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Numerical Investigation of Rotatory Systems and Heat Transfer Features of Hybrid Nanofluid

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Abstract
Squeezing flow refers to the fluid motion generate when two parallel plates move towards or away from each other. In biomedical contexts, it models the behaviour of blood or drug-laden fluids as they are forced through confined spaces such as intercellular gaps and narrow capillaries. This current investigation focuses graphene oxide based hybrid nanofluid with magnetic field and Casson rheological model to controlled drug delivery application model is applied with similarity transformation to express the governing differential equations in terms of similarity variable and solved in shooting method and MATLAB software.

Keywords: Biomedical application, Hybrid nanofluid, rotating flow, Numerical analysis, MATLAB.

1. Introduction

Nanofluids are suspensions of nanoparticles in a base fluid, engineered to enhance the fluid's properties for specific applications. In drug delivery, nanofluids can carry therapeutic agents in their dispersed nanoparticles offering a promising method for targeted and efficient treatment. The unique properties of nanofluids such as increased surface area, tenable size, and enhanced solubility make them suitable for delivering drugs in a controlled and site-specific manner. Many researchers are working to develop smart hybrid nanofluids, combining magnetic and photothermal properties for combined therapy and diagnosis (theragnostic).Ghasemi et al.[1] studied blood flow in arteries by considering cylindrical geometry by utilising various analytical methods. The authors have used third grade non-Newtonian rheological model to analyse blood flow and their results shows that thermophoresis causing an augmented concentration profile. Later GC Shit and Shreeparna Majee [2] considered wavy channel geometry to represent stenosed arteries to analyse blood flow. Their results concluded that Nusselt number decreases with magnetic field strength for higher Reynold's number. Ghadikolaei et al.[3] reported a comparative study on squeezing flow with carbon nanotubes suspended into ethylene glycol fluid.Alnahdi et al[4] studied couple stress ternary hybrid nanofluid flow in channel and explored about the possibilities of drug delivery with metallic and non-metallic nanoparticles. Carbon nanotube (CNT) based nanofluids are widely studied and frequently used in blood flow modelling or biomedical applications because of their exceptional physical and thermal properties, which make them promising for certain therapeutic and diagnostic purposes. Hussain et al [5] examined time dependent squeezing flow in rotating channel. In another work Saba et al.[6] investigated thermal characteristics of magnetite and corban nanotube particles with asymmetric channel geometry and their results portrays that single wall corban nanotubes have better impact on temperature profile as compared to multi wall corban nanotubes.

Noor et al.[7] studied behaviour of nanoparticles as drug agents in blood by analysing Casson rheological model with corban nanotubes. Ijaz Khan et al. [8] performed entropy generation analysis for squeezing flow with second grade non-Newtonian fluid. Bilal et al [9] studied iron oxide and corban nanotubes based hybrid nanofluid effects in channel squeezing flow. Recently few researchers have proved the importance of nanofluids for biotechnology applications.Nikfar et al.[10] simulated the role of rapid squeezing for intercellular drug delivery.Tamboli and Muralidharan [11] reviewed importance of magnetic field in targeted drug delivery modelling. Most recently Ratha et al.[12] scrutinised role of thermal radiation and magnetic field strength on blood in vessels by making use of mathematical model of squeezing flow between parallel surfaces.

