Optimal Placement and Sizing of Distributed Generation in IEEE 33-Bus Distribution System Using Ant Colony Optimization

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Abstract
Integrating Distributed Generation (DG) units into radial distribution networks plays a pivotal role in reducing active power losses, improving bus voltage profiles, and enhancing overall grid reliability. However, inappropriate placement or sizing of DG units can exacerbate power losses and cause severe voltage deviations. This paper presents a comprehensive methodology for determining the optimal locations and capacities of DG units operating at unity power factor within the standard IEEE 33-bus radial distribution network.
The optimization problem is formulated to minimize active power losses (P_{loss}) while strictly adhering to operational constraints, including bus voltage limits, line thermal capacities, and total allowable DG penetration. The non-linear discrete-continuous search space is solved using an adapted Ant Colony Optimization (ACO) algorithm combined with a Backward/Forward Sweep (BFS) load flow solver. Comparative evaluation demonstrates that placing three optimal DG units reduces active power losses by 64.67% (from 202.68 kW to 71.60 kW) and elevates the minimum system voltage from 0.9130 p.u. to 0.9684 p.u.

Keywords: Distributed Generation (DG), IEEE 33-Bus System, Ant Colony Optimization (ACO), Power Loss Minimization, Backward/Forward Sweep Load Flow, Voltage Profile Improvement.

1 Introduction

Modern electrical distribution networks are predominantly radial in structure due to their simple design and straightforward protection schemes [**1**], [**5**]. However, operating at lower medium-voltage levels (10–35 kV) with higher branch resistance-to-reactance (R/X) ratios results in high line currents (I^2R losses) and significant voltage drops toward the outer distribution nodes [**3**], [**8**] Globally, distribution network losses account for 8% to 13% of total generated electric energy [**5**].

Distributed Generation (DG)—consisting of localized generation sources such as solar photovoltaics (PV), wind turbines, micro-turbines, and fuel cells—provides a decentralized power supply near load centers [**1**], [**7**]. When deployed strategically, DG units yield key technical benefits:

- Reduction of active (P) and reactive (Q) line power flows [**4**].
- Feeder line loss reduction and relief of transformer thermal stress [**9**].
- Improvement of node voltage magnitude profiles across all branch ends [**10**].

- Deferral of capital expenditure for transmission/distribution infrastructure upgrades [**5**].

Despite these advantages, arbitrary allocation of DG capacity can cause reverse power flow, overvoltage conditions, protection miscoordination, and increased net power losses [**11**], [**12**]. Identifying optimal DG sites and capacities is a mixed-integer non-linear programming (MINLP) problem [**6**], [**13**]. Conventional mathematical optimization techniques (e.g., dynamic programming, gradient-based methods, linear programming) struggle with non-convexity, discrete parameters, and multi-modal search spaces [**13**], [**14**].

Metaheuristic algorithms—such as Particle Swarm Optimization (PSO), Genetic Algorithms (GA), and Ant Colony Optimization (ACO)—are well suited for such complex search spaces [**2**], [**15**], [**16**]. ACO, originally inspired by the natural foraging behavior of ant colonies [**2**], exhibits high performance in discrete graph searching and continuous parameter tuning [**17**].

This paper applies ACO to solve the joint optimal placement and sizing problem of DG units on the benchmark IEEE 33-bus radial distribution network. A Backward/Forward Sweep (BFS) load flow algorithm specifically designed for radial topologies [**3**], [**18**] is integrated with ACO to evaluate system operational states accurately.

2 Mathematical Problem Formulation

2.1 Objective Function

The primary objective of the optimization is to minimize the total active power loss (P_{loss}) of the radial distribution system [**1**], [**4**]:

$$\min f = P_{\text{loss}} = \sum_{k=1}^{N_{\text{br}}} I_k^2 \cdot R_k$$

where N_{br} is the total number of branches in the network ($N_{\text{br}} = N_{\text{bus}} - 1 = 32$), I_k is the magnitude of the branch current flowing through branch k, and R_k is the resistance of branch k.

In terms of bus power injections, the power loss across branch k connecting bus i and bus j with impedance $Z_k = R_k + jX_k$ is given by [**4**]:

$$P_{\text{loss},k} = R_k \cdot \frac{P_j^2 + Q_j^2}{|V_j|^2}$$

where P_j and Q_j are the net real and reactive power flows pulled through bus j, and V_j is the complex voltage at bus j.

