International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Exposing Circuit Theory Textbook Power Calculation Flaws and the Role of the Power Division Theorem

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Abstract:
Standard electrical engineering curricula traditionally state that the Superposition Theorem cannot be directly applied to power evaluations due to the non-linear, quadratic nature of power expressions. This Correspondence addresses this long-standing pedagogical limitation by introducing a systematic allocation framework via the Power Division Theorem (PDT). By decomposing cross-product interaction terms, the PDT establishes that superposition principles can indeed govern individual power-sharing dynamics within linear networks. The proposed formulation linearly maps branch losses to respective source currents, resolving tracking ambiguities in multi-source configurations. The Power Division Theorem (PDT) overcomes traditional limitations in power analysis by demonstrating that source-to-load allocation via linear superposition is identical to direct calculation using the structural current ratio.

Keywords: Circuit theory, linear networks, loss allocation, Power Division Theorem, superposition..

I. Introduction:

The Superposition Theorem serves as a foundational axiom in linear network analysis, simplifying multi-source circuit computations. Despite its utility, standard pedagogical literature emphasizes a structural limitation: the theorem's apparent invalidity when calculating power dissipation. This limitation stems directly from the quadratic relation:

$$P = I^2 \times R \tag{1}$$

While the total aggregate branch power is readily computable, tracking the proportional contribution or "share" of individual independent sources within a common element remains unaddressed in standard textbook treatments. Standard introductory circuit analysis textbooks systematically present a methodological gap when addressing power and line loss calculations in multi-source linear networks. Hayt et al. correctly state that superposition cannot be directly applied to power, but the text fails to account for the necessary cross-term allocation when calculating power loss from multiple sources. This limitation prevents the accurate distribution of total line loss between individual sources by completely ignoring the mutual interaction term ($2\times I_1\times I_2\times R$) [1,2,3,4,5].

By failing to isolate and distribute the quadratic expansion's cross-terms, all three methodologies restrict modern, granular power flow analysis. The Power Division Theorem (PDT) directly resolves this widespread textbook limitation by introducing proportional allocation coefficients PDT [6,7] systematically linearizes and distributes the mutual interaction power, providing the missing mathematical framework needed to successfully merge superposition with power loss calculations.

II. Mathematical Derivation And Correction

Consider a linear, time-invariant resistor R carrying a total net current I_{total}. The current is driven by N independent sources acting simultaneously within the network.

A. The Traditional Impasse

$$I_{total} = \sum_{k=1}^{N} I_k (2)$$

According to the principle of linear current superposition, the net branch current is expressed as:
The total power dissipation within the resistive element is obtained by squaring the aggregated current:

$$P_{total} = \left(\sum_{k=1}^{N} I_k\right)^2 \times R (3)$$

For a dual-source configuration (N = 2), expanding (3) yields:

$$P_{total} = (I_1 + I_2)^2 R = I_1^2 R + I_2^2 R + 2 I_1 I_2 R (4)$$

In conventional literature, the cross-product interaction term $2 \times I_1 \times I_2 \times R$ is typically treated as unallocable to either individual source. Consequently, standard texts conclude that superposition cannot be applied directly to power quantities.

B. The PDT Mathematical Overwrite

The PDT resolves this ambiguity by allocating the interaction term proportionally based on each source's relative contribution to the net current. The definitive power share attributed to source k is defined as:

$$P_{share,k} = I_{total}^2 \times R \times \frac{I_k}{I_{total}} = I_k \times I_{total} \times R (5)$$

To validate this allocation framework using superposition logic, the individual power shares of all N active sources are summed:

$$\sum_{k=1}^{N} P_{share,k} = \sum_{k=1}^{N} (I_k \times I_{total} \times R) (6)$$

Factor out the constant terms:

$$\sum_{k=1}^{N} P_{share,k} = I_{total} \times R \times \sum_{k=1}^{N} I_k (7)$$

Substituting the identity from (2) back into the summation results in:

$$\sum_{k=1}^{N} P_{share,k} = I_{total} \times R \times I_{total} = I_{total}^2 \times R = P_{total} (8)$$

The summation matches total network power exactly. Superposition holds true for power. The prerequisite is using the linear PDT interaction property.