Analysis of heat generation and absorption in squeezing flow between parallel surfaces is important for both theoretical understanding and practical applications. Whenever the fluid is squeezed, internal friction (viscosity) between fluid layers converts mechanical energy into heat. In blood flow modelling, narrow capillaries or between moving tissue surfaces, heat generation/absorption can simulate metabolic heating or heat removal by blood flow. Many research works are available in literature which are focused on heat source and sink effects. Upreti et al.[13] studied impact of uniform heat source-sink effects in squeezing flows within parallel system.Alderremy et al.[14] reported microfluidics problem and impacts of heat generation on biofluid flow with squeezing walls.Dinarvand et al.[15] studied squeezing flow with uniform heat source effect and their outcomes shous that heat source raises temperature profile. Yaseen et al.[16] studied nanofluid flow between parallel surfaces by considering uniform heat generation effects. Most of the authors discussed uniform heat source sink effects in squeezing flows, but in many applications lower wall and upper wall of channels may not be under homogenous heating. Hence impact of varying heat source-sink effect is not understood fully in squeezing flows. Some of the authors attempted to investigate this phenomenon in boundary layer flows with flat plate type geometry. Alharbi et al.[17] studied fluid flow past a moving surface with thermal radiation and exponential heat source sink effects. Ali et al.[18] examined nanofluid flow past Riga plates with varying heat source-sink term. Khan et al.[19] investigated role of varying heat generation-absorption effect for non-Fourier heat flux scenario.Nihaal et al.[20] scrutinised rotating disc geometries and significance heat generation in bioconvective fluid flows. Although a significant body of research has investigated heat transfer in squeezing flows between parallel plates, the majority of studies have focused on cases involving uniform or linear heat sources and sinks. These assumptions limit the accuracy of thermal predictions in systems with spatially variable energy generation, such as those influenced by radiative effects or nonlinear chemical reactions. The effect of exponential heat sources or sinks on the temperature distribution and flow characteristics in such constrained geometries has, however, been largely neglected in existing literature.

The current study aims to fill this research gap by formulating and solving the non-Newtonian fluid flow problem between two parallel plates, incorporating the influence of Casson model. The novelty of this work lies in the inclusion of Casson rheological model ensures more accurate representation of blood flows in drug delivery applications. This research not only enhances the classical understanding of biofluid flow models but also offers a critical benchmark for thermal design in microscale systems, radiative heat transfer modelling, and applications requiring energy-efficient fluid processing.

2. Mathematical model.

Let us consider an incompressible Casson hybrid nanofluid flow between two parallel plates where one plate is stretchable, and the other is permeable. Detailed representative diagram is shown in Figure-1 with Cartesian coordinate system (x, y), where the x-axis lies along the surface, and the y-axis is perpendicular to it. The parallel plates are aligned horizontally along the x-axis. The lower plate is subjected to stretching, and the influence of solar radiation is examined

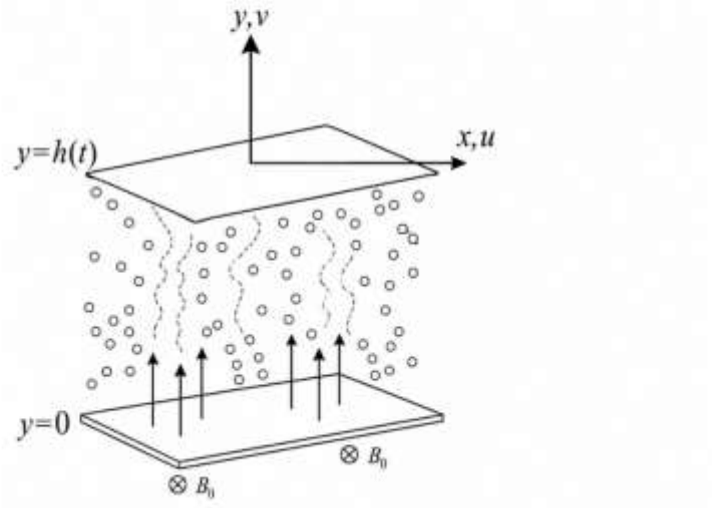

Figure 1: Representative model for the flow problem

$$\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y} \quad (1)$$

$$\rho_{hnf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + 2\Omega w\right) = -p_x + \mu_{hnf}\left(1+\frac{1}{\gamma}\right)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma_{hnf}{B_o}^2 u \quad (2)$$

$$\rho_{hnf}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -p_y + \mu_{hnf}\left(1+\frac{1}{\gamma}\right)\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

$$\rho_{hnf}\left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} - 2\Omega u\right) = -p_y + \mu_{hnf}\left(1+\frac{1}{\gamma}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) - \sigma_{hnf}{B_o}^2 w \quad (4)$$

