into consideration when characterising the system's or reliability unit's attributes. As a result, the suggested system model is examined using both classical and Bayesian frameworks in a simulation exercise. The traditional method evaluates the model parameters and reliability features, as well as their maximum likelihood estimates (MLEs), standard errors using the Monte Carlo simulation technique. In the Bayesian framework, the width of the highest posterior density (HPD) intervals and the posterior standard errors are calculated, as well as the Bayes estimates of the parameters and reliability measures. In conclusion, a comparison between the outcomes of the maximum likelihood estimation and Bayesian approaches is provided.

Hospital backup power systems provide a practical illustration of a system with defective switching components:

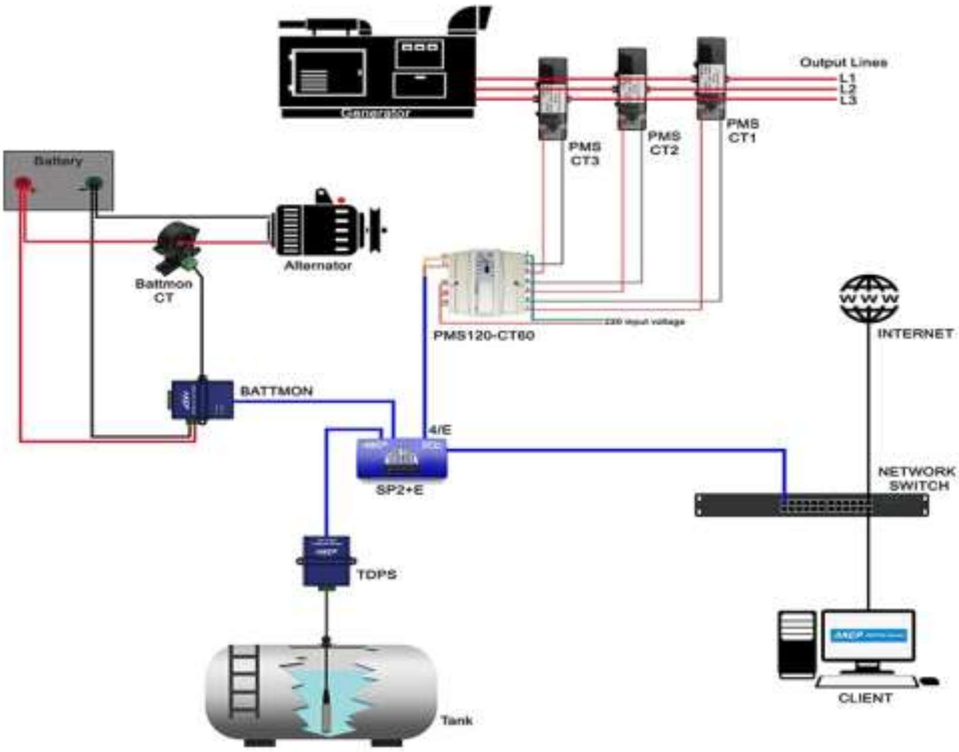
Example: Hospital Backup Power System

In order to maintain regular operations during power outages, hospitals depend on backup generators and uninterruptible power supply (UPS) systems. Usually, these systems consist of:

- **Primary Power Source:** The main electrical grid.
- **Backup Power System:** Generators or battery banks in standby mode.
- **Switching Device (Transfer Switch):** Automatically switches from the main grid to backup power when a failure is detected.

The automatic transfer switch (ATS) isn't always perfect in practical situations, though. Failures could happen because of:

1. **Delay in Switching:** The system may take a few seconds to activate, causing momentary power loss.
2. **Faulty Switching Mechanism:** The ATS may fail due to mechanical wear, electrical faults, or software errors.
3. **Partial Switching:** The backup power may engage but not at full capacity, reducing efficiency.



The failure of the switching device could have serious effects if it interrupts vital medical equipment, such as ventilators, surgical instruments, or intensive care unit monitors. This situation is consistent with reliability modeling's imperfect switching devices, in which a system's inability to smoothly move between operational modes is caused by intrinsic flaws in the switching mechanism.

ASSUMPTIONS

1. Initially, the whole system is new and operable.
2. Failures follow exponential time distribution and nothing can fail from a situation of failure.
3. An inadequate switching mechanism S has been utilised for standby A-unit online.
4. Repairs follow general time distribution and are perfect.
5. Repair facilities are always available to failed components.
6. A-units repaired only if they both fail.
7. The subsystem B is 1-out-of.-N: D and 2-out-of-N: F system. Failures are Sindependent.

LIST OF NOTATIONS

N : Number of units in subsystem B.

j, i : Subscripts, denotes the serial number of B-units $j = 1,2,- - -N$ and $i = 1,2,--(N-1)$.

$\overline{F}(s)$: Laplace transform of function F(t) .
$\sum$: Implies sum over j from 1 to N. λA :
Failure rate of online unit of subsystem A. λ_j, λ_i : Failure rate of first and second B-units.
(1-α) : Failure rate of switching device S. β : Repair rate of switching device S.
$\eta_K(m)\Delta$: First order probability that K[th] failure will be repaired in the time interval (m,m+ □), conditioned that it was not repaired up to the time m.
$P_{0,0,0}(t)$: Pr{that at time t, subsystem A, subsystem B and switching device S are all Operable}.
$P_{1,0,0}(t)$: Pr{ that at time t, one unit of subsystem A has failed}.
$P_{F,0,0}(x,t)\Delta$: Pr{ that at time t, subsystem A has failed}. Elapsed repair time lies in the Interval (x, x+ □).
$P_{0,j,0}(y,t)\Delta$: Pr{that at time t, j[th] unit of subsystem B has failed and so system works in reduced efficiency state}. Elapsed repair time for one B-unit lies within (y, y+ □).
$P_{0,F,0}(z,t)\Delta$: Pr{ that at time t, subsystem B has failed while subsystem A and switching devices are operable}. Elapsed repair time for two B- units lies within(z, z + □).
$P_{1,0,F}(t)$ etc. : Pr{ that at time t, one A-unit and switching device S have failed while B-units are operable}.
$S_k(m)$: □k (m)exp□− □ □k (m)dm□,□ k and m .
L.T. : Laplace transform
M.T.T.F. : Mean time to failure.
$D_k(m)$: □1− $\overline{S}_k$ (m)□/ m, □ k and m .
The transition diagram of the system model is shown in Figure 1(a) and Figure 1 (b)

