Integrated Geometric Cutter Selection and Machining-Time Optimization for Rough Machining of NURBS-Based Free-Form Surfaces: A Theoretical Framework

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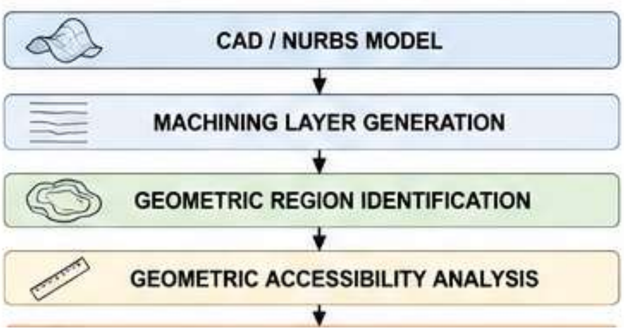
Abstract

Free-form surface machining is a challenging computer-aided manufacturing problem because local surface geometry continuously affects cutter accessibility, tool engagement, toolpath generation, and machining efficiency. In rough machining, cutter selection is particularly important because a substantial amount of material is removed and the selected cutter influences both cutting and non-cutting operations. This paper develops a theoretical framework that extends the authors' previously published NURBS/IGES-based geometric cutter-selection methodology into a global discrete cutter-assignment and machining-time optimization model. The contribution is not claimed to be the first introduction of tool selection, airtime minimization, or machining-time optimization; these topics have been addressed previously. Instead, the proposed framework explicitly connects layer- and island-based geometric cutter feasibility with discrete cutter-to-region assignment and a unified nominal process-time objective containing cutting, air-travel, and tool-change contributions. The formulation distinguishes local geometric feasibility from global process optimality and provides a small analytical counterexample showing that the largest feasible cutter for each individual region need not constitute the globally preferred cutter combination. The study is intentionally theoretical; the numerical illustration is used only to verify the mathematical behavior of the formulation and is not presented as experimental validation. The framework provides a transparent basis for future CAD/CAM implementation and for extensions involving force, energy, tool wear, and five-axis kinematics.

Keywords: Free-form surface; cutter selection; toolpath optimization; NURBS; rough machining; machining time; CAD/CAM; process planning.

1. Introduction



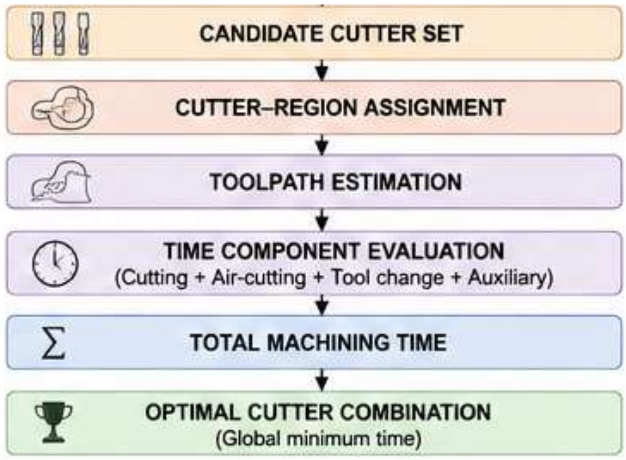


Fig. 1. Overall framework of the proposed methodology.

Free-form surfaces are widely used in aerospace, automotive, die and mold, optical, biomedical, and other high-value manufacturing applications. Their complex geometry creates substantial challenges for computer numerical control (CNC) machining because the local surface characteristics continuously modify the interaction between the cutting tool and the workpiece. Consequently, tool geometry, toolpath strategy, cutting parameters, and surface geometry are strongly coupled during machining [1], [2].

NURBS (Non-Uniform Rational B-Splines) provide a flexible mathematical representation for complex surfaces and are widely used in CAD/CAM systems. In the authors' previous study, a NURBS-based automatic cutter-selection methodology was developed for rough machining of free-form surfaces. The system read surface information from an IGES file, converted the surface representation to NURBS, and determined suitable cutter dimensions from the geometry of machining regions [3].

The previous study considered roughing, semi-roughing, and finishing stages and focused cutter selection on rough machining because a large amount of material is removed during this stage [3]. The geometric method used external boundaries, internal islands, and minimum distances between surrounding rectangular regions to determine appropriate cutter dimensions [3]. The original study reported that the developed approach produced a shorter toolpath than the conventional method used for comparison [3].

The geometric approach provides a rational basis for local cutter selection; however, geometric feasibility and global machining optimality are not identical concepts. A cutter that is locally attractive may require additional tool changes or non-cutting movements when the complete workpiece is considered. Earlier work on free-form machining demonstrated that tool selection can be coupled with machining-time estimation [4], while process-aware toolpath optimization showed that a geometrically generated path is not necessarily optimal with respect to manufacturing objectives [5].

Recent research has increasingly moved toward regional, adaptive, and optimization-based free-form machining. A comprehensive review by Mali et al. identified toolpath generation, tool geometry, cutting mechanics, and process optimization as major research themes in free-form milling [1]. Zhao et al. proposed energy-aware sub-regional milling based on geometric characteristics and curvature [7]. Herraz et al. formulated free-form machining as a partitioning-and-machining optimization problem in which surface zones and machining time are considered explicitly [8]. Kukreja et al. developed a voxel-based adaptive toolpath approach for free-form surfaces [6], while Guo et al. proposed feature-adaptive toolpath planning for freeform optics [9]. Kharat et al. investigated stochastic toolpath generation for three-axis ball-end milling [10], and Mgherony and Mikó proposed a search-based toolpath strategy that addresses variations in effective working diameter on inclined free-form surfaces [11]. Recent five-axis work has further integrated toolpath and tool-motion optimization [12].