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = \alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \frac{\frac{\partial q_r}{\partial y}}{(\rho c_p)_{hnf}} + \frac{\sigma_{hnf}}{(\rho c_p)_{hnf}}(u^2 + w^2) \quad (5)$$

The boundary conditions for upper and lower plate of the geometry are s follows

$u = u_w(x) = ax, v = 0, w = 0, T = T_1$ for the lower plate $y = 0$. (6)

$u = 0,\ v = v_w, w = 0, T = T_2$ for the lower plate $y = H$. (7)

Boundary layer equations are often complex and involve multiple independent variables. similarity variable combines these variables into a single dimensionless variable, simplifying the equations and allowing for easier analytical or numerical solutions.

So let us introduces the similarity variable transformations

$$\eta = \frac{y}{H}, u = axF'(\eta), v = -aHF(\eta), w = axG(\eta), \theta(\eta) = \frac{T-T_1}{T_1-T_2} \quad (8)$$

The non dimensionalised ordinary differential equations are obtained by introducing the above similarity transformation can be written as:

$$\left(1+\frac{1}{\gamma}\right)\frac{\partial^4 F}{\partial \eta^4} + \Delta_1\Delta_2 Re\left(F\frac{\partial^3 F}{\partial \eta^3} - \frac{\partial F}{\partial \eta}\frac{\partial^2 F}{\partial \eta^2}\right) - 2\Delta_1\Delta_2 R_o\frac{\partial G}{\partial \eta} - M\Delta_1\frac{\partial^2 F}{\partial \eta^2} = 0 \quad (9)$$

$$\left(1+\frac{1}{\gamma}\right)\frac{\partial^2 G}{\partial \eta^2} + \Delta_1\Delta_2 Re\left(F\frac{\partial G}{\partial \eta} - \frac{\partial F}{\partial \eta}G\right) + 2\Delta_1\Delta_2 R_o\frac{\partial F}{\partial \eta} - M\Delta_1 G = 0 \quad (10)$$

$$\left(\frac{k_{hnf}}{kf}\left[1+\frac{4N_r}{3}\right]\right)\frac{\partial^2\theta}{\partial\eta^2}+\Delta_3 PrReF\frac{\partial\theta}{\partial\eta}+MPrEc\left(\left(\frac{\partial G}{\partial\eta}\right)^2+(G)^2\right)=0 \qquad (11)$$

Where $\Delta_1=(1-\phi_1-\phi_2)+\frac{\phi_1\rho_1+\phi_2\rho_2}{\rho_f}$, $\Delta_2=(1-\phi_1-\phi_2)^{2.5}$ and

$$\Delta_3=(1-\phi_1-\phi_2)+\frac{\phi_1(\rho c_p)_1+\phi_2(\rho c_p)_2}{(\rho c_p)_f}$$

The surface friction coefficient and Nusselt number are important dimensionless parameters used to characterize fluid flow and heat transfer between plates.

The skin friction coefficient quantifies the ratio of the wall shear stress to the dynamic pressure of the flow. It provides a measure of the resistance to flow caused by viscous forces.

$$C_f=-\frac{\mu_{hnf}}{\rho_f {v_w}^2}\left(1+\frac{1}{\gamma}\right)\left[\frac{\partial u}{\partial y}\right]_{y=0}.$$

By utilising similarity transformation mentioned in equation (8) ,

skin friction coefficient will be $C_{fx}=\left|\frac{\left(1+\frac{1}{\gamma}\right)\left(\frac{\partial^2 F}{\partial\eta^2}\right)_{at\,\eta=0}}{\Delta_1}\right|$. (12)

The Nusselt number is a dimensionless parameter that characterizes the ratio of convective to conductive heat transfer at a boundary in a fluid flow system.

$$Nu_x=\frac{k_{hnf}h}{k_f(T_1-T_2)}\left(1+\frac{4N_r}{3}\right)\left[\frac{\partial T}{\partial y}\right]_{y=0}$$