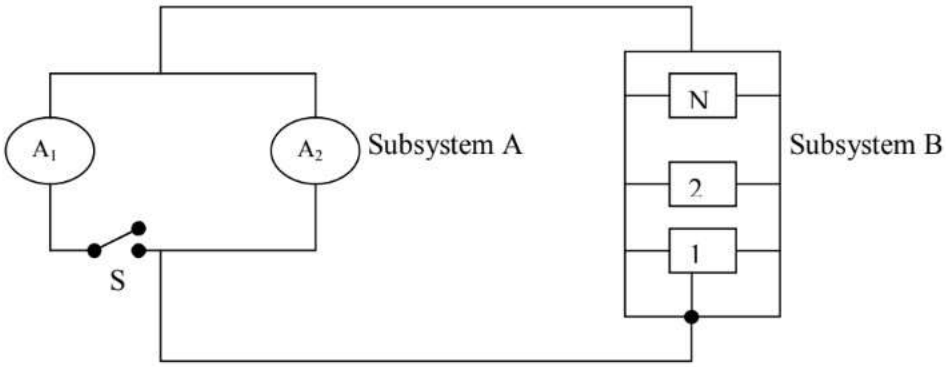


Fig-1(a): System Configuration

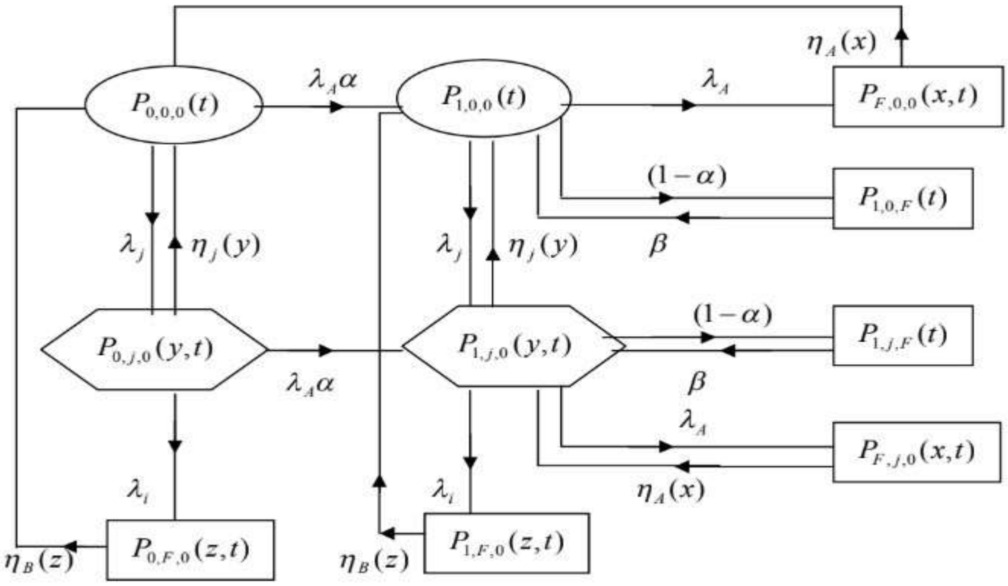


Fig-1(b): Transition-state diagram

FORMATION OF THE MATHEMATICAL MODEL

For the stochastic process under examination, we derive the following set of difference-differential equations using the basic probability consideration and continuity arguments:

$$\left[\frac{d}{dx} + \lambda_{B_1} + \lambda_A\alpha\right] P_{0,0,0}(t) - \int_0^\infty P_{F,0,0}(x,t)\eta_A(x)dx + \sum_{j=1}^{n}\int_0^\infty P_{0,j,0}(y,t)\eta_j(y)dy + \int_0^\infty P_{0,F,0}(z,t)\eta_B(z)dz \quad \ldots(1)$$

$$\left[\frac{d}{dx} + \lambda_A + \lambda_{B1} + (1-\alpha)\right] P_{1,0,0}(t) = \lambda_A\alpha P_{1,0,F}(t) + \beta P_{1,0,F}(t) + \sum_{j=1}^{n}\int_0^\infty P_{1,j,0}(y,t)\eta_j(y)dy + \int_0^\infty P_{1,F,0}(z,t)\eta_B(z)dz \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \eta_A(x)\right] P_{F,0,0}(x,t) = 0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial y} + \frac{d}{dt} + \lambda_{B2} + \lambda_A\alpha + \eta_j(y)\right] P_{0,j,0}(y,t) = 0 \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial z} + \frac{\partial}{\partial t} + \eta_B(z)\right] P_{0,F,0}(z,t) = 0 \quad \ldots(5)$$

$$\left[\frac{d}{dt} + \beta\right] P_{1,0,F}(t) = (1-\alpha)P_{1,0,0}(t) \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \lambda_A + \lambda_{B2} + (1-\alpha) + \eta_j(j)\right] P_{i,j,0}(y,t) = \lambda_A\alpha P_{0,j,0}(y,t) \quad \ldots(7)$$

$$\left[\frac{d}{dt} + \beta\right] P_{1,j,F}(t) = (1-\alpha)P_{1,j,0}(t) \quad \ldots(8)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \eta_A(x)\right] P_{F,j,0}(x,t) = 0 \quad \ldots(9)$$

$\partial\ \partial$

$[\partial z + \partial t + \eta_B]\, P_{1,F,0}(z, t) = 0$ …(10)

Boundary conditions are:

$P_{F,0,0}(0, t) = \lambda_A\ P_{1,0,0}(t)$ …(11)

$P_{0,j,0}(0, t) = \lambda_{B1}\ P_{0,0,0}(t)$ …(12) $P_{0,F,0}(0,$
$t) = \lambda_{B2}\, P_{0,j,0}(t)$ …(13)

$P_{1,j,0}(0,t) = \lambda_{B_1}\ P_{1,0,0}(t) + P_{1,j,F}(t).\beta + \int_0^{\infty} P_{F\,j,0}(x, t)\eta_A(x)dx$ …(14)

$P_{F\,j,0}(0, t) = \lambda_A\ P_{1j,0}(t)$ …(15) $P_{1,F,0}(0,$
$t) = \lambda_{B2}\, P_{1,j,0}(t)$ …(16)

Initial conditions are:

$P_{0,0,0}(0) = 1$, otherwise zero. …(17)

Solution of the Model:

By applying initial conditions (17) to the Laplace transforms of equations (1) through (16), we can get:

$[s + \lambda_{B1} + \alpha\lambda_A]\bar{P}_{0,0,0}(s) = 1 + \int_0^{\infty} \bar{P}_{F,0,0}(x, s)\eta_A(x)dx \ + \sum_{j=1}^{n}\int_0^{\infty}\bar{P}_{0,j,0}(y,s)\eta_j(y)dy$
$+\int_0^{\infty}\bar{P}_{0,F,0}(z,s)\eta_B(z)dz$ …(18)

$[s + \lambda A + \lambda B1 + (1-\alpha)]\bar{P}1,0,0(s) = \lambda A\,\alpha\,\bar{P}0,0,0(s) + \beta\bar{P}_{1,0,F}(s) + \sum_{j=1}^{n}\int_0^{\infty}\bar{P}_{1,j,0}(y,s)\eta_j(j)dy$
$+\int_0^{\infty}\bar{P}_{1,F,0}(z,s)\eta_B(z)dz$ …(19) $[+ s + \eta_A(x)]\, P$

$\partial \bar{x}\ \partial$ $\bar{}_{F,0,0}(x, s) = 0$ …(20)

$[+ s + \lambda_{B2} + \lambda_A\alpha + \eta_j(y)]\, P$

$\partial \bar{y}\ \partial$ $\bar{}_{0,j,0}(y, s) = 0$ …(21)

$-\ [+ s + \eta_B(z)]\, P$

$\partial z\ \partial$ $\bar{}_{0,F,0}(z, s) = 0$ …(22)

$[s + \beta]\bar{P}_{1,0,F}(s) = (1-\alpha)\bar{P}_{1,0,0}(s)$ …(23)

$-\ \ \partial\ \bar{}_{1,j,0}(y, s) = \lambda_A\alpha\,\bar{P}_{0,j,0}(y, s)$ …(24) $[+ s + \lambda_A + \lambda_{B2} + (1-\alpha) + \eta_j(y)]$
P

∂y

$[s + \beta]\bar{P}_{1j,F}(s) = (1-\alpha)\bar{P}_{1j,0}(s)$ …(25)

$[+ s + \eta_A(x)]\, P$

$\partial \bar{x}\partial$ $\bar{}_{F\,j,0}(x, s) = 0$ …(26)

$[+ s + \eta_B(z)]\, P$

$\partial \bar{z}\partial$ $\bar{}_{1,F,0}(z, s) = 0$ …(27)

$\bar{P}_{F,0,0}(0, s) = \lambda_A\bar{P}_{1,0,0}(s)$ …(28)

$\bar{P}_{0,j,0}(0, s) = \lambda_{B1}\bar{P}_{0,0,0}(s)$ …(29)

$\bar{P}_{0,F,0}(0, S) = \lambda_{B2}\bar{P}_{0,j,0}(s)$ …(30)