The research question addressed in this study is therefore whether the authors' earlier geometrically based cutter-selection method can be extended into a global process-planning formulation in which cutter choice itself becomes a discrete optimization variable. The objective is not to replace geometric feasibility analysis, but to use it as the first stage of a broader optimization framework.

The main contributions of this paper are: (i) a regional geometric representation for rough machining of NURBS-based free-form surfaces; (ii) a feasible cutter-set formulation based on local accessibility and an explicit geometric clearance margin; (iii) a discrete cutter-assignment model; (iv) a nominal process-time formulation that separates cutting, air-travel, and tool-change contributions; and (v) an analytical distinction between independent local cutter selection and global cutter-combination optimization. The paper deliberately remains theoretical and does not claim experimental performance improvements.

1.1 Research Gap and Originality

The originality of the present study is defined at the framework level. Yang and Han previously demonstrated interference-aware optimal tool selection for three-axis free-form machining [4]. The present paper therefore does not claim tool-selection optimization as a new problem. Its contribution is to connect the authors' NURBS/IGES layer-and-island geometric feasibility procedure [3] to an explicit discrete cutter-assignment model and to evaluate the resulting cutter combinations with a unified nominal process-time objective.

The authors' earlier study addressed a different but complementary problem: geometric cutter selection for free-form surfaces represented through IGES/NURBS data. That methodology determined suitable cutter dimensions from machining-layer boundaries, internal islands, and minimum geometric distances and was implemented as an automatic tool-selection system [3]. The present study uses that geometric feasibility concept as the first stage of a higher-level process optimization model.

A second established research direction concerns non-productive tool motion. Castelino et al. formulated airtime minimization as a toolpath-sequencing problem [13]. Accordingly, air travel is not presented here as a new optimization objective. It is included as one cost component of the proposed cutter-combination model, together with tool-change cost.

More recent studies have optimized free-form surface partitioning and associated machining time using black-box and surrogate-based approaches [8], while recent five-axis research has jointly optimized toolpaths and tool motion [12]. The present formulation addresses a narrower and complementary decision layer: the cutter diameter assigned to each geometrically feasible machining region.

Accordingly, the research gap addressed here is the lack of an explicitly documented formulation that bridges the authors' NURBS/IGES-based local geometric cutter-feasibility procedure with a discrete cutter-combination model in which nominal cutting, air-travel, and tool-change times are evaluated together. The contribution is therefore positioned as an integration and extension of established ideas rather than as a first-ever tool-selection method.

Research questions. The study addresses three questions: (Q1) how can the geometric cutter-feasibility information obtained from a NURBS/IGES free-form model be formalized as a constraint on cutter assignment; (Q2) how does the inclusion of cutting, air-cutting, and tool-change costs alter the cutter-selection problem relative to independent local selection; and (Q3) under what analytical conditions can a globally optimized cutter combination differ from the largest-feasible-cutter rule? These questions define the scope of the contribution and do not require experimental validation at the formulation stage.

The originality can be summarized in four points: (1) the local geometric feasibility information is retained as a formal constraint rather than replaced by a black-box optimizer; (2) cutter diameter is converted into an explicit discrete decision variable assigned to machining regions; (3) cutting, air-cutting, and tool-change contributions are expressed in a common total-time objective; and (4) the formulation analytically distinguishes local largest-feasible-cutter selection from global cutter-combination optimality. This last distinction leads directly to the theoretical inequality $T^* \leq T\_G$, which states that the global optimization solution cannot be worse than any feasible local-selection solution within the same search space.

Study	**Main optimization focus**	**Key limitation relative to present work**	**Role in present framework**
Ersoyoglu & Ünüvar [3]	Geometric cutter selection from IGES/NURBS; minimum cost/time	No explicit global binary cutter-assignment model combining cutter changes and air-cutting	Geometric feasibility basis
Yang & Han [4]	Optimal tool selection for minimum overall machining time; interference-aware 3-axis machining	Different geometric representation and formulation; cutter feasibility is not derived from the authors' layer/island model	Closest prior optimization study; defines the boundary of the novelty claim
Castelino et al. [13]	Airtime minimization by sequencing toolpath segments	Does not jointly optimize cutter assignment and geometric cutter feasibility	Motivates explicit air-cutting term
Herraz et al. [8]	Black-box surface partitioning and machining-time optimization	Different optimization variables and black-box formulation	Supports region-based global optimization
Zaragoza Chichell et al. [12]	5-axis toolpath, rotation and tilt optimization for a given cutter	Focuses on motion/path optimization rather than discrete cutter-set selection	Motivates future five-axis extension
Present study	Geometric feasibility + discrete cutter assignment + cutting + air-cutting + tool-change cost	Theoretical model; numerical/experimental validation remains future work	Integrated framework proposed here

Statement of contribution. The proposed framework should be understood as a geometric-to-global process-planning extension of the authors' earlier work [3]. Its contribution is the formulation of a unified decision structure in which local geometric feasibility defines the admissible cutter set, while a global objective determines the cutter combination. This positioning explicitly distinguishes the present work from earlier machining-time-based tool selection [4], airtime optimization [13], surface-partition optimization [8], and five-axis path/motion optimization [12].

2. Literature Review

Free-form surface milling has been extensively investigated from geometric, process-mechanics, toolpath, and optimization perspectives. Lasemi et al. reviewed CNC machining of free-form surfaces and identified toolpath generation, tool orientation, and tool geometry selection as central research areas [2]. Mali et al. subsequently reviewed advances over the 2009–2020 period and emphasized the continuously changing tool-workpiece contact conditions that influence effective tool radius, cutting speed, forces, wear, and part quality [1].

