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Graph Theoretical Analysis and Optimization of Urban Metro Rail Networks: A Case Study of Namma Metro

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Abstract
Urban metro rail systems are essential components of modern transportation infrastructure, and as such networks grow in scale and complexity, systematic methods for evaluating their efficiency, connectivity, and resilience become increasingly important. This paper presents a graph-theoretical framework for the analysis and optimisation of urban metro rail networks, using the Namma Metro (Bengaluru) as a case study. The network, comprising 83 stations and 82 track segments across the Purple, Green, and Yellow lines (96.1 km) is modelled as a weighted graph G = (V, E, w) with edge weights encoding travel time, distance and passenger flow. Dijkstra's algorithm is applied to determine optimal passenger routes, while degree, closeness, betweenness and eigenvector centrality measures, identify structurally critical stations. A gravity-based model estimates inter-station passenger flow, and a congestion index quantifies station-level demand concentration. A complementary coach-level analysis quantifies within-train passenger imbalance using a variance-based measure and proposes redistribution strategies to reduce it to zero. The results demonstrate that graph theory offers a rigorous and practical foundation for transportation planning, congestion mitigation, and resilience-oriented network design in rapidly growing metropolitan metro systems.

Keywords: Graph theory; metro rail networks; shortest path; Dijkstra's algorithm; centrality measures; congestion analysis; network robustness; Namma Metro.

1. Introduction

Rapid urbanisation has significantly increased travel demand in metropolitan cities, creating severe challenges related to road congestion, travel time, fuel consumption and environmental pollution. Public transportation systems, particularly metro rail networks, have emerged as sustainable solutions capable of transporting large numbers of passengers efficiently while reducing dependence on private vehicles. As metro systems continue to expand in size and complexity, effective planning and optimisation become increasingly important for ensuring reliable and efficient operation.

Graph theory has become one of the most powerful mathematical tools for analysing transportation networks. By representing stations as vertices and railway connections as edges, metro systems can be modelled as graphs that facilitate quantitative analysis of connectivity, accessibility, shortest routes, and network resilience. Such mathematical models enable transportation planners to evaluate existing infrastructure, identify vulnerable locations and propose improvements that enhance passenger mobility and operational efficiency.

The Bengaluru Metro, popularly known as Namma Metro, has experienced rapid expansion in recent years and now serves as one of India's major urban rapid transit systems. The network consists of multiple corridors connected through important interchange stations, serving thousands of passengers every day. As the network grows, maintaining efficient passenger movement, reducing congestion and ensuring network reliability become increasingly challenging. Consequently, scientific approaches based on graph theory are essential for analysing network performance and supporting data-driven planning decisions.

This paper addresses the problem of analysing and optimizing the Namma Metro network (Bengaluru) using graph-theoretic methods. The specific objectives are to

1. Construct an accurate weighted graph model of the operational network.
2. Determine optimal passenger routes via shortest-path analysis.
3. Identify structurally critical stations through multiple centrality measures.
4. Model passenger flow and station-level congestion.
5. Quantify and address passenger-load imbalance at the coach level, culminating in concrete recommendations for network improvement.

2. Literature Review

Graph theory has become an essential mathematical framework for analysing complex transportation systems due to its ability to model connectivity, accessibility and network efficiency. By representing metro stations as vertices and railway links as edges, researchers have successfully applied graph-theoretic techniques to optimise route planning, identify critical infrastructure, and evaluate network resilience.

Shortest-path algorithms are among the most widely used methods in transportation network analysis. Edsger W. Dijkstra introduced Dijkstra's algorithm [4] for determining the minimum-cost path between a source and destination in weighted graphs, making it particularly suitable for metro route planning. Similarly, Robert W. Floyd and Stephen Warshall developed the Floyd–Warshall algorithm [5], [6] to compute shortest paths between all pairs of vertices, providing an efficient framework for evaluating network-wide accessibility. These algorithms have been widely adopted in urban transportation systems because of their computational efficiency and reliability.

Centrality measures have also been extensively employed to determine the structural importance of metro stations. Linton C. Freeman introduced the concept of centrality as a quantitative measure of node importance in networks [7]. Degree centrality identifies highly connected stations, closeness centrality measures accessibility, betweenness centrality identifies stations that control passenger movement along shortest paths and eigenvector centrality evaluates the influence of stations based on the importance of their neighbouring nodes. These metrics assist transportation planners in identifying strategic interchange stations and improving network performance.