$\bar{P}_{1,j,0}(0, s) = \lambda_{B1}\bar{P}_{1,0,0}(s) + \bar{P}_{1j,F}(s).\ \beta + \int_0^{\infty}\bar{P}_{F\,j,0}(x, s)\eta_A(x)dx$ …(31)

$\bar{P}_{F\,j,0}(0, s) = \lambda_A\bar{P}_{1,j,0}(s)$ …(32)

$\bar{P}_{1,F,0}(0, s) = \lambda_{B2}\bar{P}_{1,j,0}(s)$ …(33)

Now, simplifying equation (23) and (25), we have

$\bar{P}_{1,0,F}(s) = (\underline{\qquad\qquad}^{1-\alpha}{}_{(s}{}^{)}{}_{+}P^{1}{}_{\beta}{}^{,0},{}_{)}{}^{0(s)}$ …(34)

$\bar{P}_{1,j,F} = (\overline{\quad\quad} 1-\alpha (s) + \bar{P}_{\beta,j},_{0}(s))$ …(35)

Integrate (20) by using boundary condition (28), we get

$\bar{P}_{F,0,0}(x, s) = \lambda_A \bar{P}_{1,0,0}(s) exp.\{-sx - \int \eta_A(x)dx\}$

Integrating this again w.r.t. 'x' from 0 to □, we obtain

$\bar{P}_{F,0,0}(s) = \lambda_A \bar{P}_{1,0,0}(s) \frac{1 - \bar{S}_A(s)}{s}$ or $\bar{P}_{F,0,0}(s) = \lambda_A \bar{P}_{1,0,0}(s) D_A(s)$ ….(36)

Similarly, equation (21) gives by using (29)

$\bar{P}_{0,j,0}(y, s) = \lambda_{B_1} \bar{P}_{0,0,0}(s) exp.\{-(s + \lambda_{B_2} + \alpha\lambda_A)y - \int \eta_j(y)dy\}$

$\Rightarrow \bar{P}_{0,j,0}(s) = \lambda_{B_1} \bar{P}_{0,0,0}(s) D_j(s + \lambda_{B_2} + \alpha\lambda_A)$ …(37)

Now, integrate equation (22) subjected to (30) and (37), we get

$\bar{P}_{0,F,0}(z, s) = \lambda_{B_2} \bar{P}_{0,j,0}(s) exp.\{-sz - \int \eta_B(z)dz\}$

$\Rightarrow \bar{P}_{0,F,0}(s) = \lambda_{B_2} \bar{P}_{0,j,0}(s) D_B(s)$

Or, $\bar{P}_{0,F,0}(s) = \lambda_{B_1}\lambda_{B_2} \bar{P}_{0,0,0}(s) D_j(s + \lambda_{B_2} + \alpha\lambda_A) D_B(s)$ …(38)

Again, equation (26) and (27) gives on integration, subjected to boundary conditions (32) and (33), respectively.

$\bar{P}_{F,j,0}(x, s) = \lambda_A \bar{P}_{1,j,0}(s) exp.\{-sx - \int \eta_A(x)dx\}$

$\Rightarrow \bar{P}_{F,j,0}(s) = \lambda_A \bar{P}_{1,j,0}(s) D_A(s)$ …(39) and

$\bar{P}_{1,F,0}(z, s) = \lambda_{B_2} \bar{P}_{1,j,0}(s) exp.\{-sz - \int \eta_B(z)dz\}$

$\Rightarrow \bar{P}_{1,F,0}(s) = \lambda_{B_2} \bar{P}_{1,j,0}(s) D_B(s)$ …(40)

Equation (31) becomes by using (35) and (39):

$\bar{P}_{1,j,o}(0, s) = \lambda_{B_1} \bar{P}_{1,0,0}(s) + \beta \frac{\quad}{1-(\alpha s)} + \bar{P}_{\beta 1),j,0}(s) + \lambda_A \bar{P}_{1,j,0}(s) \bar{S}_A(s)$ …(41)

Solving equation (24), we have

I.F.$= e^{[s + \lambda_A + \lambda_{B_2} + (1-\alpha)]y + \int \eta_j(y)dy}$

$\therefore \bar{P}_{1,j,0}(y, s) \times I.F. = C + \int \lambda_A \alpha \lambda_{B_1} \bar{P}_{0,0,0}(s) e^{-[s + \lambda_{B_2} + \alpha\lambda_A]y - \int \eta_j(y)dy} \times I.F.\ dy$

$= C + \alpha\lambda_A\lambda_{B_1} \bar{P}_{0,0,0}(s) \int e^{[\lambda_A + (1-\alpha) - \alpha\lambda_A]y}\, dy$

$= C + \alpha\lambda_A\lambda_{B_1} \bar{P}_{0,0,0}(s) \int e^{(1+\lambda_A)(1-\alpha)y}\, dy$

$$= C + \frac{\alpha\lambda_A\lambda_{B_1} \bar{P}_{0,0,0}(s) e^{(1+\lambda_A)(1-\alpha)y}}{(1+\lambda_A)(1-\alpha)}$$

where C is constant of integration. Putting y = 0 & dy = 0, we get

$\bar{P}_{1,j,0}(0, s) = C + \frac{\alpha\lambda_A\lambda_{B_1}}{(1+\lambda_A)(1-\alpha)} \bar{P}_{0,0,0}(s)$

$\therefore C = \bar{P}_{1,j,0}(0, s) - \frac{\alpha\lambda_A\lambda_{B_1}\bar{P}_{0,0,0}(s)}{(1+\lambda_A)(1-\alpha)}$

$\therefore \bar{P}_{1,j,0}(y, s) \times I.F. = \bar{P}_{1,j,0}(0, s) - \frac{\alpha\lambda_A\lambda_{B_1}\bar{P}_{0,0,0}(s)}{(1+\lambda_A)(1-\alpha)}[1 - e^{(1+\lambda_A)(1-\alpha)y}]$

$\therefore \bar{P}_{1,j,0}(y, s) = \bar{P}_{1,j,0}(0, s) e^{-[s + \lambda_A + \lambda_{B_2} + (1-\alpha)]y - \int \eta_j(y)dy}$

$- \frac{\alpha\lambda_A\lambda_{B_1}\bar{P}_{0,0,0}(s)}{(1+\lambda_A)(1-\alpha)} \{e^{-[s + \lambda_A + \lambda_{B_2} + (1-\alpha)]y - \int \eta_j(y)dy} - e^{-[s + \lambda_{B_2} + \alpha\lambda_A]y - \int \eta_j(y)dy}\}$

$\therefore \bar{P}_{1,j,0}(s) = \bar{P}_{1,j,0}(0, s) D_j(s + \lambda_A + \lambda_{B_2} + (1-\alpha))$

$\alpha\lambda$ $\frac{A\ B_1}{(1+\lambda_A)(1-\alpha)}$ λ $\bar{P}_{0,0,0}(s)$
$-\{D_j(s+\lambda_A+$ $\lambda_{B2}+(1-\alpha))-D_j(s+\lambda_{B2}+\alpha\lambda_A)\}$
$\{1-[\beta$ ____ $(s1+-\beta\alpha)+\lambda A\bar{S}A(s)]\ Dj\ (s+\lambda A+\lambda B2+(1-\alpha))\}\ \bar{P}1,j,0(s)=\lambda B\ 1\bar{P}1,0,0(s)Dj\ (s+$
$\lambda A+$
$\alpha\lambda\ \lambda$ $P^{-0,0,0}(s)$ $\frac{A\ B_1}{(1+\lambda_A)(1-\alpha)}$
$\lambda_{B2}+(1-\alpha))$ $+\{D_j(s+\lambda_{B2}+\alpha\lambda_A)-D_j(s+\lambda_A+\lambda_{B2}+(1-\alpha))\}$
$\therefore \bar{P}_{1,j,0}(s)=A(s)\bar{P}_{1,0,0}(s)+B(s)\bar{P}_{0,0,0}(s)$ ….. (42)
where, $A(s)=$ $\{1-[\beta(s\ 1+-\beta\alpha\lambda)B+1\lambda DAjS(\bar{s}+(s\lambda)A]_D+_j\lambda(Bs+2+\lambda(A1+-\lambda\alpha B)2)+(1-\alpha))\}$
____ A
and B(s)= ____ $(1+\lambda\alpha A\lambda)A(\lambda 1B-1\alpha\})D\ \{1j(-s[+\beta\lambda(s1B+-2\beta\alpha+)\alpha\lambda\lambda A)S-$
$D(sj)(]sD+j\lambda(sA++\lambda\lambda AB+2+\lambda B(12-+\alpha(1)-)\}\alpha))\}$
$+A$ A