Tool selection has traditionally been associated with interference avoidance, accessible surface area, tolerance, and machining time. Yang and Han developed a methodology that integrates interference detection, optimal tool selection, and toolpath generation for three-axis free-form machining [4]. Their work is particularly relevant to the present study because it explicitly relates tool selection to estimated machining time.

The authors' earlier work also treated cutter selection as a geometric process-planning task. The machining layer was intersected by horizontal planes and the external boundary and internal islands were determined [3]. Minimum distances between boundaries were then used to determine suitable cutter dimensions [3]. This provides the geometric feasibility basis retained in the present framework.

Toolpath optimization has subsequently been formulated with process objectives beyond pure geometric feasibility. Lazoglu et al. proposed force-minimal toolpath generation for free-form surfaces and demonstrated that anisotropic surface geometry strongly influences cutting forces and therefore toolpath quality [5].

Regional decomposition has become increasingly important. Zhao et al. demonstrated that geometric characteristics and curvature affect machining efficiency, surface quality, and energy consumption and proposed a multi-objective sub-regional milling framework [7]. Herraz et al. introduced a black-box optimization model consisting of a clustering component that partitions a free-form surface into zones and independent machining-time components for the resulting zones [8]. Their results support the general principle that surface partitioning and process optimization should be treated as coupled decisions.

Adaptive toolpath research provides additional support for region-specific treatment. Kukreja et al. proposed a voxel-based method for generating gouge-free adaptive toolpaths [6]. Guo et al. related feature-adaptive toolpath spacing to local freeform characteristics in ultra-precision diamond milling [9]. Kharat et al. showed that free-form toolpath strategies involve a trade-off among path length, cutting load, machining time, and scallop height [10]. Mgherony and Mikó demonstrated that local surface inclination changes the effective working diameter of a ball-end cutter and consequently changes local cutting conditions [11]. In five-axis machining, Zaragoza Chichell et al. recently proposed simultaneous optimization of toolpaths and tool positioning, highlighting the continuing movement toward integrated process optimization [12].

These studies show that free-form machining research has progressed from isolated geometric operations toward integrated optimization. The specific contribution of the present work is to connect the authors' earlier local geometric cutter-feasibility model with a global discrete cutter-combination objective containing cutting, air-cutting, and tool-change terms.

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3. Geometric Representation of the Free-Form Surface

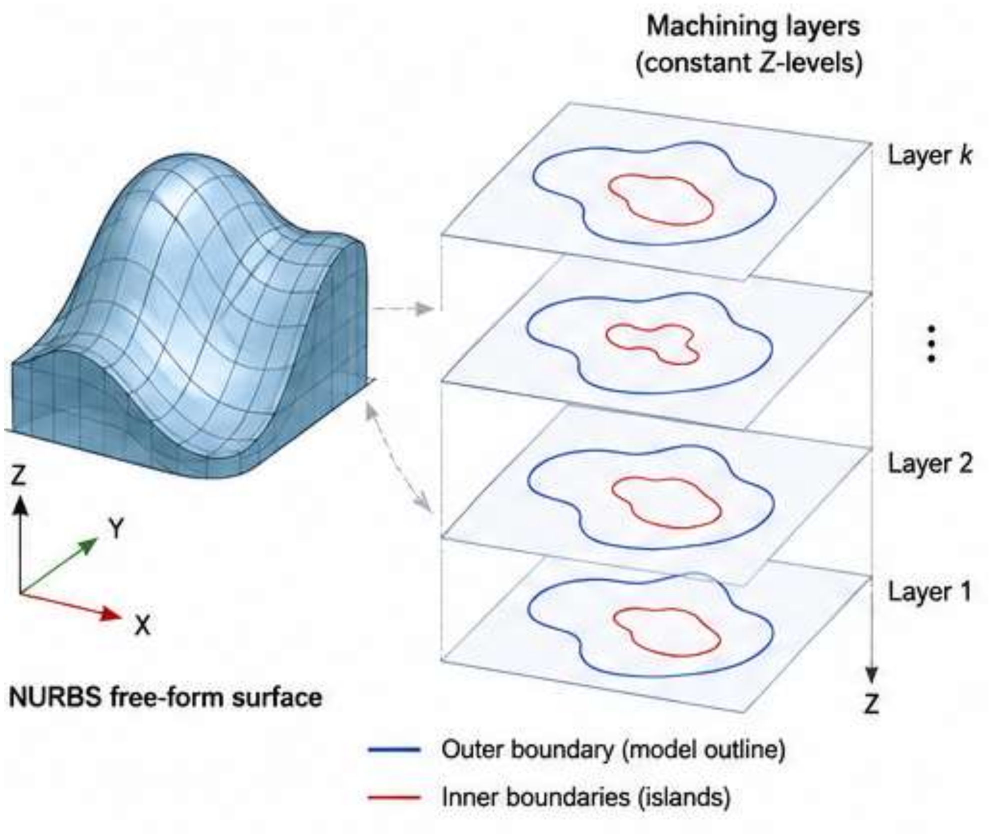

Fig. 2. NURBS free-form surface and machining-layer decomposition.

A tensor-product NURBS surface is represented by:

$$S(u,v) = [\Sigma_{i=0}^{n} \Sigma_{j=0}^{m} N_{i,p}(u) N_{j,\varphi}(v) w_{ij} P_{ij}] \,/\, [\Sigma_{i=0}^{n} \Sigma_{j=0}^{m} N_{i,p}(u) N_{j,\varphi}(v) w_{ij}] \quad (1)$$

where P_{ij} are control points, w_{ij} are weights, $N_{i,p}(u)$ and $N_{j,\varphi}(v)$ are normalized B-spline basis functions, and p and q are polynomial degrees. This representation follows the NURBS-based formulation used in the authors' earlier cutter-selection study [3].