Several researchers have applied graph theory to metro systems worldwide. Sybil Derrible and Kennedy analysed metro networks using complex network theory and demonstrated that interchange stations significantly influence network robustness and operational efficiency [8]. Their studies also showed that metro systems generally exhibit resilience to random failures but are more vulnerable to disruptions at

highly central stations. Similar findings have been reported in studies examining passenger flow, congestion and infrastructure resilience in urban transportation networks [9], [14].
Recent studies have focused on evaluating network robustness under disruption scenarios. Robustness analysis investigates the ability of transportation systems to maintain connectivity following failures caused by technical faults, maintenance activities or emergency situations. Research has shown that transportation networks generally exhibit high resilience against random failures but remain vulnerable to failures at strategically important interchange stations [9]. Identifying these critical stations enables transportation authorities to implement preventive maintenance strategies and improve emergency preparedness.
Passenger flow analysis has also gained considerable attention in recent years. Increasing passenger demand often leads to congestion at interchange stations, affecting travel efficiency and commuter satisfaction. Graph-based passenger flow models allow researchers to identify bottlenecks, estimate congestion levels and evaluate alternative routing strategies that distribute passenger demand more uniformly throughout the network [13]. Such analyses contribute significantly to operational planning and infrastructure development.
Collectively, these studies establish graph theory as a well-validated framework for transportation network analysis, spanning shortest-path routing, centrality-based structural assessment and robustness evaluation across a range of metro systems worldwide.

3. Research Gap

Although numerous studies have investigated metro networks using graph-theoretic approaches, most have focused on isolated aspects such as shortest-path routing, centrality analysis or network robustness. Comprehensive studies that integrate these analytical techniques with passenger flow, congestion assessment, network efficiency and resilience evaluation remain limited, particularly for the Bengaluru Metro (Namma Metro) network. Furthermore, the impact of proposed network enhancements on overall system performance has received comparatively little attention. Therefore, there is a need for a unified analytical framework that simultaneously evaluates network structure, operational efficiency, and resilience. This study addresses this gap by integrating shortest-path algorithms, centrality measures, passenger flow analysis, congestion assessment, network efficiency, and robustness evaluation to support informed decision-making for metro network planning and optimisation.

4. Methodology: Network Modeling

The Namma Metro network is represented as a weighted, undirected graph $G = (V, E, w)$ where V is the set of stations, E the set of direct track connections between adjacent stations and w an edge-weight function assigning travel time (minutes), physical distance (km) or estimated passenger flow to each edge. As of the study period, the operational network comprises 83 stations and 82 track segments across three lines: the Purple Line (37 stations, Challaghatta–Whitefield), the Green Line (32 stations, Madavara–Silk Institute) and the Yellow Line (16 stations, RV Road–Bommasandra, inaugurated August 2025), spanning approximately 96.1 km. Majestic (Kempegowda) station is modelled as a single vertex connecting the Purple and Green lines and RV Road as a single vertex connecting the Green and Yellow lines; each interchange thus reduces the vertex count from 85 line-station assignments to 83 unique stations.

The metro network data were compiled from publicly available OpenStreetMap and Google Maps, official station information and operational details published by Bangalore Metro Rail Corporation Limited (BMRCL) [15], [16].

Table 1 summarizes the resulting graph's structural statistics.

Parameter	Value
Total stations, $\|V\|$	83
Total track segments, $\|E\|$	82
Number of lines	3 (Purple, Green, Yellow)
Interchange stations	2
Total network length	~96.1 km
Maximum degree, $\Delta(G)$	4
Minimum degree, $\delta(G)$	1
Pendant vertices	5

Table 1. Network graph summary statistics for Namma Metro.

Because $|E| = |V| - 1$ and the graph is connected, the modelled network graph is a tree. Thus, there are no redundant paths in the present topological representation. Consequently, $\kappa(G) = \lambda(G) = 1$, indicating that the network contains articulation vertices and bridge edges whose removal can disconnect the network. This structural property is investigated further in Section 5.4.

4.1 Shortest Path Analysis

Shortest path analysis was performed using Dijkstra's algorithm and the Floyd–Warshall algorithm.

Dijkstra's algorithm computes a minimum-weight path from a selected source station to reachable destinations, where the edge weight is defined according to the selected criterion, such as travel time or distance. The algorithm iteratively updates tentative distances while selecting the nearest unvisited vertex until optimal routes are obtained.

