International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Exploring the Radioactive Decay Mode for ^{191}Tl nuclei

M. G. Srinivas

Government College for Women, Kolar - 563101, Karnataka, India

Abstract
The radioactive decay characteristics of the proton-rich nucleus ^{191}Tl were systematically investigated by examining the competition among the energetically possible decay modes including alpha-decay, cluster decay, beta-decay, and spontaneous fission. The half-lives of the respective decay channels were evaluated using established theoretical and semi-empirical models, namely the Royer formula for α-decay, the Universal Decay Law (UDL) for cluster radioactivity, empirical relations for β^+-decay, β^--decay and the Xu*et al.*, formalism for spontaneous fission. The calculated results indicate that cluster decay possesses the largest positive (Q)-value but remains highly suppressed because of the large Coulomb barrier associated with heavy-particle emission. Although α-decay is energetically allowed, its relatively long half-life limits its contribution to the decay process. The comparison of calculated half-lives demonstrates that β^+-decay has the shortest half-life among all competing channels, establishing it as the dominant decay mode of ^{191}Tl. In contrast, spontaneous fission and cluster decay exhibit extremely long half-lives and are therefore insignificant. The present study highlights the crucial role of decay energy, Coulomb barrier penetration, and nuclear structure in determining the dominant radioactive decay mechanism and provides a comprehensive theoretical description of the decay behavior of ^{191}Tl.

Keywords : Radioactive decay; Alpha-decay; Proton radioactivity; Beta-decay; Electron capture; Spontaneous fission; Quantum tunneling; Nuclear stability.

1. INTRODUCTION

Radioactive decay is a process of emission of particle or electron capture brings the changes in the atomic number of the radionuclide, whereas decay by gamma-ray emission does not[1]. Radioactive decay is believed to be exponential and for slowly decaying isotopes it is non-exponential[2]. Thallium (Tl) was discovered by the English chemist William Crookes in 1861 and in the year 1862 metallic thallium was first prepared by a French scientist C Lamy. Pure thallium is a soft, malleable and bluish-white metal and that is found in trace amounts in the earth’s crust[3]. Thallium isotopes have been used to investigate paleoceanographic processes in the Cenozoic, utilized as a monitor of the marine manganese oxide burial flux over million year time scales, used to calculate the magnitude of hydrothermal fluid circulation through ocean crust [4].

Radioactivity is the process of spontaneous disintegration of certain atomic nuclei into the emission of radiations in the form of particles such as alpha, beta, clusters or electromagnetic waves[5]. It was first discovered by Henry Becquerel [6], its exhibits major role in the field of nuclear physics since it finds applications in various fields such as medicine, scientific research, energy production. The α-decay process was first explained by Gamow in the year 1920[7], later it was independently explained by Condon and Gurney[8] as a quantum tunneling effect. Dong et al.,[9], Royer et al.,[10], Sobiczewski, Adam et al.,[11] investigated α decay half-lives using various models and formulae.

Beta decay involves the emission of β^+ or β^- particle from proton-rich or neutron-rich nuclei respectively. Semi emperical formulae were introduced to evaluate half-lives of β^+-decay and β^--decay processes [12, 13]. Various proximity potentials have been employed in order to study cluster decay, α decay and heavy particle radioactivity [14–16]. Systematic calculations were carried out in the evaluation of spontaneous fission half-lives of heavy nuclei Swiatecki's formula with new parameters [17].

Thallium is a rare element and is more toxic to human than mercury, lead, cadmium[3]. ^{176}Tl was discovered by Kettunen etal. ,in the year 2004[18], Tl isotopes from mass number A=176-217 have been investigated so far, out of which two nuclei were found to be stable, 13 nuclei were neutron rich and 27 nuclei were observed as neutron deficient isotopes[19]. Tl isotope signatures remain largely consistent across Zn mining and metallurgical waste despite wide variations in Tl concentrations. Slightly lower values of ^{205}Tl values in fly ash suggest kinetic fractionation during Tl evaporation, while contaminated soils preserve the isotopic fingerprint of the pollution source [20]. Y.S Horowitz et al.,[21] described the theory of heavy charged particle response with Tl materials. A detailed decay spectroscopy study of neutron-deficient ^{178}Tl identified -delayed fission for the first time and revealed an asymmetric fission-fragment mass distribution in^{178}Hg also its α-decay provided information on the low-energy excited states of the daughter nucleus ^{174}Au [22].