The machining process is represented by a set of horizontal or otherwise predefined roughing layers:

$$\mathcal{L} = \{L_1, L_2, \dots, L_{nl}\} \quad (2)$$

For layer k, the material-removal region is represented as:

$$R_k = R_k^{ext} - \bigcup_{h=1}^{Mk} R_{kh}^{island} \quad (3)$$

where R_k^ext is the external boundary and R_{kh}^island denotes the h-th internal island. This representation generalizes the layer-and-island structure used in the previous study [3].

4. Machining Regions and Geometric Cutter Feasibility

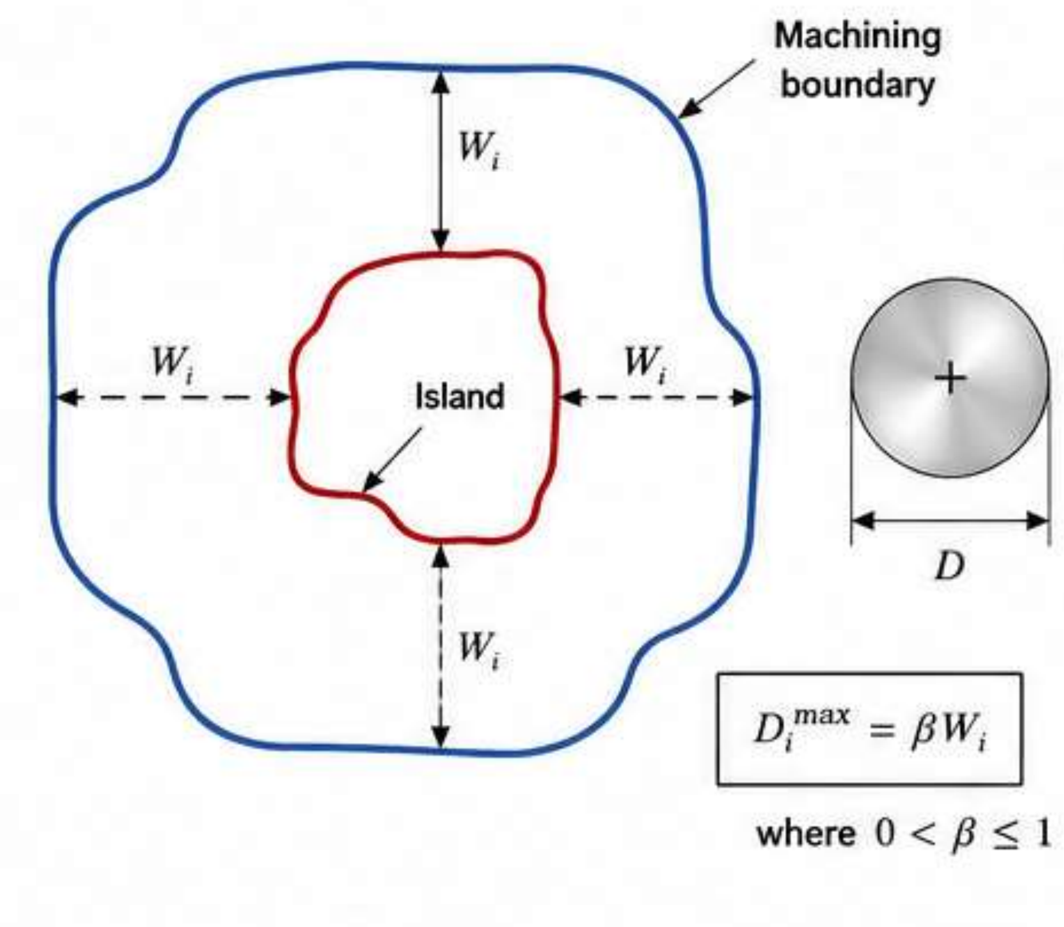

Fig. 3. Geometric determination of the feasible cutter diameter.

The complete machining region is divided into N_R regions:

$\mathcal{R} = \{R_1, R_2, \ldots, R_{\mathrm{nr}}\}$ (4)

For each region i, four descriptors are introduced:

$A_{\mathrm{i}} = machining\ area;\ W_{\mathrm{i}} = minimum\ accessible\ width;\ L_{\mathrm{i}} = characteristic\ length;\ K_{\mathrm{i}} = curvature\ parameter$ (5)

$D = \{D_1, D_2, \ldots, D_{\mathrm{nD}}\}$ (6)

$0 < \beta \leq 1$ (7)

$D_{\mathrm{i}}^{\mathrm{max}} = W_{\mathrm{i}} - 2c_{\mathrm{i}},\ c_{\mathrm{i}} \geq 0$ (8)

The minimum accessible width W_i is the principal geometric quantity used to establish cutter feasibility. Let the available cutter set be:

$\mathcal{D}_{\mathrm{i}}^{\mathrm{f}} = \{D_{\mathrm{j}} \in D \mid D_{\mathrm{j}} \leq D_{\mathrm{i}}^{\mathrm{max}}\}$ (9)

A non-dimensional clearance factor β may be retained as an implementation parameter, but it is not used to define the fundamental geometric limit. Instead, the maximum feasible cutter diameter is defined from the minimum accessible width W_i and an explicit clearance margin c_i. The margin represents the combined geometric safety and tolerance allowance.

$x_{\mathrm{ij}} = \{1, if\ cutter\ D_{\mathrm{j}}\ is\ assigned\ to\ region\ i;\ 0, otherwise\}$ (10)

The feasible cutter set is therefore:

$\Sigma_{\mathrm{j=1}}^{\mathrm{ND}} x_{\mathrm{ij}} = 1,\ i = 1, \ldots, N_{\mathrm{r}}$ (11)

Equation (11) defines geometric feasibility only. A cutter in D_i^F is a candidate; it is not automatically the globally optimal cutter. This distinction is the basis for the optimization layer introduced in the present work.