Simplifying equation (19) subjected to relevant relations, we obtain

$(s+\lambda_A+\lambda_{B1}+(1-\alpha))\bar{P}_{1,0,0}(s)=\lambda_A\alpha P^{-0,0,0}(s)+\frac{\beta(1-\alpha)\bar{P}_{1,0,0}(s)}{(s+\beta)}+\lambda_{B_2}[A(s)\bar{P}_{1,0,0}(s)+$
$B(s)\bar{P}_{0,0,0}(s)]S_B(s)+\sum_{j=1}^{n}\bar{P}_{1,j,0}(0,s)S_j\left(s+\lambda_A+\lambda_{B_2}+(1-\alpha)\right)+$ $-$ α ____
$\lambda(1A+\lambda\lambda BA1)P(\bar{0}1,-0,0\alpha()s)\{\bar{S}_j(s+$
$\alpha\lambda_A+\lambda_{B_2})-S_j(s+\lambda_A+\lambda_{B_2}+(1-\alpha))\}\therefore \bar{P}_{1,0,0}(s)=\frac{C(s)}{0(s)}\bar{P}_{0,0,0}(s)$ (43)

Lastly, solving equation (18) subjected to relevant relations, we have
$\bar{P}_{0,0,0}(s)=\frac{1}{E(s)}$

Therefore, finally we obtain the following L.T. of various state probabilities of fig-1 (b), in terms of E(s):

$\bar{P}_{0,0,0}(s)=\frac{1}{E(s)}$ (44)
$\bar{P}_{1,0,0}(s)=\frac{C(s)}{0(s)E(s)}$ (45)
$\bar{P}_{F,0,0}(s)=\frac{\lambda_A C(s)}{0(s)E(s)}D_A(s)$ (46)
$\bar{P}_{0,j,0}(s)=\frac{\lambda_{B1}D_j(s+\alpha\lambda_A+\lambda_{B2})}{E(s)}$ (47)
$\bar{P}_{0,F,0}(s)=\frac{\lambda_{B1}\lambda_{B2}D_j(s+\alpha\lambda_A+\lambda_{B2})D_B(s)}{E(s)}$
(48)
$\bar{P}_{1,0,F}(s)=\frac{(1-\alpha)C(s)}{(s+\beta)0(s)E(s)}$ (49)
$\bar{P}_{1,j,0}(s)=\frac{1}{E(s)}[A(s)\frac{C(s)}{0(s)}+B(s)]$ …(50) $\bar{P}_{1,j,F}(s)$
$=\frac{(1-\alpha)}{(s+\beta)E(s)}[A(s)\frac{C(s)}{0(s)}+B(s)]$ …(51)
$\bar{P}_{F,j,0}(s)=\frac{\lambda_A D_A(s)}{E(s)}[A(s)\frac{C(s)}{0(s)}+B(s)]$ …(52)
$\bar{P}_{1,F,0}(s)=\frac{\lambda_{B2}D_B(s)}{E(s)}[A(s)\frac{C(s)}{0(s)}+B(s)]$ …(53)

where

A(s) = $\{1-[\beta(s\ 1+-\beta\alpha\lambda)B+1\lambda D_A jS(_A s+(s\lambda)A]D+j\lambda(Bs+2+\lambda(A1+-\lambda\alpha B)2)+(1-\alpha))\}$
…(54)
B(s) = $\alpha\lambda A\{1\lambda-B[1\beta\{(Ds1+j-\beta(\alpha s)+\alpha\lambda A+$
$(\lambda sB)2]D)-j(Dsj+(\lambda s+A+\lambda A\lambda+B2\lambda+B(21+-(1\alpha-))\alpha\})\}$
…(55) $+\lambda ASA$

C(s)= $\lambda A\alpha+\lambda_{B_2}B(s)S_B(s)+\sum_{j=1}^{\infty}\frac{A\ B_1}{(\ \ )}\bar{S}_j\left(s+\right.$ $(1+\alpha\lambda\lambda A)\lambda\ 1-\alpha\ \ \alpha\lambda A+\lambda B2)$

$+\sum_{j=1}^{n}\left[\frac{\beta(1-\alpha)B(s)}{(s+\beta)}+\right.$ $\lambda A \bar{S}A(s)B(s)-($______$1+\alpha\lambda\lambda AA)\lambda(B 11-\alpha)]$ $S j^{-}(s+\lambda A+\lambda B 2$ $+(1-\alpha))$… (56) $0(s)=s+\lambda A+\lambda B 1+($______$1s-+\alpha\beta)s-\lambda B 2A(s)S\bar{B}(s)$

$-\sum_{j=1}^{n}\left[\lambda_{B_1}+\frac{\beta(1-\alpha)B(s)}{(s+\beta)}+\right.$ $\lambda_A A(s)\bar{S}_A(s)]\bar{S}_j(s+\lambda_A+\lambda_{B2}+1-\alpha)$ …(57)

and E(s)= $s+\alpha\lambda_A+\lambda_{B_1}-\frac{\lambda_A C(s)}{0(s)}S_A(s)-\sum_{j=1}^{n}\lambda_{B_1}S_j(s+$ $\alpha\lambda_A+\lambda_{B2})-\sum^{n}{}_{j=1}$
$\lambda_{B1}\lambda_{B2}D_j$
$(s+\alpha\lambda_A+\lambda_{B2})S_B^{-}(s)$ …(58)

It worth noticing that Sum of equations (44) through (53) = 1/s (59)

Ergodic System Behaviour Using Abel's Lemma, viz., lim $P(t)=\lim s\ \bar{P}(s)=P$ (say), provided the limit on L.H.S
$t\to\infty$ $s\to 0$
exists, we obtain the following ergodic behaviour of consideration system from equations (44) through (53):

1
$P_{0,0,0}={}_E$______$'(0)$ …(60)
$C(0)$
$P_{1,0,0}={}_o$______$(0)E'(0)$ …(61)
$P_{F,0,0}={}_o$______$\lambda_{(0}{}^{A})^{C}{}_{E}{}^{(,0}{}_{(0)}M_A$ …(62)
$P_{0,j,0}=\lambda$______$^{B\,1}D_j{}_E{}^{(\alpha,(\lambda_0{}^A)^{+\lambda B\,2)}}$ …(63)
$P_{0,F,0}=\lambda$______$^{B\,1\lambda B\,2}D_{E}{}^{j,(\alpha_{0)}\lambda A+\lambda B\,2)}M_B$ …(64)
$(1-\alpha)C(0)$
$P_{1,0,F}={}_o$______$(0)E'(0)\beta$ …(65)
1 $A(0)C(0)$
$P_{1,j,0}={}_E$______ ______$'(0)$ [$o(0)$ + $B(0)]$
…(66)
$(1-\alpha)$ $A(0)C(0)$
$P_{1,j,F}={}_{\beta E}$______ ______$'(0)$ [$o(0)$ + $B(0)]$
…(67)
$P_{F,j,0}=\lambda_E{}^{A,M}{}_{(0}{}^{A}{}_{)}[A$______$(_o{}^{0)}(^C{}_{0)}{}^{(0)}+B(0)]$ …(68)
$P_{1,F,0}=\lambda$______ ______$_E{}^{B\,2,}(^M{}_{0)}{}^{B}\,[A(_o{}^{0)}(^C{}_{0)}{}^{(0)}+B(0)]$ …(69)
where , $E'(0)=[\frac{d}{ds}E(s)]_{s=0}$
$M_i=-S_i^{-}(0)$ = Mean time to repair i^{th} subsystem

PARTICULAR CASE

When repairs ~ exponential time distribution

In this particular scenario, we can obtain the L.T. of transition-state probabilities of fig-1(b) from equations (44) through (53) by setting:

$\bar{S}_j(j)=\frac{\eta_i}{}$, $\forall_{j+\eta}$i i and j.