Despite significant progress in understanding radioactive decay mechanisms such as α-decay, β-decay, cluster decay, and spontaneous fission, most studies have focused on heavy and super heavy nuclei or on the experimental characterization of thallium isotopes.

Although several Tl isotopes have been investigated for their decay properties, isotopic signatures, and spectroscopic behavior, a systematic study of the decay-mode competition in the proton-rich nucleus (^{191}Tl) remains scarce. In particular, the relative probabilities decay, β-decay, cluster decay, and spontaneous of α- fission have not been comprehensively compared using a unified theoretical frame work. This lack of information limits our understanding of the stability and decay characteristics of nuclei near the proton drip line. Motivated by this research gap, the present work investigates all energetically allowed decay channels of (^{191}Tl) by evaluating their Q-values and half-lives using established theoretical and semi-empirical models. The study aims to identify the dominant decay mode and provides reliable theoretical predictions that can guide future experimental investigations of proton-rich thallium nuclei.

2. THEORETICAL FRAMEWORK

A. Alpha Radioactivity

The Royer semi-empirical formula [23] is one of the most widely used expressions for calculating α-decay half-lives. It was proposed by G. Royer by fitting a large set of experimental α-decay data. For even-even nuclei the expression for alpha-decay is written as;

$$\log_{10}T_{1/2} = -25.752 - 1.15055A^{1/6}\sqrt{Z} + \frac{1.5913Z}{\sqrt{Q_\alpha}}, \quad (1)$$

For even-odd nuclei;

$$\log_{10}T_{1/2} = -34.156 - 0.87478\, A^{1/6}\sqrt{Z} + \frac{1.6923Z}{\sqrt{Q_\alpha}}, \quad (2)$$

For odd-even nuclei;

$$\log_{10}T_{1/2} = -34.156 - 0.87478\, A^{1/6}\sqrt{Z} + \frac{1.6923Z}{\sqrt{Q_\alpha}}, \quad (3)$$

For odd-odd nuclei;

$$\log_{10}T_{1/2} = -31.186 - 0.98047\, A^{1/6}\sqrt{Z} + \frac{1.6744Z}{\sqrt{Q_\alpha}}, \quad (4)$$

B. Universal decay of Cluster radioactivity

The Universal Decay Law (UDL) proposed by Qi etal., [24] is widely used for both α-decay and cluster radioactivity. The general expression is

$$\log_{10}T_{1/2} = a\,\chi' + b\,\rho' + c \quad (5)$$

where $\chi' = Z_c Z_d\sqrt{\frac{A}{Q_c}}$, $\rho' = \sqrt{AZ_cZ_d(A_d^{1/3} + A_d^{1/3})}$ and $A = \frac{A_dA_c}{A_d+A_c}$

C. β± - decay

The β⁻ decay half lives[12] are expressed as ;

$$\log_{10}T_{1/2} = (C_1Z+C_2)N + C_3Z+C_4+ \mathrm{Shell}(Z,N) \quad (6)$$

where Shell(Z,N) is the shell correction[12],and the

The β^+ decay half lives[13] are evaluated using the following equation;

$$\log_{10}T_{1/2} = (C_1Z+C_2)N + C_3Z+C_4 \quad (7)$$

parameter C_i is evaluated as explained in literature[12,13]

D. Spontaneous fission

By using the semi-empirical formula given by Xu et al., [25], the spontaneous fission half-lives are evaluated and is given by ;

$$T_{1/2} = \exp\left(2\pi\left[C_o + C_1A + C_2Z^2 + C_3Z^4 + C_4(N-Z)^2 - \left(0.13323\frac{Z^2}{A^{\frac{1}{3}}} - 11.64\right)\right]\right) \quad (8)$$