5. Cutter Assignment and Toolpath-Cost Model

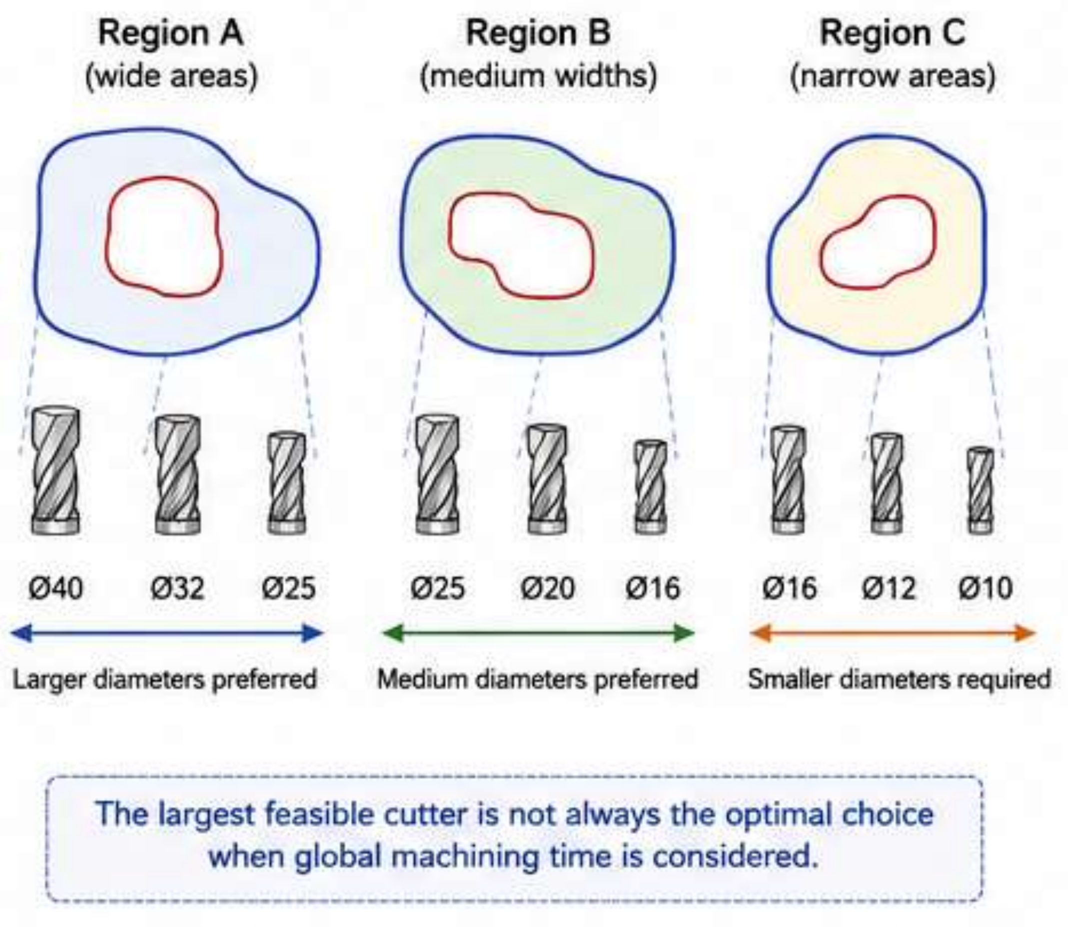

Fig. 4. Candidate cutter selection for different machining regions.

A binary decision variable is introduced:

$x_{ij} = 0 \ if \ D_j > D_i^{max}$ (12)

Each region must receive exactly one cutter:

$s_{ij} = \lambda_i D_j$ (13)

Geometric infeasibility is imposed by:

$n_{ij} = [W_i / s_{ij}]$ (14)

For the theoretical illustration, the effective step-over is represented by Eq. (13), where λ_i is a dimensionless regional step-over coefficient. This is a nominal approximation, not a universal milling law. In a machine-specific implementation, λ_i should be obtained from a scallop-height, surface-tolerance, force, or adaptive-toolpath model that accounts for local curvature and effective cutter geometry.

$L_{ij}^{cut} = \Sigma_{p=1}^{n_{ij}} L_{ijp}$ (15)

where λ_i is a dimensionless regional step-over coefficient. The relationship in Eq. (15) is a modeling assumption rather than a universal milling law. In an implementation, λ_i may instead be obtained from a scallop-height, surface-tolerance, cutting-force, or adaptive-CAM model.

The approximate number of passes across region i is:

$$T_{ij}^{cut} = L_{ij}^{cut}/V^{f}_{,ij} \quad (16)$$

The corresponding cutting-path length is:

$$T^{cut} = \Sigma_{i=1}^{NR}\Sigma_{j=1}^{ND} x_{ij}T_{ij}^{cut} \quad (17)$$

The cutting time for the region is:

$$T^{air}(x,z) = L^{air}(x,z)/V^{rapid} \quad (18)$$

and total cutting time is:

$$x_{ij} \le y_j \quad (19)$$

Actual machining also contains non-cutting movements. Because the required air travel depends on the selected regions and their sequence, it is represented as the decision-dependent quantity $L^{air}(x,z)$, where z denotes sequencing/transition variables.

$$N^{tool} = \Sigma_{j=1}^{ND} y_j \quad (20)$$

To model tool use and tool changes, let y_j be a binary variable equal to one when cutter j is used. Let z_{jk} be a binary transition variable equal to one when the process changes from cutter j to cutter k.