The Floyd–Warshall algorithm computes shortest paths between every pair of stations simultaneously through dynamic programming. This approach is particularly useful for evaluating network-wide accessibility and constructing complete shortest-path matrices.

The shortest-path analysis enables route planning under the selected edge-weight criterion. In this study, the reported examples use travel time as the primary route-weight measure.

4.2. Centrality Analysis

Four centrality measures are used to quantify station importance:

1. Degree Centrality measures the number of directly connected neighbouring stations and identifies highly connected locations.
2. Closeness Centrality evaluates how efficiently a station can access all other stations within the network.
3. Betweenness Centrality quantifies the frequency with which a station appears on shortest paths connecting different station pairs.
4. Eigenvector Centrality measures station importance by considering both direct connectivity and the influence of neighbouring stations.

The combined interpretation of these metrics provides a comprehensive assessment of network hierarchy and critical station identification.

4.3 Congestion and Passenger Flow Analysis

Passenger movement characteristics were analysed through congestion indices and passenger flow distribution. Stations exhibiting consistently high passenger volumes and centrality values were identified as potential congestion hotspots.

The analysis enables transportation planners to identify bottlenecks, optimise passenger distribution and improve operational efficiency during peak travel periods.

Passenger Flow (Gravity Model):

$F(s,t) = k \cdot P(s)P(t) / [d(s,t)]^2$

Where:

- $F(s,t)$ = estimated passenger flow between stations s and t
- $P(s)$, $P(t)$ = passenger volumes at stations s and t
- $d(s,t)$ = shortest-path travel time between s and t
- k = calibration constant

Congestion Index:

$CI(v) = P(v) / P_{max}$

Where:

- $P(v)$ = passenger volume at station v
- P_{max} = passenger volume at the busiest station in the network
- $0 \leq CI(v) \leq 1$

5. Results and Discussion

5.1 Shortest-Path Analysis

As an illustrative case, the shortest travel time from Central College to Ragigudda is 18 minutes across 9 stops, routed through Majestic, Chickpete, K.R. Market, National College, Lalbagh, South End Circle, Jayanagar and RV Road.

Table 2 reports shortest-path results for a broader set of representative station pairs.

Origin	Destination	Shortest Path (min)	Hops	Transfer
Whitefield	Majestic	43	22	No
Whitefield	Nagasandra	72	34	Yes
Baiyappanahalli	MG Road	10	5	No
Majestic	Silk Institute	32	15	No
Challaghatta	Nagasandra	59	27	Yes
Banashankari	Baiyappanahalli	39	17	Yes

Table 2. Shortest-path results for selected station pairs.

5.2 Passenger Flow and Congestion Analysis

Applying the gravity model to MG Road and Majestic (passenger volumes 22,156 and 28,650 respectively; travel time 10 minutes; k = 0.001) yields an estimated flow of approximately 6,348 passengers, reflecting the strong interaction between these two heavily used stations. Station-level congestion indices, computed relative to Majestic (34,948 passengers, CI = 1.000), classify Baiyappanahalli, Indiranagar, MG Road and KR Puram as medium-risk stations ($0.50 \leq CI < 0.75$), with the remaining sampled stations falling below the medium threshold. Majestic alone is classified highrisk, confirming that congestion in the network is concentrated overwhelmingly at the primary interchange rather than distributed across multiple hubs.

5.3 Critical Station Identification

Centrality analysis identifies Majestic as the highest-ranked station among the reported centrality measures (Table 3), consistent with its role as the Purple–Green interchange. Chickpete, K.R. Market, National College, Lalbagh, and Jayanagar also rank highly in the reported analysis, while RV Road is the second interchange connecting the Green and Yellow lines.

Rank	Station	Degree centrality	Closeness centrality	Betweenness centrality	Eigen vector centrality
1	Majestic	0.049	0.098	0.736	0.577
2	Chickpete	0.024	0.095	0.463	0.333
3	K.R. Market	0.024	0.092	0.455	0.193
4	National College	0.024	0.090	0.447	0.112
5	RV Road	0.037	0.079	0.445	0.025

Table 3. Top-ranked stations by centrality measure.

The high betweenness-centrality values of these stations indicate that many network-wide shortest paths pass through them. Consequently, disruption at such locations can have a disproportionate effect on network connectivity and route accessibility.