3. RESULTS AND DISCUSSIONS

The radioactive decay properties of the ^{191}Tl nucleus were systematically investigated by evaluating the half lives of all energetically possible decay modes, including α-decay, cluster decay, β^+-decay, β^--decay and spontaneous fission (SF). The corresponding half-lives were calculated using well-established theoretical and semi-empirical models such as the Royer formula for α-decay, the Universal Decay Law (UDL) for cluster radioactivity, and the empirical formulas for β^+-decay, β^--decay, and the Xu *et al.*, formalism for spontaneous fission. The decay energies (Q-values) required for α and cluster radio activity were obtained from nuclear mass data [26], and the calculations were performed only for energetically allowed decay channels.

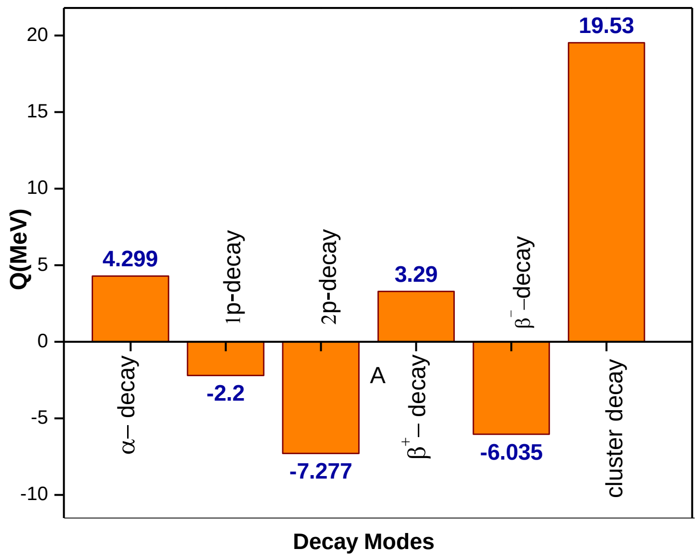

FIG.1. Comparison of the decay energies (Q-Values) for different decay modes of ^{191}Tl nuclei

Figure1 presents the calculated decay energies ((Q)-values) for the energetically possible decay modes of the 191Tl nucleus. Among all the investigated channels, cluster decay exhibits the largest positive (Q)-value of approximately 19.53 MeV, indicating a substantial energy release during the decay process. However, although a large (Q)-value generally enhances the barrier penetrability, the emission of a heavy cluster is strongly hindered by the significantly wider and higher Coulomb barrier, resulting in extremely long half-lives compared with lighter-particle emission. The $\beta^{`}$-decay channel possesses a positive(Q)-value of about 3.29 MeV, suggesting that it is energetically allowed, where as the (Q)-value for α-decay is approximately 4.299 MeV, which is sufficiently large to permit quantum tunnelling through the Coulomb barrier. The calculated (Q)-values for $\beta^{'}$-decay, one-proton (1p) and two-proton (2p) emission are negative, indicating that these decay channels are energetically forbidden for ^{191}Tl. Thus, $\beta^{'}$-decay and proton radioactivity is not expected to occur in this nucleus.

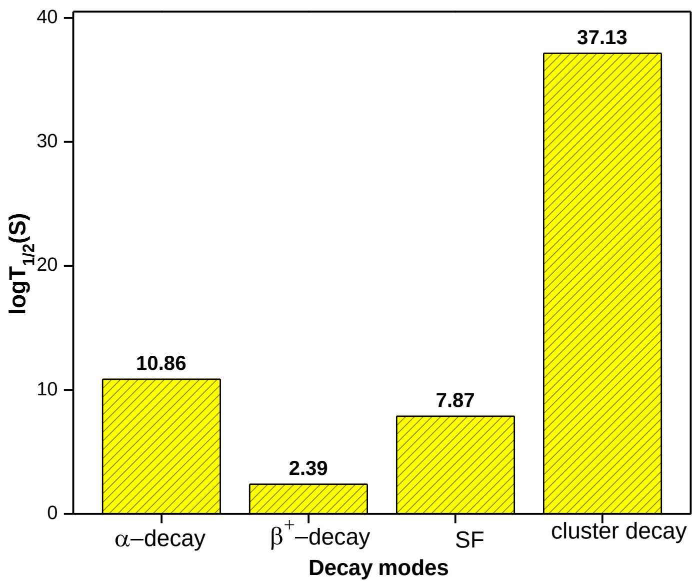

FIG.2. A comparison of logarithmic half-lives of different possible decay modes of ^{191}Tl nuclei