$$N^{tc} = N^{tool} - 1 \quad (21)$$

The number of different cutters is:

$$T^{tc} = \Sigma_{j=1}^{ND}\Sigma_{k=1}^{ND} z_{jk}T_{jk}^{tc} \quad (22)$$

If one cutter is initially loaded, the minimum number of tool changes for a selected set is $N^{tool}-1$. For a prescribed or optimized cutter sequence, however, the actual cost is represented more generally by the transition expression in Eq. (22).

$$T^{total}(x,y,z) = T^{cut}(x) + T^{air}(x,z) + T^{tc}(z) + T^{aux} \quad (23)$$

With transition-dependent tool-change time T_{jk}^{tc}, the tool-change contribution is given by Eq. (22). If all changes have the same duration T_0^{tc}, the expression reduces to $(N^{tool}-1)T_0^{tc}$.

$$T^{total} = \Sigma_i\Sigma_j x_{ij}T_{ij}^{cut} + L^{air}(x,z)/V^{rapid} + \Sigma_j\Sigma_k z_{jk}T_{jk}^{tc} + T^{aux} \quad (24)$$

The total estimated machining time is therefore:

$$min_{(x,y,z)}\, T^{total}(x,y,z) \quad (25)$$

Substitution yields:

$$D_i^{G} = max(\mathcal{D}_i^{f}) \quad (26)$$

The proposed optimization problem is:

$$D* = arg\ min\_D\ T^{total}(D) \quad (27)$$

6. Local Geometric Selection versus Global Optimization

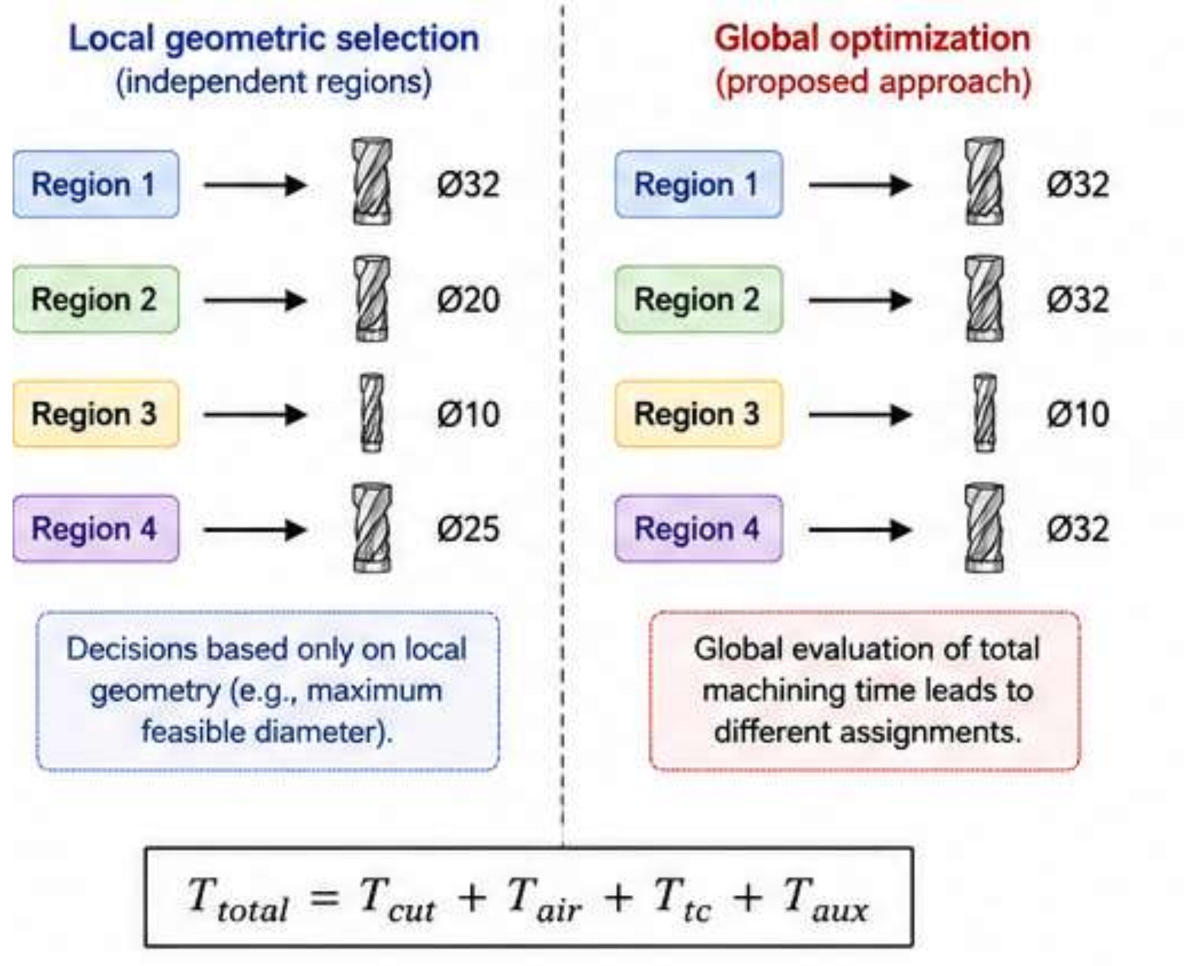

Fig. 5. Comparison between local geometric selection and global optimization.

The local geometric strategy used in the previous methodology can be written as:

$D_i^G \neq D_i *$ (28)

That rule selects the largest feasible cutter independently for every region. The proposed framework instead determines the complete cutter assignment that minimizes the global objective:

$\Delta T = T^{Atotal} - T^{Btotal}$ (29)

Consequently, in the general case:

$\Delta T < 0$ (30)

Equation (30) is the central theoretical proposition of this paper. A large cutter may reduce the number of passes in a region but may simultaneously increase the number of cutter changes or alter non-cutting movements. Conversely, a smaller cutter can be globally attractive when the same tool can efficiently serve several regions.