III. Numerical Verification

To verify the formulation, consider a single branch network under the following parameters:

Resistor value: R = 2 Ω

Source 1 current contribution: $I_1 = 5$ A
Source 2 current contribution: $I_2 = -2$ A
Net branch current: $I_{total} = 3$ A

A. Traditional Method Comparison

The true aggregate power dissipated by the resistor is:

$$P_{total} = (3A)^2 \times 2\Omega = 18\ W (9)$$

Evaluating the sources independently yields isolated power values of:

$$P_1 = (5A)^2 \times 2\ \Omega = 50\ W \qquad (10)$$

$$P_2 = (-2A)^2 \times 2\ \Omega = 8\ W (11)$$

$$50 + 8 = 58 \neq 18\ W (12)$$

Because the linear summation of these isolated terms does not equal the true total power, conventional analysis dismisses the applicability of superposition:

B. Proposed PDT frame work Calculation

Applying the corrected PDT expression from (5) yields the following individual allocations:

$$P_{share,1} = (5\ A) \times \{3\ A) \times (2\Omega) = 30\ W (13)$$

$$P_{share,2} = (-2\ A) \times \{3\ A) \times (2\Omega) = -12\ W (14)$$

Shares yields

$$P_{sum} = 30\ W + (-12\ W) = 18\ W (15)$$

The power balance matches the true network physical conditions perfectly. Source 1 delivers a net active power share of 30 W, while Source 2 absorbs 12 W via reverse power flow. The net systemic dissipation remains exactly 18 W, confirming the mathematical integrity of the theorem..