Figure 2 compares the calculated logarithmic half-lives $\log_{10}T_{1/2}$ of the possible radioactive decay modes of the ^{191}Tl nucleus, namely α-decay, cluster decay, β^{+}-decay, cluster-decay, and spontaneous fission (SF). The calculated half-lives exhibit significant variations among the competing decay channels, reflecting the combined influence of decay energy, Coulomb barrier penetration, and nuclear structure effects. The β^{+}-decay mode possesses the shortest half-life, with 2.39s, indicating that it is the dominant decay mechanism for ^{191}Tl. This prediction is consistent with the proton-rich nature of the nucleus, where proton-to-neutron conversion through positron emission or electron capture drives the nucleus toward the valley of stability. In contrast, the calculated α-decay half-life is considerably longer of 10.86s, implying that although α-decay is energetically allowed, it is strongly suppressed by the Coulomb barrier. The cluster decay channel exhibits the largest half-life of 37.13S, demonstrating that heavy-cluster emission is highly improbable despite its large decay energy because of the extremely small barrier penetrability associated with the emission of a heavier fragment. The spontaneous fission half-life is also substantially longer than that of β^{+}-decay mode, confirming that spontaneous fission is not a competitive decay process for this medium-heavy nucleus. Overall, the comparison of half-lives clearly establishes β^{+}-decay as the most probable decay mode, while α-decay, cluster decay, and spontaneous fission contribute negligibly to the decay of ^{191}Tl, highlighting the strong dependence of decay competition on both the available decay energy and quantum tunnelling probability.

4. CONCLUSIONS

The radioactive decay modes of the proton-rich nucleus ^{191}Tl were systematically investigated using established theoretical and semi-empirical models. The calculated (Q)-values indicate that α-decay and cluster decay are energetically allowed, whereas proton-emission channels are forbidden. However, the half-life calculations show that β^{+}-decay possesses the shortest half-life, establishing it as the dominant decay mode of ^{191}Tl. In contrast, α-decay, cluster decay, and spontaneous fission are strongly suppressed because of their significantly longer half-lives. These results demonstrate that the competition among decay modes is governed not only by the available decay energy but also by the Coulomb barrier penetration probability and nuclear structure effects. The present study provides a theoretical under-standing of the decay characteristics of ^{191}Tl and may serve as a useful reference for future experimental and theoretical investigations of proton-rich nuclei in this mass region.

REFERENCES

1. G. B. Saha, “Radioactive decay, in Fundamentals of Nuclear Pharmacy”,(Springer, 2010) pp. 11–31.
2. P. J. Aston, “Is radioactive decay really exponential?”, EPL (Europhysics Letters) **97**, 52001 (2012).
3. A. J. Peter and T. Viraraghavan, “Thallium: a review of public health and environmental concerns”, Environment international **31**, 493 (2005).
4. S. G. Nielsen, M. Rehkamper, and J. Prytulak, “Investi-gation and application of thallium isotope fractionation”, Reviews in Mineralogy and Geochemistry**82**,759(2017).
5. S. Vallabhajosula, “Radioactivity, in Molecular Imaging and Targeted Therapy:

