

# Coupled Thermal Radiation-Viscous Dissipation Interaction in MHD Buoyancy-Induced Heat and Mass Transfer Flow over a Stretching Porous Surface

Kamba Indimuli Brian<sup>1</sup>, Richard Opiyo<sup>2</sup>, David Ambogo<sup>3</sup>

<sup>1</sup>Research Assistant and Masters Student, Department of Pure and Applied Mathematics, Maseno University, Kenya

<sup>2,3</sup>Supervisor, Department of Pure and Applied Mathematics, Maseno University, Kenya

## Abstract

This paper examined the coupled effects of thermal radiation and viscous dissipation on steady two-dimensional MHD buoyancy-induced heat and mass transfer over a stretching porous surface. The governing continuity, momentum, energy and species equations included thermal and solutal buoyancy, Lorentz force, Darcy resistance, Rosseland radiation and viscous dissipation. Similarity transformations reduced the model to a nonlinear boundary-value problem and the resultant model was solved by a shooting method together with the fourth-order Runge-Kutta integration. For  $0 \leq Rd \leq 1$  and  $0 \leq Ec \leq 0.20$ , the dimensionless Nusselt number decreased from 0.828429 at  $(Rd, Ec) = (0, 0)$  to 0.525282 at  $(Rd, Ec) = (1, 0.20)$ , an overall reduction of 36.6%. The interaction index remained positive, with values of 0.008044 at  $(Rd, Ec) = (0.50, 0.10)$  and 0.024011 at  $(Rd, Ec) = (1.00, 0.20)$ , indicating a non-additive thermal response. The concentration field changed more weakly because  $Rd$  and  $Ec$  entered the species transport indirectly through the coupled thermal and momentum fields. The results are relevant to heat and mass transfer processes involving stretching porous surfaces, including coating, extrusion and related transport operations where thermal radiation and viscous dissipation act simultaneously.

**Keywords:** Thermal radiation, viscous dissipation, MHD flow, stretching porous surface, Nusselt number, interaction index, response surface, heat transfer.

## 1. Introduction

Thermal radiation and viscous dissipation modify the energy balance in electrically conducting flows. Radiation alters radiative heat transport, while viscous dissipation converts mechanical energy into internal heat [9,12,13]. These effects become important in MHD heat and mass transfer over stretching and porous surfaces, where magnetic forces, buoyancy and porous resistance couple momentum, temperature and concentration fields [1,5,7,8]. Studies of radiation in MHD flows have reported changes in thermal boundary-layer behaviour [7,8], while investigations involving viscous dissipation have shown its contribution to fluid heating and heat-transfer characteristics [3,5]. Porous-medium resistance also changes the momentum field and therefore influences the coupled transport response [1,5].

Despite these separate investigations, the literature cited above provides limited treatment of the simultaneous  $Rd$ - $Ec$  interaction for buoyancy-induced MHD heat and mass transfer over a stretching