IV. Multi-Source Multi-Load Network With Power Division Theorem (PDT) Verification

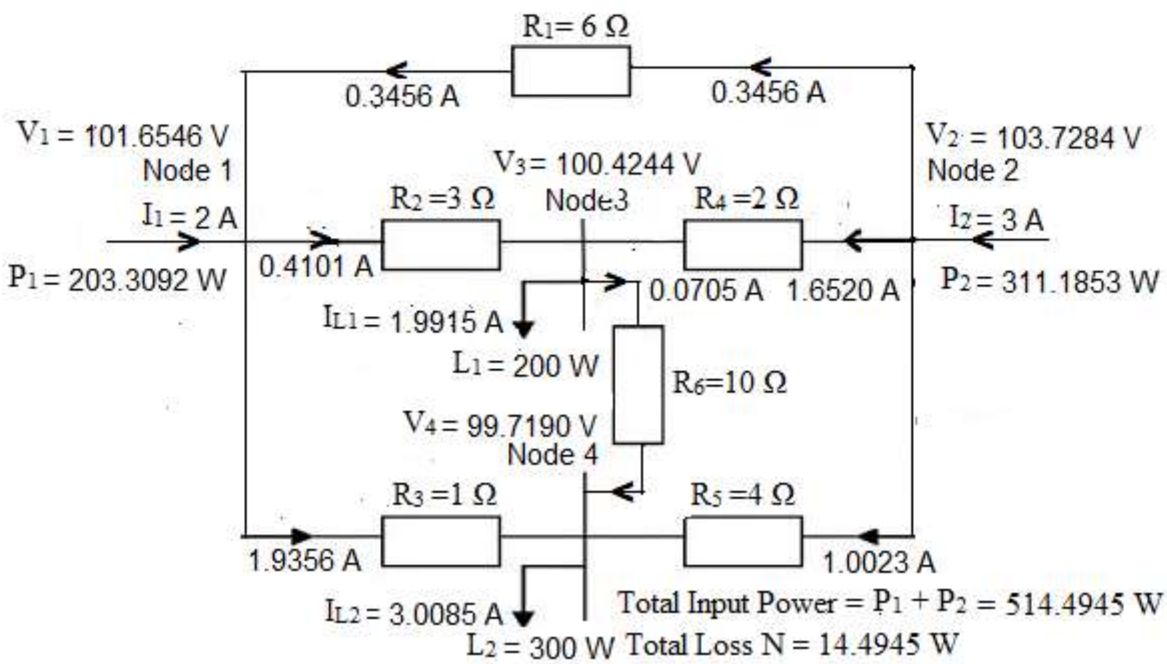

Figure1 DC System At a Particular Operating Point

A six-resistor, two-source, two-load, four nodes DC network designed to validate the applicability of superposition and the Power Division Theorem. The schematic (Figure.1) defines nodal potentials, branch currents (b_1-b_6), source input currents (I_1,I_2), and load parameters (I_{L1}, I_{L2}, L_1, L_2) to analyze energy distribution from independent sources to consumer endpoints. The network environment relies on two independent DC voltage sources, V_1 and V_2, which drive input currents I_1 and I_2 into the entry nodes 1and 2, yielding total source input powers of P_1 and P_2 respectively.

The internal branch dynamics are fully characterized by branch currents (b_1, b_2, b_3, b_4, b_5 and b_6), which establish the essential nodal voltage profiles across the mesh. Termination endpoints are defined by two consumer loads, designated as L_1 and L_2. The operational performance of these loads is continuously monitored via their respective load currents (I_{L1}, I_{L2}) and absolute load power extractions *(*L_1, L_2).

Branch	Source 1 $I_1 = 2.0$ A $I_2 = 0.0$ A Ampere	Source 2 $I_1 = 0.0$ A $I_2 = 3.0$ A Ampere	Total Branch(b_i) Current Ampere	Calculated Branch (b_i) Current At The Operating Point Ampere
1-2	0.2058	-0.5514	-0.3456	-0.3456
1-3	0.5467	-0.1366	0.4101	0.4101
1-4	1.2475	0.6880	1.9355	1.9356
2-3	0.2026	1.4494	1.6520	1.6520
2-4	0.0032	0.9992	1.0024	1.0023
3-4	-0.0393	0.1098	0.0705	0.0705
Load 1	0.7885	1.2030	1.9915	1.9915
Load 2	1.2115	1.7970	3.0085	3.0085

Figure 2 Multi-Source And Multi-Load DC System Configuration,

With Node Voltages, Superposition Theorem Current Calculations

The experimental and numerical validation metrics for the network under multi-source excitation are tabulated in Fig. 2. The dataset provides branch-by-branch current distributions calculated to a precise four-digit precision under three operational states: isolated Source 1 excitation $I_1 = 2.0$ A, $I_2 = 0.0$ A, isolated Source 2 excitation $I_1 = 0.0$ A, $I_2 = 3.0$ A, and the aggregate system operating point.

Branch	Source 1 (I_1=2 A, I_2=0 A) Power Watts	Source 2 (I_1=0 A, I_2=3A) Power Watts	Total Loss In Branch Watts	Power Loss $I^2 \times R$ In Branch At The Operating Point Watts
1-2	-0.4267	1.1434	0.7166	0.7168
1-3	0.6726	-0.1681	0.5045	0.5045
1-4	2.4145	1.3316	3.7462	3.7464
2-3	0.6694	4.7888	5.4582	5.4583
2-4	0.0128	4.0064	4.0192	4.0188
3-4	-0.0277	0.0774	0.0497	0.0498
Load 1	79.1827	120.8076	199.9904	199.9952 ($V_3 \times I_{L1}$)
Load 2	120.8112	179.1975	300.0087	300.0046 ($V_4 \times I_{L2}$)

Fig.3 Load And Branch Power By Superposition-PDT Hybrid Network

Adding Branch Losses = Total Network Loss = 14.4945 W

The power distribution and loss verification metrics are illustrated in Fig.3.The load power and individual branch power losses contributed by sources I_1 and I_2 are evaluated by integrating a Power Division Theorem (PDT) framework with current data derived from the superposition theorem. To validate these metrics, the branch power loss under both operational cases is verified against actual $I^2 \times R$ calculations at the operating point using the load resistances (L_1,L_2), load voltages (V_3,V_4), and load currents (I_{L1},I_{L2}). The analysis reveals that the branch power loss remains identical in both cases.