RELIABILITY AND M.T.T.F. OF THE SYSTEM

From equation (44), we get

$$\bar{R}(s) = \frac{1}{s + \alpha\lambda_A + \lambda_{B1}}$$

$$\therefore R(t) = exp.\{-(\lambda_{B1} + \alpha\lambda_A)t\} \qquad \dots(70)$$

and M.T.T.F. $= \int_0^\infty R(t)\,dt$

$$= \frac{1}{\alpha\lambda_A + \lambda_{B1}} \qquad \dots(71)$$

PROFIT FUNCTION FOR THE SYSTEM

The system under consideration's profit function is provided by

$$P(t) = C_1 \int_0^t P_{up}(t)dt - C_2 t \qquad \dots(72)$$

where, and C_1 are C_2 the revenue and repair costs per unit time, respectively.

NUMERICAL COMPUTATION

For a numerical computation, let us consider the values $\lambda_A = 0.003$, $\lambda_{B2} = 0.05$, $\alpha = 0.6$ and $t = 0,1,2$ …………Using these values in the equations (70) and (71), we compute the tables-1, 2, 3 and 4. Corresponding graphs have been shown in fig-2, 3, 4 and 5, respectively.

RESULTS AND DISCUSSION

Table -1 gives the value of reliability function for different t and its graph has been drawn in fig-2. We observe that reliability of considered system decreases approximately in constant manner. Table-2, 3 and 4 gives the values of M.T.T.F. with different failure rates and the corresponding graphs have been shown in fig-3, 4 and 5 respectively. We observe that value of MTTF decreases smoothly as we make increase in the values of □ and λ_A . But when we increase the failure rate λ_{B1}, , the value of MTTF decreases catastrophically in the beginning and thereafter it decreases constantly.

t	R(t)
0	1
1	0.9654
2	0.89342
3	0.76284
4	0.71234
5	0.64328
6	0.60623
7	0.54197
8	0.51297
9	0.43874
10	0.40234

Table.1: reliability function for different t

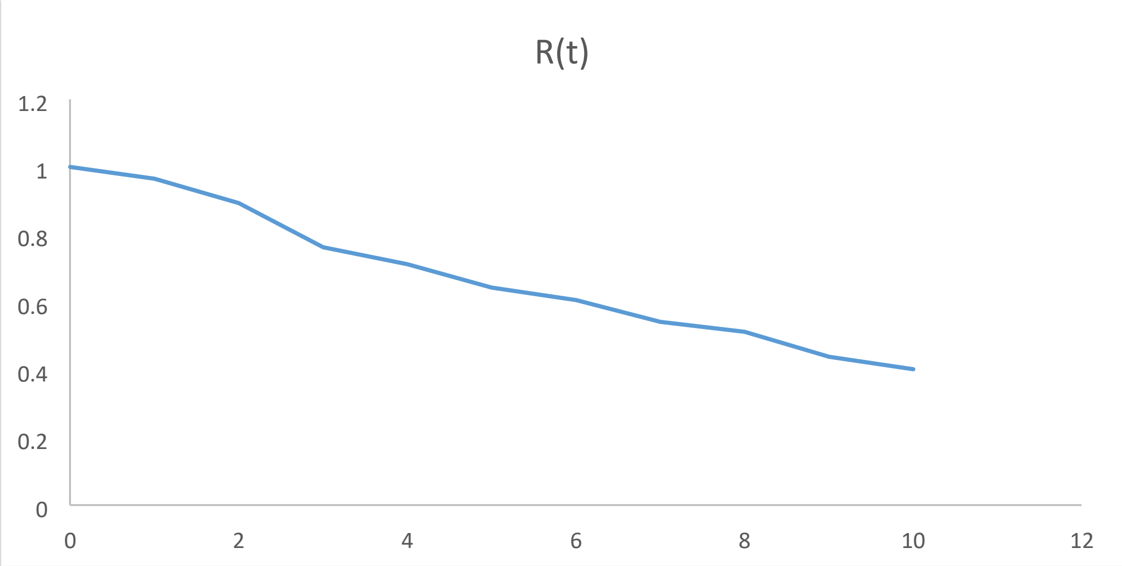


Fig.2

Interpretation:

1. **Reliability Decreases Over Time**: The system starts at full reliability (R(0)=1) but steadily declines, reaching about **40% reliability at t=10**.
2. **Potential Exponential Decay**: The pattern suggests a typical **exponential reliability decline**, which is common in electrical and mechanical systems subjected to wear, resistance changes, or material fatigue.
3. **Impact of Imperfect Switching**: The failures could stem from **contact wear, increased resistance, leakage currents, or thermal degradation** in the switching components.
4. **System Maintenance and Design Considerations**: To prolong system lifespan, **better switch materials, redundancy, or preventive maintenance strategies** should be implemented.

α	MTTF
0.1	19.56072
0.2	18.77354
0.3	17.56984
0.4	15.34567
0.5	13.23478
0.6	12.11245
0.7	11.24786
0.8	10.98345
0.9	9.23456
1	8.34256

Table 2 MTTF for different value of

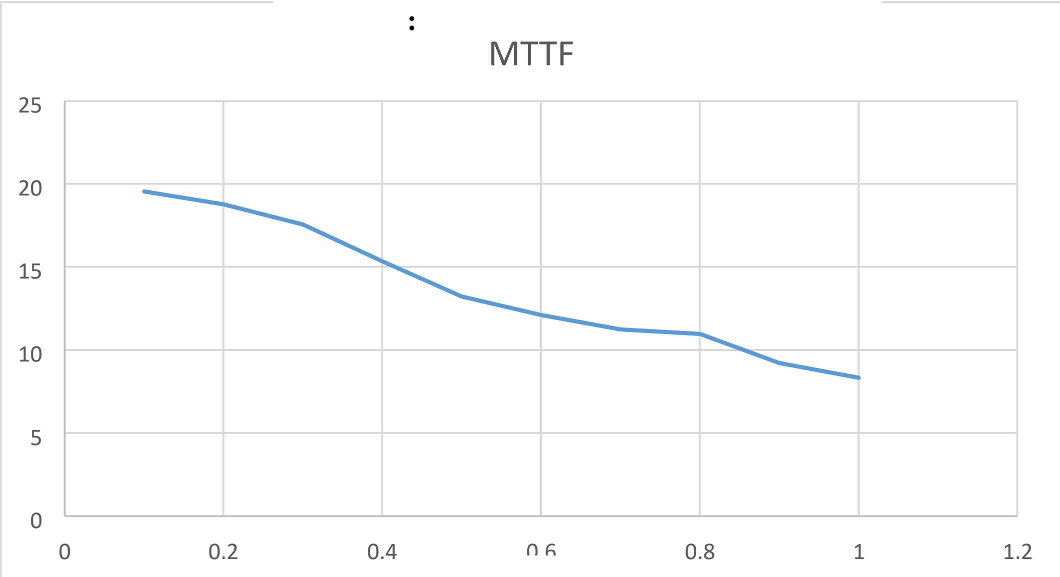


Fig.3

- As the failure rate (α) increases, the MTTF decreases.
- This trend is expected: higher failure probabilities lead to more frequent failures, reducing the time until a failure occurs.
- Lowering the failure rate significantly increases system reliability, suggesting that efforts to improve switching device performance would yield substantial reliability gains.
- Understanding this relationship helps in designing more reliable systems by focusing on minimizing the failure rate of critical components.