For two feasible strategies A and B, define the total-time difference:

$|\Delta T^{cut} + \Delta T^{air}| > \Delta T^{tc}$ (31)

Strategy A is preferred when ΔT < 0. If strategy A uses more cutters than strategy B, the additional tool-change time must be compensated by a sufficiently large reduction in cutting and air-cutting time:

$N_{solutions} = N^{DNR}$ (32)

This relation explains analytically why the locally largest feasible cutter is not necessarily the globally optimal cutter.

7. Proposed Optimization Procedure

The proposed computational procedure is organized into ten stages: (1) acquisition of the CAD/NURBS surface; (2) generation of roughing layers; (3) identification of external boundaries and internal islands; (4) calculation of regional geometric descriptors; (5) generation of candidate cutters; (6) elimination of geometrically infeasible cutters; (7) estimation of cutting passes and cutting distance; (8) estimation of air-cutting and tool-change time; (9) evaluation of feasible cutter combinations; and (10) selection of the minimum-time combination.

The unpruned theoretical search space is:

$$N_{\mathrm{solutions}}{}^{\mathrm{f}} = \prod_{\mathrm{i}=1}^{\mathrm{NR}} |\mathcal{D}_{\mathrm{i}}{}^{\mathrm{f}}| \quad (33)$$

After geometric feasibility pruning, the search space becomes:

$$n_{\mathrm{ij}} = \lceil W_{\mathrm{i}}/(\lambda_{\mathrm{i}} D_{\mathrm{j}}) \rceil \quad (34)$$

Equation (34) shows that the geometric stage has a second role: it reduces the combinatorial search space. For small problems, exhaustive enumeration is possible. For larger problems, the formulation can be solved with mixed-integer optimization, branch-and-bound, dynamic programming, or metaheuristic search. The present paper does not claim a computationally superior solver; the solver choice is deliberately left open for future implementation.

8. Numerical Illustration and Analytical Verification

To demonstrate the behavior of the formulation without introducing experimental claims, a small hypothetical problem is considered. Three machining regions and two cutter options are used. The example is intentionally synthetic and serves only as an analytical counterexample to independent largest-feasible-cutter selection.

Region	Feasible cutters	Cutting time with Ø10 (s)	Cutting time with Ø20 (s)
R1	Ø10, Ø20	100	70
R2	Ø10, Ø20	100	70
R3	Ø10	40	—

Assume 5 s of common air-travel time. If the tool-change time between Ø10 and Ø20 is 30 s, the local largest-feasible-cutter rule gives 70+70+40+30+5 = 215 s, whereas the single-Ø10 strategy gives 100+100+40+5 = 245 s. If the tool-change time increases to 80 s, the local strategy becomes 265 s while the single-tool strategy remains 245 s. The global preference is therefore reversed without changing geometric feasibility. This simple counterexample demonstrates the mechanism represented by the objective function.

$$|\Delta T^{\mathrm{cut}} + \Delta T^{\mathrm{air}}| < \Delta T^{\mathrm{tc}} \;\Rightarrow\; single - tool\ strategy\ is\ preferred \quad (38)$$

The numerical illustration is not a prediction of machining performance and does not constitute experimental validation. Its purpose is to demonstrate that the proposed objective can mathematically produce a different global decision from the independent largest-feasible-cutter rule when tool-change cost becomes sufficiently large.