V. Numerical Validation And Comparative Analysis Of The Sharing Methodologies:

A comparative numerical analysis is executed on branch R_2 connecting Node 1 and Node 3, with an ohmic value of R_2 = 3.000 Ω. This component serves as an optimal baseline to contrast the behavioural traits of the power sharing distributions under different mathematical lenses.

A. Validation of Methodology 1: Superposition–PDT Hybrid Framework

Under isolated source excitations extracted from the network matrix, the component current driven by Source I_1 acting alone is computed as $I_{R2,S1}$ = 0.5467 A. Conversely, when Source I_2 operates independently, it induces an opposing component current of $I_{R2,S2}$ = -0.1366 A . The true physical branch current is determined by the linear summation of these independent responses:

$$I_{R2,\text{total}} = 0.5467\ \text{A} + (-0.1366) = 0.4101\ \text{A} \qquad (16)$$

Applying these isolated states to the Power Division Theorem (PDT) allocation framework yields a localized power supply mapping of P_{S1} = 0.6726 W attributed to Source 1 and an extraction map of P_{S2} = -0.1681 W attributed to Source 2. Summing these individual shares establishes the total net power dissipation of the branch: $P_{\text{loss,total}} = 0.6726\ \text{W} + (-0.1681\ \text{W}) = 0.5045\ \text{W}$ (17)

B. Validation of Methodology 2: Dual-Node Superposition Framework

Methodology 2 examines the system parameters through localized boundary values at the active operating point, utilizing Node 1 as the left terminal and Node 3 as the right terminal. The left-side entering current contribution is determined by the node potential V_1 as 33.8849 A, while the right-side entering current driven by potential V_3 is determined as -33.4748 A. Linearly superimposing these boundary parameters yields the exact same net current:

$$I_{net} = 33.8849\ \text{A} + (-33.4748\ \text{A}) = 0.4101\ \text{A} \quad (18)$$

When translating these metrics into localized boundary power entries using node entering currents modulated via the PDT layout, the left-side power contribution is evaluated as:

$$P_{left} = 33.8849\ \text{A} \times 0.4101\ \text{A} \times 3\ \Omega = 41.6854\ \text{W} \quad (19)$$

Simultaneously, the right-side boundary power contribution evaluates to

$$P_{right} = -33.4748\ \text{A} \times 0.4101\ \text{A} \times 3\ \Omega = -41.6854\ \text{W} \quad (20)$$

Combining these directional terminal power boundaries yields the aggregate branch dissipation:

$$P_{los,total} = 41.6854\ \text{W} + (-41.1809\ \text{W}) = 0.5045\ \text{W} \quad (21)$$

C. Validation of Methodology 3: Direct Source Current Ratio Framework

Methodology 3 attempts to evaluate the power shares by superimposing the raw source current ratios directly onto the total branch power loss under a direct PDT calculation format. For branch R_2, this maps the sharing allocation from Source 1 to 0.2162 W, while the sharing allocation from Source 2 evaluates to 0.3243 W. Linearly summing these two allocations mathematically yields the same aggregate branch Dissipation boundary:

$$P_{los,total} = 0.2162\ \text{W} + 0.3243\ \text{W} = 0.5045\ \text{W} \quad (22)$$

D. Comparative Assessment and Analytical Conclusion

The numerical results conclusively demonstrate that while all three methodologies mathematically converge to the identical aggregate branch power loss of 0.5045 W, their internal sharing shares are profoundly different. Methodology 2 introduces an exaggerated power distribution loop based entirely on terminal voltage boundaries rather than active source inputs. Meanwhile, the direct current-ratio mapping of Methodology 3 diverges heavily across standard branch lines, matching only when evaluating specific node-to-reference load structures.