5.4 Network Robustness and Failure Analysis

The tree topology of the modelled network implies zero topological redundancy. Because every edge of a tree is a bridge, removal of an internal edge can disconnect the graph. Robustness should therefore be assessed quantitatively using an explicit failure metric rather than described only qualitatively.

Building on this finding, an interchange congestion optimisation model was formulated to reduce the load concentrated at Majestic. The model indicates that diverting passenger volume away from Majestic. Example, via an additional interchange or alternative routing options achieves a substantially more balanced congestion profile across the network's major stations, directly mitigating the vulnerability identified in this section.

5.5 Proposed Network Improvements

Four graph-theoretically motivated improvements are proposed:

1. an additional interchange station in East Bengaluru can act as second transfer point and reduces dependence on Majestic.
2. A southern loop connecting Silk Institute and Challaghatta, introducing the network's first structural cycle and thereby raising $\kappa(G)$ and $\lambda(G)$ above 1.
3. Prioritized capacity enhancement at Majestic itself — additional fare gates, widening passenger transfer corridors and improved signboards for wayfinding making passenger movement smoother and faster.

6 Analysis and Optimization of Coach-Level Passenger Distribution

6.1 Introduction

While station-level congestion provides important insights into passenger movement across the metro network, congestion also occurs within individual train coaches. In many metro systems, passengers tend to concentrate in specific coaches located near station entrances, escalators, lifts and interchange access points. As a result, some coaches become overcrowded while others remain underutilized. This phenomenon, referred to as **uneven passenger distribution**, negatively affects passenger comfort, boarding efficiency, train dwell times and overall operational performance, particularly during peak commuting hours. To address this issue, a graph-theoretic and statistical framework is developed to analyse passenger distribution across metro coaches and identify strategies for achieving more balanced occupancy levels.

6.2 Mathematical Model

Consider a metro train consisting of n coaches represented by the graph,

$$G = (V, E)$$

where $V = \{C_1, C_2, \dots, C_n\}$ represents the set of coaches and E represents passenger movement between adjacent coaches. Each vertex is assigned a weight corresponding to the number of passengers occupying that coach. Let P_i denote the passenger load of coach C_i and let K_i denote its maximum passenger capacity. The load factor of coach C_i is defined as

$$L_i = \frac{P_i}{K_i}$$

where

- $L_i = 1$ indicates full capacity
- $L_i < 1$ indicates available capacity
- $L_i > 1$ represents overcrowding.

A perfectly balanced train satisfies

$$L_1 = L_2 = \cdots = L_n.$$

The average load factor is given by

$$\bar{L} = \frac{1}{n}\sum_{i=1}^{n} L_i$$

and the variance-based congestion measure is

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (L_i - \bar{L})^2.$$

The optimization objective is minimizing σ^2, so that passenger occupancy becomes an evenly distributed as possible among all coaches. A value of $\sigma^2 = 0$ indicates perfect balance, a small value indicates good distribution and a large value indicates severe imbalance with localized overcrowding.

6.3 Numerical Example

A representative six-coach metro train operating during peak hours is considered. The passenger occupancy, coach capacity and corresponding load factors are presented in Table 4.

Coach	Passengers	Capacity	Load Factor
C1	280	300	0.933
C2	270	300	0.900

Coach	Passengers	Capacity	Load Factor
C3	240	300	0.800
C4	180	300	0.600
C5	150	300	0.500
C6	130	300	0.433

Table 4. Sample passenger distribution across coaches during peak hours.

The average load factor is

$$\bar{L} = \frac{0.933 + 0.900 + 0.800 + 0.600 + 0.500 + 0.433}{6} = 0.694.$$

Thus, on average each coach is approximately **69.4% occupied**.
The squared deviations from the mean load factor are

$$0.05712,\ 0.04244,\ 0.01124,\ 0.00884,\ 0.03764,\ 0.06812$$

giving

$$\sum_{i=1}^{6} (L_i - \bar{L})^2 = 0.22540.$$

Hence,

$$\sigma^2 = \frac{0.22540}{6} = 0.03757.$$

Since $\sigma^2 > 0$, the passenger distribution is uneven, indicating that the front coaches are more crowded while the rear coaches remain underutilized.