λA	**MTTF**
0.001	19.5678
0.002	18.24567
0.003	18.11294
0.004	17.95478
0.005	17.32897
0.006	16.92345
0.007	16.34985
0.008	16.11748
0.009	15.89744

Table.3: MTTF for different value of λ_A

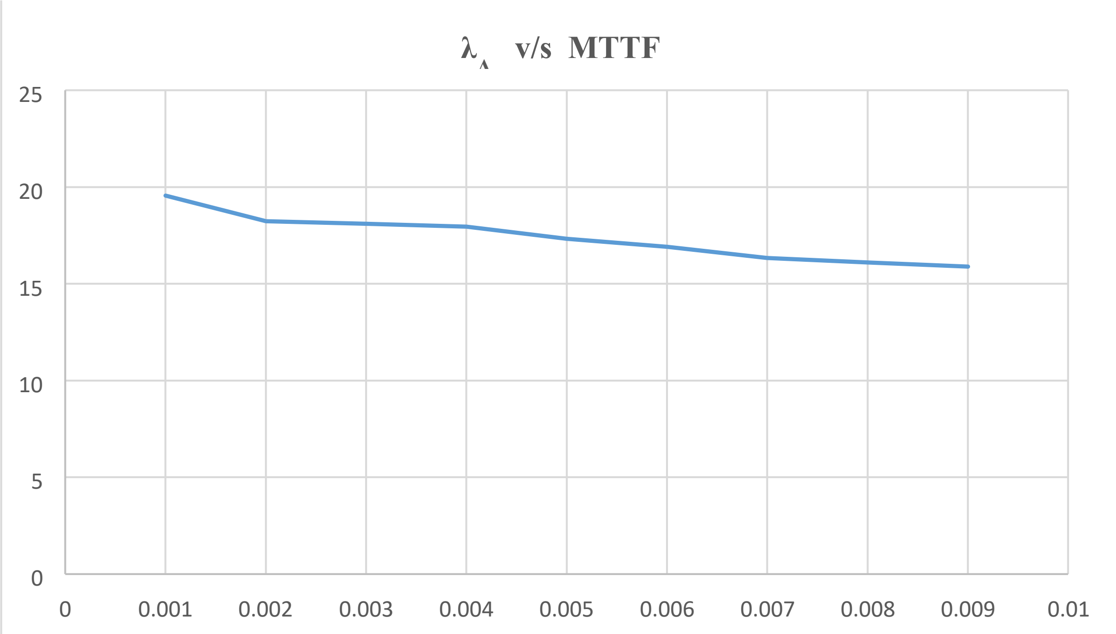


Fig.4.

The analysis of the failure rate (λ) and Mean Time to Failure (MTTF) for the online unit of Subsystem A demonstrates a clear inverse relationship between these parameters. As the failure rate increases from 0.001 to 0.009, the MTTF decreases from 19.5678 to 15.89744. This trend indicates that higher failure rates lead to a reduction in system reliability and longevity.

The observed pattern aligns with the principles of reliability theory, suggesting that the MTTF is significantly influenced by the failure rate of the subsystem. Therefore, minimizing the failure rate is crucial for enhancing the operational lifespan and reliability of Subsystem A. Future efforts should focus on improving design robustness and implementing effective maintenance strategies to achieve lower failure rates, thereby ensuring a higher MTTF and improved system performance.

λ B1	**MTTF**
0.01	90.4356
0.02	75.3897
0.03	66.8234
0.04	43.9847
0.05	31.3345
0.06	20.6789
0.07	15.7893
0.08	11.8934

0.09	9.3456
1.11	5.3183

Table.4: MTTF for different value of λ_{B1}

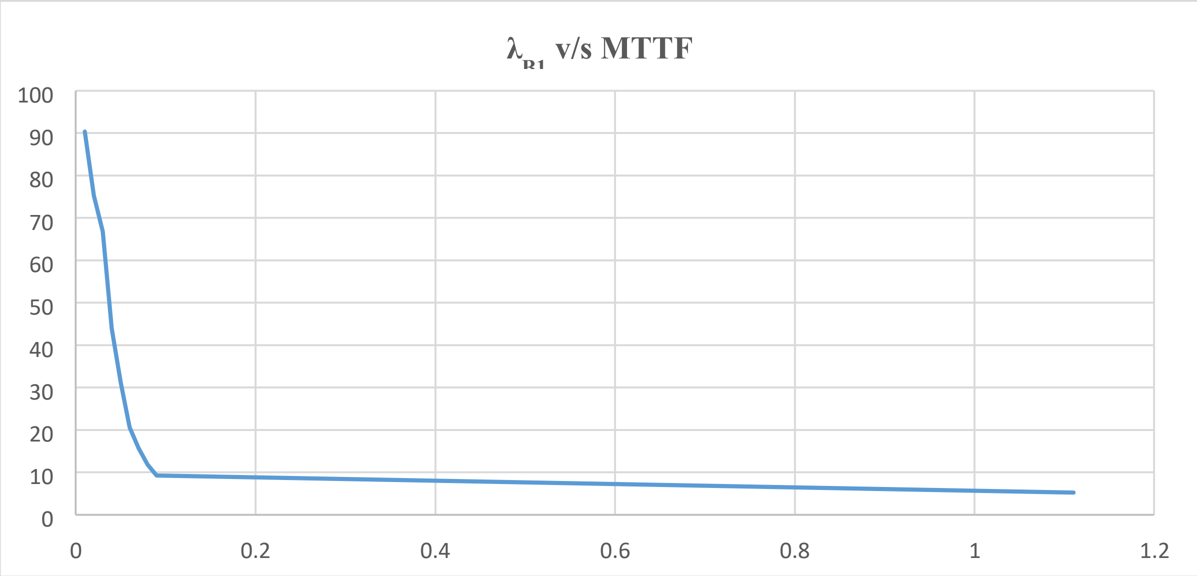


Fig.5.

The analysis of the failure rate (λ_{B1}) and Mean Time to Failure (MTTF) for the first and second B-units reveals a significant inverse relationship between these parameters. As the failure rate increases from 0.01 to 1.11, the MTTF sharply declines from 90.4356 to 5.3183. This trend indicates that higher failure rates substantially reduce the reliability and operational lifespan of the B-units. The data suggests that even moderate increases in the failure rate cause a notable decrease in MTTF, highlighting the sensitivity of B-units to reliability issues. For instance, a rise in (λ_{B1}) from 0.01 to 0.05 leads to a drop in MTTF from 90.4356 to 31.3345, emphasizing the impact of failure rate on system performance. To improve the reliability of B-units, it is essential to focus on reducing the failure rate through enhanced design, regular maintenance, and the use of higher-quality components. Implementing these measures can help extend the MTTF, ensuring better performance and longevity of the B-units.

Bayesian Approach of MTSF and Profit function analysis:

Classical estimation:

In this section, we propose a Bayesian technique for the situation when the repairable system's failure rate and repair rate □ □$_{A\ B'}$ i and □are unknown and need to be determined from suitable prior distributions.

Let U□□1=(U ,U11 12,....U1n1),U2 =(U21,U22,....U2n2) and U□3 =(U31,U32.....U3n3)be the random samples drawn from there pdfs. All samples come from exponential populations. The following formula can be used to obtain the likelihood function of □$_{A\ B,}$□$_{i}$ and □:

□$_A$ □ Gamma a ,b($_{1\ 1}$)

□$_{Bi}$ □ Gamma a ,b($_{2\ 2}$)

□□ Gamma a ,b(3 3)

Here, a and b_i i (i =1,2,3) respectively denote the scale and shape parameters.