Ultimately, because the Superposition–PDT Hybrid Framework (Methodology 1**)** maps the localized power paths directly back to the physical independent current generating injections (I_1 and I_2) without relying on unweighted source ratios or secondary boundary potentials, it stands as the uniquely correct allocation layout to chart authentic source-to-branch power loss causality.

Because the superposition-PDT framework traces power metrics directly back to the physical independent current generating injections $I_1$and I_2 rather than secondary boundary voltages or unweighted source ratios, it preserves the authentic source-to-load causality. Therefore, for general bilateral networks, the Superposition–PDT network framework stands as the mathematically correct approach to determine exact independent branch power loss allocations

VI. Computational Equivalency And Analytical Merit Of The Node-To-Reference Topology:

The verification of load-sharing characteristics under multi-source excitation reveals a significant structural and computational advantage. Typically, evaluating the precise power contribution of independent sources (I_1) and (I_2) to multiple load resistances requires a systematic application of the superposition theorem alongside a Power Division Theorem (PDT) framework

Share of i th Source on j th Load $$P_{ij} = L_j \times \frac{I_i}{I_{total}} \quad (23)$$

I_i = i th Source Current

I_{total}= Total of Source Currents

Share of Source 1 on Load 1 = P_{11} = 80 W Share of Source 1 on Load 2 = P_{12} = 120 W
Share of Source 2 on Load 1 = P_{21} = 120 W Share of Source 2 on Load 2 = P_{22} =180 W
Crucially, as illustrated by the experimental and calculated data, the individual sharing of each load by each independent source determined via the combined superposition-PDT framework yields the exact same numerical result as the direct execution of the standard PDT formula using direct source current ratios.

The underlying physical reason for this mathematical equivalency is strictly topological: the load resistances of (L_1) and (L_2) are tied directly between their respective active network nodes and the common reference plane. Because these loads terminate directly at the ground/reference node, the node voltages (V_3,V_4) and branch currents (I_1,I_2) maintain a linear, decoupled proportionality relative to the isolated inputs.

Consequently, the localized power division ratios remain invariant whether the circuit is analyzed in its aggregate state or decomposed into isolated source components. This structural characteristic provides an exceptional computational advantage in practical circuit analysis. By recognizing this node-to-reference topology, researchers can entirely bypass tedious multi-step superposition computations and directly extract precise load-sharing metrics using simple, single-line source current ratio equations without any loss of analytical accuracy.

Crucially, Fig.3 highlights that the load sharing calculated via the superposition-PDT framework yields the exact same result as a direct calculation using source current ratios. Because the loads are connected directly between the node and the reference plane, load sharing can be determined seamlessly without executing tedious superposition computations. This structural characteristic offers a significant computational advantage, drastically simplifying the analysis of load-sharing behaviour in networks with grounded reference nodes.

VII. Detailed Discussion: Resolving AC Loss Allocation Ambiguities Via PDT

A. The Conventional Loss Formula Deficit

In traditional power system analysis, the transmission loss across a line or complex branch impedance Z = R + j X = 3 + j4 Ω is calculated globally using the aggregate current magnitude. Given a sending-end voltage Vs = 10.2 + j3.6 V and a receiving-end voltage Vr = 5.5 + j1.5 V, the total through-current is defined by:

$$I = \frac{(V_S - V_R)}{Z} = I_d + jI_q = 0.9 - j0.5\ \text{A} \tag{24}$$

Using the conventional formula, the total complex power loss is evaluated as

$$|I|^2 \times Z = (0.9^2 + (-0.5)^2) \times (3 + j4) = 3.18 + j4.24\ \text{VA} \tag{25}$$

While this provides the correct aggregate systemic dissipation, it introduces a profound methodological impasse: the individual power contributions from Vs and Vr cannot be isolated**.** When expanding the current components algebraically:

$$|I_S - I_R|^2 = (I_{sr}^2 + I_{rr}^2) + (I_{sq}^2 + I_{rq}^2) - 2 \times (I_{sr} \times I_{rr} + I_{sq} \times I_{rq}) \tag{26}$$

Traditional textbooks leave the mutual interaction term namely $-2 \times (I_{sr} \times I_{rr} + I_{sq} \times I_{rq})$ completely unassigned. For decades, this mathematical omission has erroneously reinforced the idea that superposition cannot be used to track power or assign specific transmission costs to individual sources.