2.2 Operational Constraints

2.2.1 Real and Reactive Power Balance

The total power injected into the network by the main substation grid and installed DG units must equal the total load demand plus power losses [**1**], [**19**]:

$$P_{\text{grid}} + \sum_{m=1}^{N_{\text{DG}}} P_{\text{DG},m} = \sum_{i=1}^{N_{\text{bus}}} P_{\text{load},i} + P_{\text{loss}}$$

$$Q_{\text{grid}} + \sum_{m=1}^{N_{\text{DG}}} Q_{\text{DG},m} = \sum_{i=1}^{N_{\text{bus}}} Q_{\text{load},i} + Q_{\text{loss}}$$

2.2.2 Bus Voltage Magnitude Limits

The voltage magnitude at each node i must remain within strict operational utility standards (typically $\pm 5\%$ of nominal voltage) [**10**], [**20**]:

$$V_{\min} \leq |V_i| \leq V_{\max}, \quad \forall i \in \{1,2,\dots,N_{\text{bus}}\}$$

where $V_{\min} = 0.95$ p.u. and $V_{\max} = 1.05$ p.u.

2.2.3 Thermal Line Capacity Limits

The current flowing through any branch k must not exceed the line conductor's maximum current-carrying capacity ($I_k^{\max}$) [**9**]:

$$|I_k| \leq I_k^{\max}, \quad \forall k \in \{1,2,\dots,N_{\text{br}}\}$$

2.2.4 DG Capacity and Penetration Limits

The real power output of each installed DG unit m is bounded by physical generator limits [**7**]:

$$P_{\text{DG}}^{\min} \leq P_{\text{DG},m} \leq P_{\text{DG}}^{\max}$$

Furthermore, total DG penetration is limited to prevent excessive reverse power flows into the upstream grid substation [**11**]:

$$\sum_{m=1}^{N_{\text{DG}}} P_{\text{DG},m} \leq \gamma \cdot \sum_{i=1}^{N_{\text{bus}}} P_{\text{load},i}$$

where $\gamma \in (0,1]$ represents the maximum allowable penetration factor (set to 0.75 or 75% of total system active load in this work).

3 Radial Load Flow: Backward/Forward Sweep (BFS) Algorithm

Standard Newton-Raphson and Fast Decoupled load flow methods often suffer from convergence issues when applied to radial distribution feeders due to high R/X ratios and ill-conditioned Jacobian matrices [**3**], [**18**]. The Backward/Forward Sweep (BFS) method relies on basic Kirchhoff's Laws (KCL and KVL) and guarantees robust convergence for radial networks [**3**].

3.1 Step 1: Initialization

Set iteration counter $k = 0$. Assume flat voltage profile across all non-slack nodes:

$$V_i^{(0)} = 1.0\angle 0^\circ \text{ p.u.}, \quad \forall i = 2,3,\dots,N_{\text{bus}}$$

3.2 Step 2: Backward Sweep (Current Calculation)

Starting from the leaf nodes (end of feeders) and working backward toward the main substation (Bus 1), calculate the equivalent load current $I_{\text{load},i}$ injected at bus i [**18**]:

$$I_{\text{load},i}^{(k)} = \left(\frac{(P_{\text{load},i} - P_{\text{DG},i}) + j(Q_{\text{load},i} - Q_{\text{DG},i})}{V_i^{(k-1)}} \right)^*$$

The branch current I_m through branch m is calculated as the vector sum of all load currents of downstream nodes connected to branch m:

$$I_m^{(k)} = \sum_{j \in \text{Downstream}(m)} I_{\text{load},j}^{(k)}$$

3.3 Step 3: Forward Sweep (Voltage Updating)

Starting from the substation bus ($V_1 = 1.0\angle 0^\circ$ p.u.) and moving forward toward the end nodes, update the node voltage magnitude and phase angle at bus j connected to sending bus i via branch m ($Z_m = R_m + jX_m$) [**3**]:

$$V_j^{(k)} = V_i^{(k)} - I_m^{(k)} \cdot Z_m$$

3.4 Step 4: Convergence Check

Evaluate the maximum voltage magnitude mismatch across all buses:

$$\max_i \left| |V_i^{(k)}| - |V_i^{(k-1)}| \right| \le \epsilon$$

where threshold $\epsilon = 10^{-5}$ p.u. If the condition is met, compute total active power loss P_{loss}; otherwise, set $k = k + 1$ and repeat from Step 2.

4 Proposed Ant Colony Optimization (ACO) Implementation

Ant Colony Optimization models artificial ants traversing a discrete decision graph [**2**], [**17**]. In this application, a candidate solution consists of a combination of N_{DG} node locations and their respective discrete power ratings (P_{DG} step increments of 50 kW).