ML Estimation:

Then the joint likelihood function is:

L = L U ,U ,U(□□□1 2 3 □ □A, Bi ,□ =□)An1□Bin2□n3e−(□ +□A 1T BiT2+□T3)

[73]

n_i

Where, T_i =□U , i_{ij} =1,2,3 is sufficient for □ □A B' i and □.

j 1=

Using the standard maximisation likelihood method, the ML estimates state that (□ □ˆA B, ˆ i and □ˆ)the parameters are □ □A B' i and □ are:

□ =ˆA n T1 1

□ =ˆBi n T2 / 2 [74]

□=ˆ n T3 3

The MTSF and Profit function estimates, say Mˆ and Pˆ , can now be estimated utilising the invariance property of ML estimates. The sampling distribution for asymptotic:

□ □ −□ˆA A □

□ □

□□ −□ˆBiBi □□ $N_3(0, I^{-1})$ [75]

□ □ □ □−□ˆ

□

□ □

Where I denotes the Fisher information matrix with diagonal elements

$I_{11} = n_1/\lambda^2$, $I_{22} = n_2/\lambda^2$, $I_{33} = n_3/\lambda^2$ and non- diagonal elements are all zero.

□A □Bi □

Gamma distribution with G a ,b(1 1) density is a suitable prior distribution for $□_A$.

p($□_A$)= b1a1 □Aa 1 b1− −e1 □A ; □ □A 0

[76]

□a1

denoted by □A □G a ,b(1 1), where □a_1 is the gamma function and a_1□0,b_1□0 are specified parameters.

E(□ =A) a1 1b and Var(□ =A) a b1 12 .

Bayesian Estimation:

According to Bayesian theory and using equation (73) and (76), the posterior distribution of $□_A$ given T_1 is given by

(T b1+ 1)n a1 1+ □An a 11 1+ −e−□A 1 1(T b+)

□ □(A 1 T)= ―――――――――――――――――― [77]

□ +(n a1 1)

where is a gamma distribution's density with the parameters (n_1+ a_1) and (T_1+ b_1).

The prior distributions for $□_{Bi}$ and □ are similarly assumed to be G a ,b(i i), i = 2,3 . We assume that each system parameter's prior distribution is independent.

As a result, it is assumed that the prior distributions of each parameter made up the joint distribution of □ □A B' i and □. Using a similar process, we are able to derive the joint posterior distribution provided by,

$$(\square\ \square\ \square_{A}, {}_{B}i\ |T, T, T_{1}\ \ {}_{2}\ \ {}_{3}) \square\square G\,n({}_{i} + a, T_{ii} + b_{i}) \quad \prod_{i=1}^{3} \tag{78}$$

The posterior distributions of MTSF and Profit function for the repairable system can be obtained by taking MTSF and Profit function, and from these distributions, the posterior mean (PM) and highest posterior density (HPD) intervals can be computed. The sample means of the respective draws are used as the Bayes estimates of the parameter and reliability characteristics under the assumption of a square error loss function. The 'boa' package of R-software has been used to determine the width of HPD intervals.

Simulation study and Comparisons:

A brief numerical research is also carried out to analyse the behaviour of the suggested system. The following measures are used to compare the effectiveness of the Bayes and ML estimating methods:

- Standard error (SE)/ Posterior standard error (PSE)
- Width of the confidence interval (CI)/ highest posterior density (HPD) interval.

From exponential distribution, random samples of sizes n = = = =n_1 n_2 n_3 100 were generated and after that, ML estimates for the model's parameters, MTSF, and profit function were determined. We created four data sets with n=30, 50, 75, and 100 observations each, and we used these data sets to test the MLE and Bayes estimates' sequences of parameters for their stationary distributions using various starting values. It was noted that all Markov chain records reached their stationary state after a few burn-in periods. The MCMC runs of the parameters □ □$_{A\,B}$, i and □ are plotted in fig.

Bayesian estimation of the parameters we take gamma priors have been obtained by setting the values of prior parameters as E(□ =$_A$) a_1/b_1 ,E(□ =$_{Bi}$) a_2 $_2$b , E(□ =) a_3 $_3$b The results of the simulation study were summarized in Tables (5-6):

Comparison Result of Bayesian and Classical estimation:

Table 5: Estimates of MTSF for various values of □ and □$_A$=0.24

□	□**Bi**	True MTSF	MLE MTSF	Bayes MTSF (Gamma Prior)	SE	PSE	CI width	HPD width
0.01	0.37	109.57	194.76	186.43	21.71	19.81	85.1	42.55
0.025	0.37	70	63.53	61.32	8.39	5.87	22.99	11.5
0.034	0.37	54.79	60.29	55.84	8.36	4.58	17.95	8.97
0.075	0.37	34.63	45.74	37.02	8.46	2.93	11.48	5.74
0.1	0.37	28.45	37.69	31.58	9.7	2.78	10.82	5.41

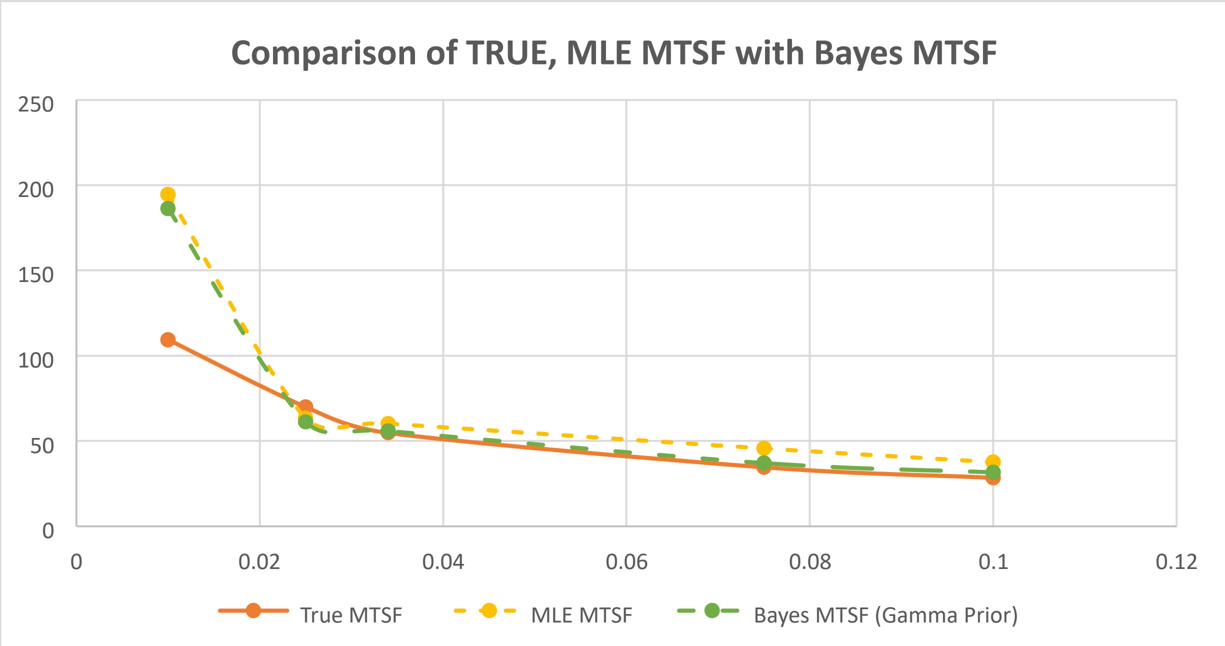


Fig.6.