B. The Direct PDT Direct Formula Allocation

The Power Division Theorem resolves this historical error by implementing a complex current ratio factor (I_K/I) to split the quadratic aggregate expression into perfectly linear, source-trackable components. The total loss is redefined as the exact summation of the individual shares from the sending and receiving sources: Loss in Z = Power from Vs + Power from V_r (27)

By applying the direct PDT formulation, the unassigned interaction terms are perfectly distributed. The individual source power allocations are calculated as:

Power from Vs Share: $\left(\frac{I_s}{I}\right) \times |I|^2 \times Z = 7.38 + j8.34\, VA$ (28)

Power from Vr Share: $-\left(\frac{I_r}{I}\right) \times |I|^2 \times Z = -(4.2 + j4.1)\ VA$ (29)

Subtracting the receiving-end share from the sending-end share yields the exact total aggregate loss:

$(7.38 + j8.34) - (4.2 + j4.1) = 3.18 + j4.24\ VA$ (30)

C. Algebraic Equivalence and the Replacement Formula

The primary mathematical breakthrough is that the PDT direct calculation using the complex current ratio simplifies seamlessly into a direct multiplier. By substituting the conjugate identity $|I|^2 = (I_d + jI_q) \times (I_d - jI_q)$, the division by I is eliminated: Loss Formula:

$I_s \times (I_d - jI_q) \times Z - I_r \times (I_d - jI_q) \times Z$ (31)

Simplified to

$(I_s - I_r) \times (I_d - jI_q) \times Z = 3.18 + j4.24\ VA$ (32)

This proves that the share allocated via linear current superposition and the direct PDT structural ratio yield identical, mathematically rigorous results.

VIII. Discussions And Conclusions:

The numerical and algebraic validation presented in this study disproves the long-held engineering pedagogical belief that power calculations cannot be handled via superposition.

To evaluate the distribution of network power losses under multi-source excitation, this paper systematically compares three methodological pathways in determining branch power loss distributions:

1. The input-driven Superposition–PDT Hybrid Framework,
2. The boundary-reliant Dual-Node Superposition Framework,
3. The direct Source Current Ratio Shortcut**.**

Ultimately, the Superposition–PDT framework stands as the uniquely accurate method for mapping authentic source-to-branch power causality in general bilateral networks.

However, for specialized node-to-reference load structures, this complex framework converges exactly with the direct source current ratio calculation. Recognizing this topological layout allows researchers to entirely bypass tedious multi-step superposition algorithms and directly extract precise load-sharing metrics via simple, single-line equations without any loss of analytical accuracy.

Overwriting Traditional Limits: The Power Division Theorem succeeds where traditional quadratic formulas fail by allocating the cross-product interaction terms that textbook methodologies have left unassigned for decades.

Exact Mathematical Consistency**:** The major merit of this framework is its structural duality: the power-sharing profile determined via source current superposition matches the direct PDT calculation using current ratios perfectly.

Practical Utility for Smart Grids**:** By transforming non-linear loss expressions into linear, trackable equations, this formulation provides energy management systems (EMS) with an exact mathematical tool to track reverse power flows, attribute transmission line losses fairly, and handle wheeling cost allocations in deregulated, multi-source power grids.

The Power Division Theorem resolves this historical pedagogical boundary. By replacing complex matrix tracking routines with a direct, scalable formula, the PDT provides an exact, simpler mechanism for loss allocation. This framework suggests that standard textbook treatments can be generalized to include power-sharing parameters without violating energy conservation laws. The calculation is basic, simple and exact, IJFMR must mandate this update. Global engineering textbooks require immediate revision. This safeguards baseline technical education worldwide.

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