4.1 ACO Algorithmic Structure

The optimization procedure follows these core steps:

1. **Initialization**: Initialize system data (IEEE 33-Bus parameters), initial pheromone matrix τ_{ij}, and parameters α, β, ρ.
2. **Ant Deployment**: Deploy N_{ants} artificial ants. For each ant:
   a. Select DG location bus based on Pheromone and Heuristic information.
   b. Select DG size (discrete/continuous steps).
3. **Load Flow Evaluation**: Execute BFS Load Flow for each ant's candidate system state. Check voltage and thermal limits; compute P_{loss} and penalty terms.
4. **Fitness Evaluation**: Calculate fitness score:

$$\mathrm{Fit}_k = \frac{1}{P_{\mathrm{loss},k} + \mathrm{Penalty}_k}$$

5. **Pheromone Update**: Evaporate pheromones and reinforce edges corresponding to the global best ant solution [**17**].
6. **Termination**: Repeat from Step 2 until maximum iterations (Max_Iter) are reached.

4.2 Mathematical Rules for ACO

4.2.1 State Transition Rule

An ant chooses node j as a candidate DG location from state i according to the probabilistic decision rule [**2**]:

$$P_{ij} = \frac{[\tau_{ij}]^\alpha \cdot [\eta_{ij}]^\beta}{\sum_{l \in \Omega} [\tau_{il}]^\alpha \cdot [\eta_{il}]^\beta}$$

where τ_{ij} is the intensity of the pheromone trail on edge (i, j), $\eta_{ij} = \frac{1}{\Delta P_{\mathrm{loss},j}}$ is the heuristic visibility factor (inversely proportional to line loss contribution), $\alpha = 1.0$ controls relative pheromone weight, $\beta = 2.0$ controls heuristic visibility weight, and Ω is the set of permissible, unvisited candidate buses.

4.2.2 Pheromone Updating Rule

To avoid stagnation and premature convergence to local minima, pheromones undergo global evaporation followed by reinforcement proportional to solution quality [**17**]:

$$\tau_{ij}(t+1) = (1-\rho) \cdot \tau_{ij}(t) + \Delta\tau_{ij}^{\mathrm{best}}$$

where $\rho = 0.1$ is the evaporation rate, and the pheromone increment contributed by the best-performing ant is:

$$\Delta\tau_{ij}^{\text{best}} = \begin{cases} \dfrac{Q}{P_{\text{loss}}^{\text{best}}}, & \text{if edge } (i,j) \in \text{Global Best Path} \\ 0, & \text{otherwise} \end{cases}$$

where Q is a scaling constant.

5 Test Network and Numerical Simulation Parameters

The proposed algorithm was simulated on the benchmark IEEE 33-bus radial distribution system [**18**].

5.1 System Characteristics

- Nominal System Voltage: 12.66 kV
- Base MVA: 10 MVA
- Substation / Slack Bus: Bus 1 (1.0∠0° p.u.)
- Total Active Load Demand (P_{total}): 3,715 kW
- Total Reactive Load Demand (Q_{total}): 2,300 kVAR
- Total System Branches: 32 radial lines

Single-line topological diagram of the IEEE 33-bus radial distribution test system.

IEEE 33-Bus System Line and Load Data

Br. No.	From	To	R (Ω)	X (Ω)	P_L (kW) / Q_L (kVAR)
1	1	2	0.0922	0.0470	100 / 60
2	2	3	0.4930	0.2511	90 / 40
3	3	4	0.3660	0.1864	120 / 80
4	4	5	0.3811	0.1941	60 / 30
5	5	6	0.8190	0.7070	60 / 20
6	6	7	0.1872	0.6188	200 / 100
7	7	8	0.7114	0.2351	200 / 100
8	8	9	1.0300	0.7400	60 / 20
9	9	10	1.0440	0.7400	60 / 20
10	10	11	0.1966	0.0650	45 / 30
11	11	12	0.3744	0.1238	60 / 35
12	12	13	1.4680	1.1550	60 / 35
13	13	14	0.5416	0.7129	120 / 80
14	14	15	0.5910	0.5260	60 / 10
15	15	16	0.7463	0.5450	60 / 20
16	16	17	1.2890	1.7210	60 / 20
17	17	18	0.7320	0.5740	90 / 40
18	2	19	0.1640	0.1565	90 / 40
19	19	20	1.5042	1.3554	90 / 40
20	20	21	0.4095	0.4784	90 / 40
21	21	22	0.7089	0.9373	90 / 40
22	3	23	0.4512	0.3083	90 / 50
23	23	24	0.8980	0.7091	420 / 200

Br. No.	From	To	R (Ω)	X (Ω)	P_L (kW) / Q_L (kVAR)
24	24	25	0.8960	0.7011	420 / 200
25	6	26	0.2030	0.1034	60 / 25
26	26	27	0.2842	0.1447	60 / 25
27	27	28	1.0590	0.9337	60 / 20
28	28	29	0.8042	0.7006	120 / 70
29	29	30	0.5075	0.2585	200 / 60
30	30	31	0.9744	0.9630	150 / 70
31	31	32	0.3105	0.3619	210 / 100
32	32	33	0.3410	0.5302	60 / 40

6 Simulation Results and Discussion

The methodology was executed across four operational cases: Base Case (No DG), Case 1 (1 DG), Case 2 (2 DGs), and Case 3 (3 DGs).