9. Theoretical Properties of the Model

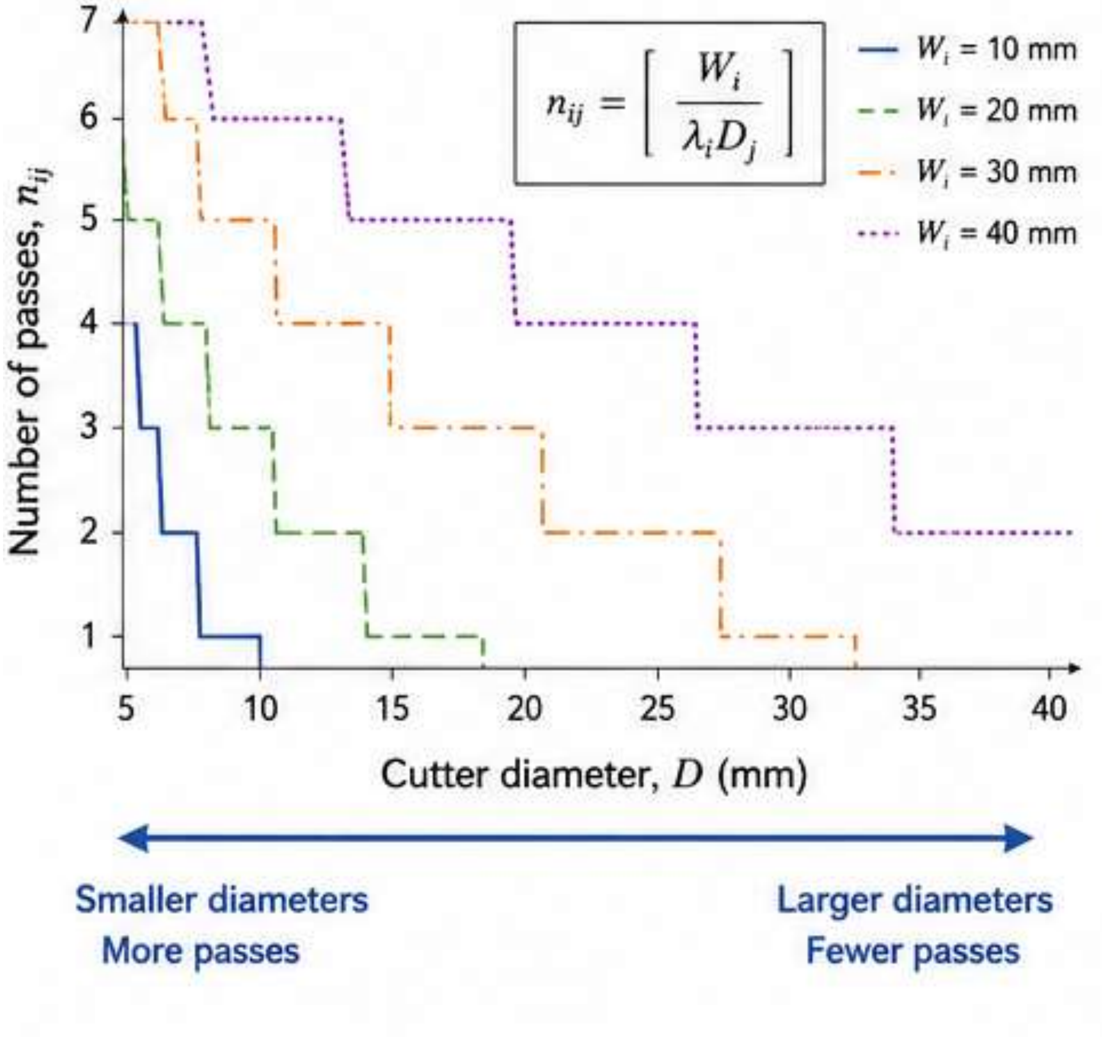

Fig. 6. Relationship between cutter diameter and number of machining passes.

For a fixed region and constant λ_i, the number of passes is:

$D_a > D_\beta \Rightarrow n_{ia} \leq n_{i\beta}$ (35)

If D_a > D_b and both are feasible, then:

$T* \leq T^G$ (36)

Thus, under the simplified analytical model, increasing cutter diameter cannot increase the calculated number of passes while the cutter remains geometrically feasible.

Let T_G denote the total time obtained by the local largest-feasible-cutter rule and T* the optimum of the global problem. Because the local rule generates one feasible solution within the global search space:

$T* \leq T^G$ (37)

Equation (37) is a mathematical property of the optimization formulation and is not an experimentally measured improvement. It does not justify any percentage reduction in machining time without numerical or experimental validation.

10. Discussion

The proposed framework should be interpreted as an extension of the authors' earlier geometric method rather than as a rejection of that method. The earlier study solved the local problem of determining a suitable cutter dimension from the minimum dimensions of machining regions [3]. The present study adds a global process-planning layer in which the combination of cutters is evaluated against a total machining-time objective.

This distinction is consistent with the development of free-form machining research. Tool-selection studies have related cutter choice to machining time [4]; process-aware toolpath optimization has incorporated cutting-force objectives [5]; regional methods have considered geometric characteristics, curvature, and energy [7], [8]; adaptive methods have incorporated local surface features [6], [9], [11]; and recent five-axis research has simultaneously optimized toolpaths and tool motion [12].

The model also clarifies an important practical trade-off. If only local geometry is considered, the largest feasible cutter appears attractive because it reduces the number of passes. Once global process cost is introduced, however, cutter changes and non-cutting movements become explicit terms in the objective. The optimum is therefore determined by the complete machining strategy rather than by cutter diameter alone.

The proposed model is deliberately formulated at a level that can be translated into a CAD/CAM process-planning module. Surface boundaries and islands can be extracted from the CAD model; candidate cutters can be obtained from the machine-tool library; feed rates and rapid-traverse parameters can be obtained from process planning; and tool-change time can be obtained from the machine-tool specification. The optimization layer can then return the cutter combination and associated cost estimate.

11. Limitations and Future Work

The present study is theoretical. The step-over relationship in Eq. (15) is a normalized modeling assumption and should not be interpreted as a universal milling law. In a practical implementation, step-over should be calculated from a specified surface-tolerance, scallop-height, force, or adaptive-toolpath criterion.

The current formulation does not explicitly model cutting forces, tool deflection, tool wear, spindle-power transients, machine acceleration, controller interpolation, collision detection, or detailed feed-rate scheduling. Tool-change time is represented by an average parameter. These simplifications are appropriate for establishing the optimization structure but must be refined for machine-specific implementation.

Future work should include numerical benchmarking using synthetic and industrial free-form geometries, followed by experimental validation. Additional objectives can include cutting force, energy consumption, tool wear, surface error, and tool life. A five-axis extension can introduce tool orientation, collision, and machine-kinematic constraints. Such an extension would be consistent with recent optimization-based five-axis free-form machining research [12].

12. Conclusions

A theoretical framework for integrated cutter selection and toolpath optimization in rough machining of NURBS-based free-form surfaces has been presented. The framework extends the authors' previous NURBS-based geometric cutter-selection methodology by treating cutter diameter as an explicit discrete process-planning variable.

The principal conclusions are: (1) geometric feasibility and global machining optimality are distinct concepts; (2) larger feasible cutters generally reduce the estimated number of passes within the proposed analytical model; (3) multiple cutter diameters introduce a tool-change cost; (4) air-cutting movements contribute to total machining time; (5) cutter assignment can be formulated as a discrete optimization problem; and (6) the globally optimal cutter combination need not consist of the individually largest feasible cutter for every region.

The most important theoretical result is the transition from $D_i^G = \max(D_i^f)$ to D* = arg min_D $T^{total}(D)$. The numerical illustration demonstrates, without experimental inference, that the preferred cutter combination can change when tool-change cost becomes sufficiently large. The study does not claim a quantitative machining-time reduction for a physical machine. Numerical benchmarking and experimental validation remain necessary before such quantitative claims can be made.

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