Radiopharmaceuticals and Clinical Applications"(Springer, 2023) pp. 49–62.
6. R. Mould, "Antoine-henri becquerel 1852-1908, Nowotwory". Journal of Oncology **58**, 554 (2008).
7. G. Gamov, "Quantum theory of the atomic nucleus", Z.Phys **51**, 204 (1928).
8. R. W. Gurney and E. U. Condon, "Quantum mechanics and radioactive disintegration", Physical Review **33**,127 (1929).
9. T. Dong and Z. Ren, "New calculations of α-decay half-lives by the viola-seaborg formula", The European Physical Journal A -Hadrons and Nuclei **26**, 69 (2005).
10. G. Royer, "Alpha emission and spontaneous fission through quasi-molecular shapes", Journal of Physics G: Nuclear and Particle Physics **26**, 1149 (2000).
11. A.Sobiczewski and K.Pomorski, "Description of structure and properties of super heavy nuclei", Progress in Particle and Nuclear Physics **58**, 292 (2007).
12. S.Zong-Qiang, S.Liang-Ping, M.Ying, H.Ji-Gang,and Q.Jian-Fa , "Competition between α-decay and β-decay for heavy and super heavy nucle"i, Chinese Physics C**38**, 124101 (2014).
13. X.Zhang and Z.Ren, "New exponential law of β+-decay half-lives of nuclei far from β-stable line", Physical Review C**73**, 014305 (2006).
14. A. M. Nagaraja, H. C. Manjunatha, N. Sowmya, N. Manjunath, and S. A. C. Raj, "Cluster radioactivity of super-heavy nuclei $^{290-310}$120 using different proximity functions", The European Physical Journal Plus **135**, 814 (2020).
15. H. C.Manjunatha, N.Sowmya, P. S.DamodaraGupta,K. N. Sridhar, A. M. Nagaraja, L. Seenappa, and S. Alfred Cecil Raj, "Investigation of decay modes of super-heavy nuclei", Nuclear Science and Techniques **32**,1(2021).
16. H. C. Manjunatha, G. R. Sridhara, N. Sowmya, P. S. Gupta, N. Manjunatha, and H. B. Ramalingam, "Alpha decay half-lives of actinides using different proximity potentials", Indian Journal of Physics **96**, 2485 (2022).
17. C.Xu and Z.Ren, "Systematical law of spontaneous fission half-lives of heavy nuclei", Physical Review C-Nuclear Physics **71**, 014309 (2005).
18. H.Kettunen, T.Enqvist, T.Grahn, P.Greenlees, P.Jones, R.Julin, S.Juutinen, A.Keenan, P.Kuusiniemi, M. Leino,*et al.*, "Decay studies of Au $^{170, 171}$, Hg $^{171–173}$, and Tl ^{176}", Physical Review C—Nuclear Physics**69**, 054323 (2004).
19. C. Fry and M. Thoennessen, "Discovery of the thallium, lead, bismuth, and polonium isotopes", Atomic Data and Nuclear Data Tables **99**, 365 (2013).
20. A.Vanek, Z.Grosslova, M.Mihaljevic, V.Ettler,J.Trubac, V.Chrastny, V.Penizek, L.Teper, J.Cabala,A.Voegelin,*etal.*,"Thallium isotopes in metallurgical wastes/contaminated soils: A novel tool to trace metal source and behavior", Journal of Hazardous Materials **343**, 78 (2018).
21. Y. S. Horowitz, O. Avila, and M. Rodriguez-Villafuerte, "Theory of heavy charged particle response (efficiency and supralinearity) in Tl materials", Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms**184**, 85 (2001).
22. V.Liberati, A.Andreyev, S.Antalic, A.Barzakh, T.E.Cocolios, J.Elseviers, D.Fedorov, V.Fedoseeev, M.Huyse, D.Joss,*etal.*,"β-delayed fission and α-de-cay of ^{178}

Tl", Physical Review C—Nuclear Physics **88**, 044322 (2013).

23. G.Royer, "Analytic expressions for alpha-decay half-lives and potential barriers", Nuclear Physics A**848**, 279 (2010).
24. C. Qi, F. Xu, R. J. Liotta, and R. Wyss, "Universal decay law in charged-particle emission and exotic cluster radioactivity", Physical review letters**103**, 072501 (2009).
25. C. Xu, Z.Ren, and Y.Guo, "Competition between α decay and spontaneous fission for heavy and super heavy nuclei", Physical Review C—Nuclear Physics**78**, 044329 (2008).
26. P. Moller, J. Nix, W. Myers, and W. Swiatecki, "Nuclear ground-state masses and deformations", Atomic Data and Nuclear Data Tables **59**, 185 (1995).