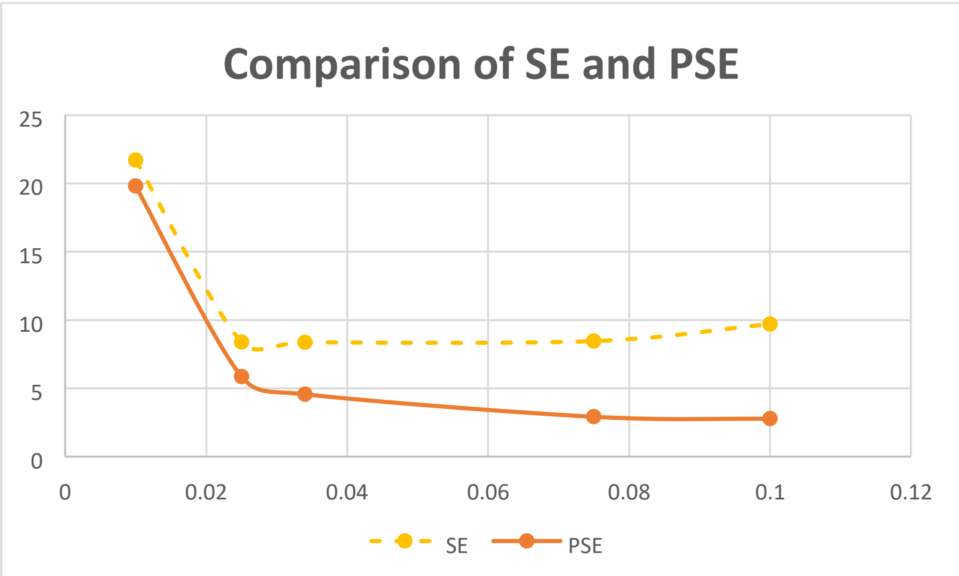


Fig.7.

The above table and figure show the following conclusions:

As the failure rate α increases:

- Both MLE and Bayesian estimators become more accurate and more precise.
- Bayesian method is preferred, especially for small a1 values, because it reduces bias and improves stability.
- PSE is higher at low failure rates (like 0.01), indicating less reliable estimation.
- Estimation error and confidence interval widths shrink, suggesting more reliable predictions at higher failure intensities.

Table 6: Estimates of Profit for various values of □ and $^{□}{}_{A}$=1.64

□	□**Bi**	True Profit	MLE Profit	Bayes Profit (Gamma Prior)	SE	PSE	CI width	HPD width
0.01	0.45	0.9436	0.8945	0.9126	0.03587	0.02345	0.0427	0.0303
0.025	0.45	0.8384	0.7389	0.8183	0.02225	0.01284	0.1153	0.0654
0.034	0.45	0.7238	0.6342	0.7038	0.01863	0.01274	0.1578	0.0792
0.075	0.45	0.6586	0.4936	0.6237	0.01437	0.01134	0.1743	0.0678
0.1	0.45	0.4326	0.3548	0.4027	0.1289	0.01073	0.1789	0.05437

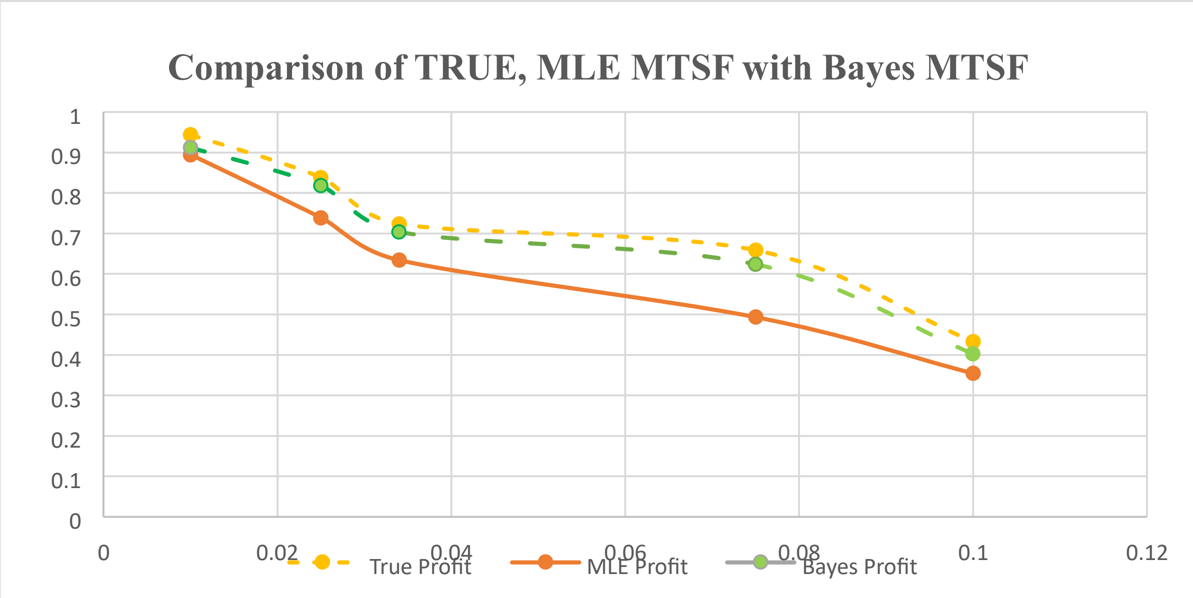


Fig.8.

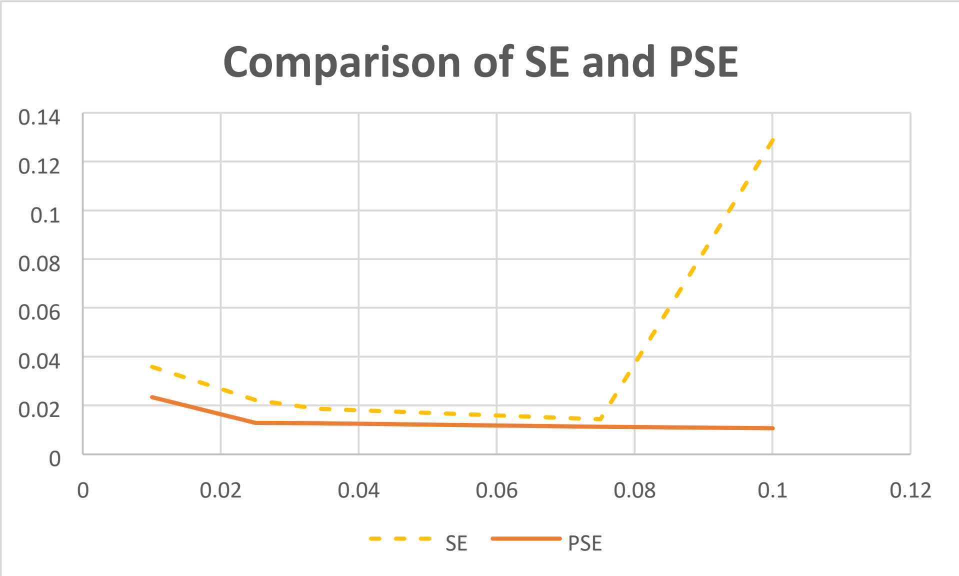


Fig.9.

Conclusion:

- Both MLE and Bayes profit estimates decrease as the failure rate (□) increases.
- Bayesian estimation (with Gamma prior) tends to provide estimates closer to the True Profit when compared to MLE, especially at higher failure rates.
- The SE values are small across all cases, suggesting that both estimation methods are statistically stable and have low variability. PSE values are extremely small (almost near zero), indicating a very high precision in the estimation of profits.
- Both CI Width and HPD Width tend to increase slightly as failure rate increases.
- However, Bayesian HPD intervals are narrower than classical confidence intervals, indicating more certainty around Bayesian estimates.
- Narrower HPD widths suggest that Bayesian methods are more efficient and provide more informative intervals compared to MLE-based methods.
- As failure rate increases (from 0.01 to 0.10), the estimated profits decrease sharply.
- Bayesian estimation remains slightly more stable and trustworthy than MLE at higher failure rates.

Future aspects of this study:

In order to improve reliability assessment of systems with defective switches, future research will try to create more realistic models, incorporate dynamic behaviour, optimise maintenance plans, and make use of contemporary computational approaches.

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