In non-dimensional form Nusselt number can be expressed as

$$Nu=\left|\frac{k_{hnf}}{k_f}\left(1+\frac{4N_r}{3}\right)\left[\frac{\partial\theta}{\partial\eta}\right]_{\eta=0}\right|$$

3. Results and discussions.

In the present numerical investigation, the authors examine the magnetohydrodynamic (MHD) flow and heat transfer characteristics of a hybrid nanofluid in a rotating system confined between two parallel plates. The governing nonlinear boundary-layer equations are solved numerically using the shooting method implemented in MATLAB. The upper plate is considered permeable, whereas the lower plate is modelled as a linearly stretching sheet. To establish the accuracy and reliability of the numerical formulation, the MATLAB-based computational procedure is validated against previously published results available in the literature. The effects of the governing physical parameters, including the Casson parameter, radiation parameter, rotation parameter, Reynolds number (Re), magnetic parameter (M), and suction/injection parameter (S), are systematically investigated. Their influence on the flow and thermal characteristics is quantified through the skin-friction coefficients and Nusselt number, while the corresponding variations in the hydrodynamic and thermal boundary layers are illustrated through the primary velocity, secondary velocity, and temperature profiles. The working hybrid nanofluid is formulated by dispersing graphene oxide (GO) and molybdenum disulfide (MoS_2) nanoparticles in engine oil as the base fluid. The rotation of the parallel-plate system plays a significant role in modifying both the hydrodynamic and thermal transport characteristics owing to the presence of rotational body forces, particularly the Coriolis and centrifugal forces. These rotational effects induce a coupling between the primary and secondary flow components, thereby altering the momentum transport within the boundary layer. Figures 2 and Figure 3 illustrate the influence of the rotation parameter on the primary and secondary velocity profiles, respectively. It is observed that an increase in the rotation parameter suppresses the primary velocity profile, whereas it enhances the secondary velocity profile. This contrasting behaviour can be attributed to the increasing strength of the Coriolis force associated with the system rotation, which redistributes momentum between the primary and secondary flow directions.

From Figure 4, it is observed that the magnitude of the secondary velocity decreases progressively with increasing magnetic parameter. The observed reduction can be attributed to the Lorentz force generated by the interaction between the imposed magnetic field and the electrically conducting hybrid nanofluid. The Lorentz force acts in opposition to the fluid motion and consequently introduces an additional resistive force within the momentum equations. As the magnetic parameter increases, the strength of this electromagnetic resistance becomes more pronounced, leading to greater damping of the secondary velocity and a corresponding reduction in the maximum value

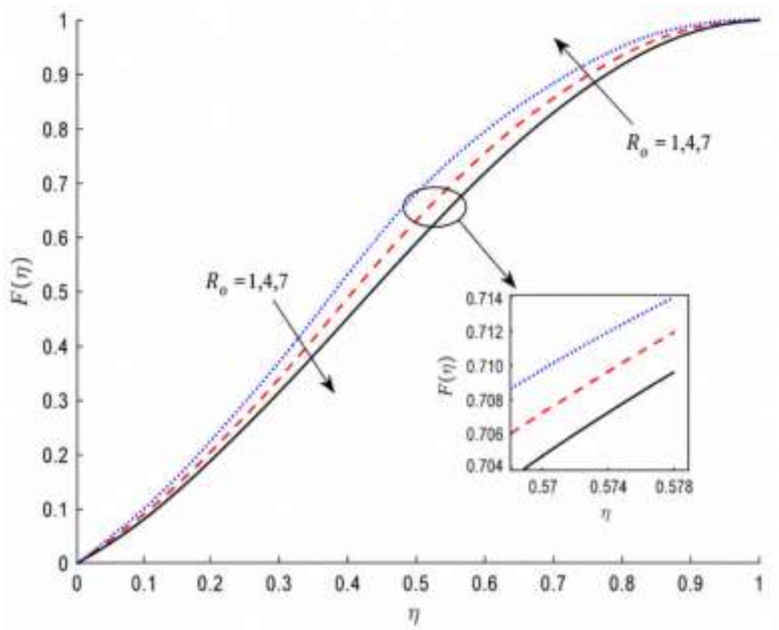

Figure 2: Role of rotation parameter on primary velocity profile.