Optimization Strategy

The total passenger count is

$$P_{\text{total}} = 280 + 270 + 240 + 180 + 150 + 130 = 1250.$$

The ideal passenger occupancy per coach is therefore

$$P_{\text{ideal}} = \frac{1250}{6} = 208.33$$

The corresponding ideal load factor is

$$L_{\text{ideal}} = \frac{208.33}{300} = 0.694$$

Accordingly, passengers should be redistributed from coaches **C1–C3** to coaches **C4–C6** until each coach contains approximately **208.33 passengers**, as shown in Table 5.

Coach	Actual Passengers	Ideal Passengers	Adjustment
C1	280	208.33	−71.67
C2	270	208.33	−61.67
C3	240	208.33	−31.67
C4	180	208.33	+28.33

Coach	Actual Passengers	Ideal Passengers	Adjustment
C5	150	208.33	+58.33
C6	130	208.33	+78.33

Table 5. Passenger redistribution required for balanced coach occupancy.

Under perfect redistribution, each coach carries 208 passengers, giving a uniform load factor $L_1 = L_2 = \ldots\ldots = L_6 = 0.694$ across all six coaches and consequently,

$$\sigma^2 = 0$$

which represents the theoretical optimum for coach-level passenger distribution.

6.4 Results and Discussion

The analysis demonstrates that passenger occupancy is concentrated in the front coaches, while the rear coaches remain underutilized. This imbalance leads to localized congestion despite the train operating below its overall carrying capacity. The proposed optimization redistributes passengers uniformly among the coaches, reducing the variance of passenger occupancy from **0.03757** to **0.**

To reduce this imbalance, the study proposes several optimization strategies, including **real-time occupancy information**, **platform guidance systems**, **station infrastructure modifications** and **AI-based passenger information systems** that direct passengers towards less crowded coaches. These measures are expected to improve coach occupancy balance, reduce platform congestion, shorten train dwell times, enhance passenger comfort and increase the overall operational efficiency of the Bengaluru Metro network.

7. Future Scope

The rapid expansion of urban transportation systems presents numerous opportunities for extending the present research. Future investigations may incorporate real-time passenger demand, train scheduling information and operational delays to develop dynamic graph models capable of supporting intelligent transportation management.

Machine learning and artificial intelligence techniques may be integrated with graph-theoretic models to predict passenger demand, estimate congestion levels and recommend adaptive routing strategies under varying operational conditions. Graph Neural Networks (GNNs) and reinforcement learning algorithms offer particularly promising approaches for analysing large-scale transportation systems and supporting automated decision-making.

Future studies may also evaluate network resilience under more realistic disruption scenarios, including simultaneous station failures, infrastructure maintenance, natural disasters and emergency evacuations. Such investigations would contribute to the development of more robust and resilient metro systems capable of maintaining efficient service during unexpected operational events.

Another promising direction involves the integration of multimodal transportation networks by combining metro systems with buses, suburban railways, airports and pedestrian infrastructure into a unified graph model. This integrated approach would facilitate city-wide route optimisation and improve overall urban mobility.

Comparative studies involving metro networks from different metropolitan cities could further validate the proposed methodology and identify structural characteristics associated with highly efficient transportation systems. Such analyses would provide valuable guidance for future metro expansion projects and smart-city initiatives.

Finally, optimisation techniques based on advanced graph algorithms, metaheuristic methods and network science may be employed to determine optimal locations for future stations, new metro corridors and additional interchange facilities. These developments would significantly contribute to sustainable transportation planning and enhance the long-term efficiency, resilience and accessibility of urban metro networks.

8. Conclusion

This study developed a graph-theoretic framework for analysing the Bengaluru Metro (Namma Metro) network using shortest-path algorithms, centrality measures, passenger-flow analysis, congestion assessment and network robustness evaluation. The results demonstrate that graph-based methods provide an effective mathematical approach for evaluating network accessibility and identifying structurally important stations.

The analysis identified Majestic as the most critical station among those evaluated, owing to its high centrality and interchange role. The network's tree structure also indicates limited topological redundancy, making important connections vulnerable to disruption. Passenger-flow and congestion analysis further showed that demand is concentrated at major interchange stations, particularly Majestic, while the coach-level analysis demonstrated that passenger redistribution can significantly improve occupancy balance.

Overall, the study establishes graph theory as a useful framework for metro network planning, congestion management, route evaluation, and resilience assessment. The proposed methodology can support transportation authorities in identifying critical infrastructure and improving passenger movement, and it can be extended in future studies using real-time passenger data and dynamic operational conditions.

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