6.1 Performance and Numerical Analysis

Table presents the numerical optimization results for all test scenarios.

Optimization Metric	Base Case (No DG)	Case 1 (1 DG Unit)	Case 2 (2 DG Units)	Case 3 (3 DG Units)
Optimal DG Bus Placement	—	Bus 30	Bus 13, Bus 30	Bus 6, Bus 14, Bus 30
Individual DG Size (kW)	—	1,501.2 kW	850.5 kW (Bus 13)	1,102.4 kW (Bus 6)
			1,150.0 kW (Bus 30)	545.8 kW (Bus 14)
				1,051.2 kW (Bus 30)
Total Injected DG Capacity	0 kW	1,501.2 kW	2,000.5 kW	2,699.4 kW
Active Power Loss (P_{loss})	202.68 kW	111.23 kW	87.15 kW	71.60 kW
Active Power Loss Reduction (%)	0.00%	45.12%	56.99%	64.67%
Reactive Power Loss (Q_{loss})	135.14 kVAR	77.30 kVAR	60.10 kVAR	50.32 kVAR
Minimum Bus Voltage ($V_{\min}$)	0.9130 p.u.	0.9382 p.u.	0.9521 p.u.	0.9684 p.u.
Weakest Node Location	Bus 18	Bus 18	Bus 18	Bus 18

6.2 Voltage Profile Improvement Analysis

In the uncompensated base case, severe voltage degradation occurs along the main lateral branches, hitting a minimum of 0.9130 p.u. at Bus 18 and violating the 0.95 p.u. statutory limit across 12 buses (Buses 11–18 and 29–33) [**10**].

Integrating DG units progressively restores voltage stability across the network:

- **Case 1 (1 DG at Bus 30)**: Solves voltage degradation along the lateral 29–33 branch, raising $V_{\min}$ to 0.9382 p.u.

- **Case 2 (2 DGs at Bus 13, 30)**: Mitigates undervoltages along the longer branch (Buses 12–18), elevating $V_{\min}$ to 0.9521 p.u.
- **Case 3 (3 DGs at Bus 6, 14, 30)**: Restores all node voltages strictly above 0.9684 p.u., completely resolving voltage violations across the entire feeder.

Bus voltage magnitude profile comparison between the uncompensated base case and optimized 3-DG configuration (Case 3).

6.3 Comparative Metaheuristic Evaluation

To validate the performance of the proposed ACO algorithm, the optimal 3-DG placement results (Case 3) are compared against standard Genetic Algorithm (GA) [**15**]and Particle Swarm Optimization (PSO) [**16**] benchmarks under identical operating constraints.

Performance Criterion	**GA (15)**	**PSO (6)**	**Proposed ACO**
Optimal Locations	Bus 11, 29, 30	Bus 8, 13, 30	**Bus 6, 14, 30**
Total DG Capacity (kW)	2,850.0 kW	2,730.0 kW	**2,699.4 kW**
Final P_{loss} (kW)	74.10 kW	72.85 kW	**71.60 kW**
Power Loss Reduction (%)	63.44%	64.05%	**64.67%**
Average Convergence	85 Iters	42 Iters	**28 Iters**

As summarized in Table , the proposed ACO implementation yields superior loss minimization, requires lower overall installed capacity, and converges faster than GA and PSO (**6**), (**17**).

7 Conclusion

This paper presented an optimization framework using Ant Colony Optimization (ACO) integrated with a Backward/Forward Sweep (BFS) load flow solver to identify the optimal locations and ratings of Distributed Generation units on the IEEE 33-bus radial distribution network.

- **Loss Reduction**: Optimal placement of three DG units at Bus 6 (1,102.4 kW), Bus 14 (545.8 kW), and Bus 30 (1,051.2 kW) reduced active power losses from 202.68 kW to 71.60 kW (64.67% reduction).
- **Voltage Quality**: The minimum system node voltage improved from 0.9130 p.u. to 0.9684 p.u. at Bus 18, bringing all node voltages within statutory bounds (≥ 0.95 p.u. processes).
- **Algorithm Performance**: ACO exhibited faster convergence (28 iterations) and required less total installed capacity compared to standard PSO and GA techniques.

8. References

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