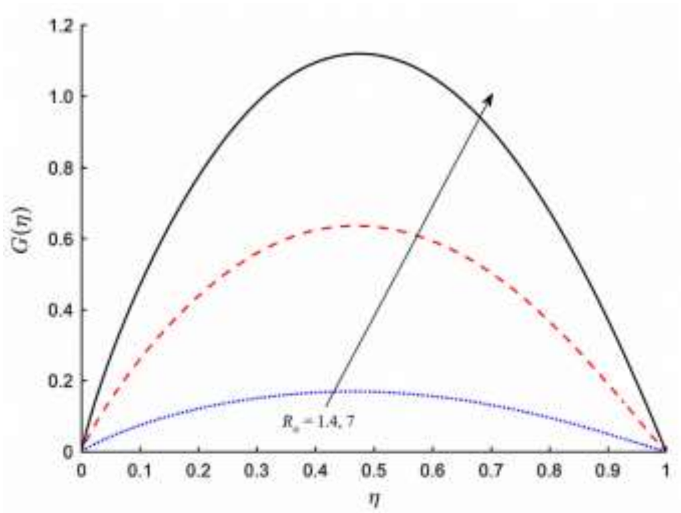

Figure 3: Impression of rotation parameter on secondary velocity profile.

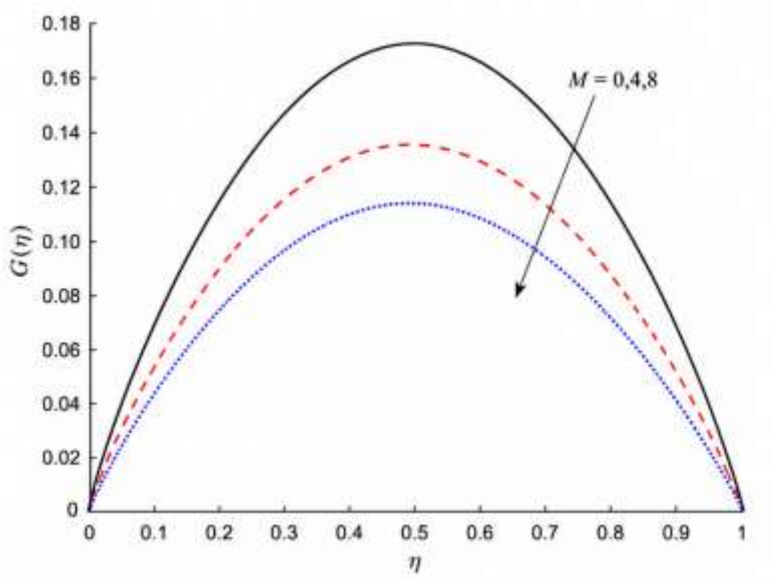

Figure 4: Effect of magnetic parameter on secondary velocity profile.

Table 1: Thermophysical properties of nanoparticle

Fluid Property	Graphene Oxide (GO)	Copper (Cu)	Water (H_2O)
Density $\rho(kg/m^3)$	1800	8933	997.1
Heat Capacity $C_p(J/kgK)$	717	385	4179
Thermal conductivity k(W/mK)	5000	401	0.613

4. Concluding remarks

The governing nonlinear equations describing the flow and heat-transfer characteristics of the rotating hybrid nanofluid were solved numerically using the shooting method implemented in MATLAB.

- The rotation of the parallel-plate system significantly influences the flow field by introducing Coriolis and centrifugal effects, which establish a strong coupling between the primary and secondary velocity components.
- The magnetic parameter has a pronounced damping effect on the secondary velocity profile.
- The magnetic field therefore provides an effective mechanism for controlling the secondary flow intensity in the rotating system.

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