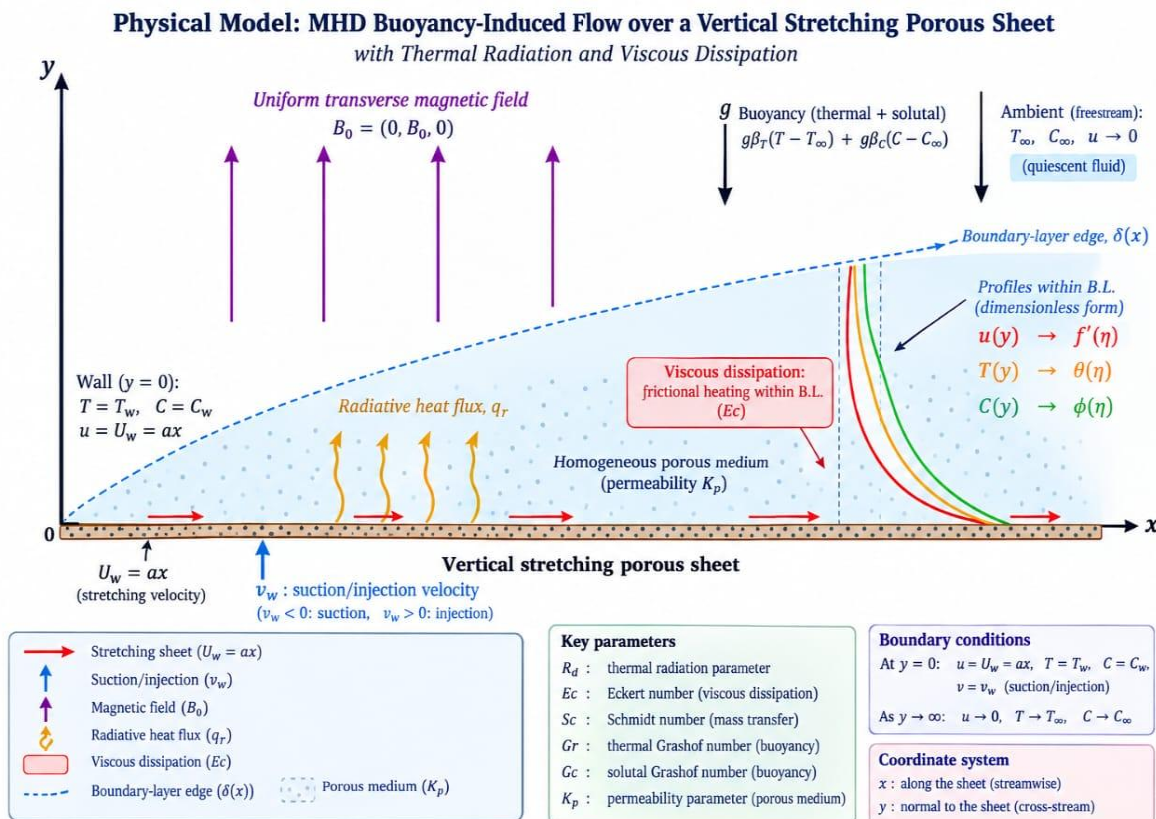
porous surface. In particular, the combined response of the dimensionless Nusselt number and the non-additive interaction between radiation and viscous dissipation required explicit quantification. The present paper addressed this gap by varying  $Rd$  and  $Ec$  jointly and examining temperature profiles, the dimensionless Nusselt number, a two-parameter response surface and an interaction index.

## 2. Mathematical Formulation

Figure (1) presents the physical configuration of the steady two-dimensional MHD flow over a vertical stretching porous surface. The  $x$ -axis is directed along the surface and the  $y$ -axis is normal to it. The sheet stretches with  $U_w(x) = ax$ , while a uniform transverse magnetic field  $B_0$  acts in the  $y$ -direction. The ambient temperature and concentration are  $T_\infty$  and  $C_\infty$ , respectively. The model assumes an incompressible Newtonian electrically conducting fluid and includes thermal and solutal buoyancy, Lorentz force, Darcy resistance, thermal radiation and viscous dissipation.

### 2.1 Model Assumptions

The mathematical model consists of the continuity, momentum, energy, and species concentration equations. The analysis assumes steady flow, two-dimensional motion and constant fluid properties except for the density variation retained in the buoyancy terms. The Boussinesq approximation was used for thermal and solutal buoyancy so that density variations are retained only where they contribute to buoyancy forces. The magnetic Reynolds number was taken to be sufficiently small so that the induced magnetic field is neglected. Thermal radiation was described by the Rosseland diffusion approximation under the optically thick assumption. Chemical reaction was excluded, and Soret and Dufour effects were also not included in the base model.



**Figure 1: Conceptualized physical configuration of the MHD flow over a stretching porous surface.**

## 2.3 Continuity equation

The Continuity equation expresses mass conservation [14]. For a compressible fluid, it is written as;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

Expanding the divergence term for  $\mathbf{V} = u, v, w$  gives,

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \quad (2)$$

Under the assumptions adopted in the present study, the continuity equation reduced to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Equation (3) represents the dimensional continuity equation used in the study.

To automatically satisfy continuity, a stream function  $\psi(x,y)$  was introduced as;

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

Substitution of the stream-function definition into equation (3) gives,

$$\frac{\partial}{\partial x} \frac{\partial \psi}{\partial y} + \frac{\partial}{\partial y} \frac{\partial \psi}{\partial x} = 0 \quad (4)$$

which follows from equality of mixed partial derivatives,

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0 \quad (5)$$

therefore, the continuity equation was satisfied identically.

## 2.4 Momentum Equation

The momentum equation describes the motion of a fluid under the action of various forces acting on the fluid flow, in our case, the thermal buoyancy force, solutal buoyancy force, the Lorentz force and the darcy resistance from porous medium

For flow along the stretching surface,  $x$ -momentum equation is,

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F_x \quad (6)$$

Here,  $F_x$  represents the net body force per unit volume in the  $x$ -direction.

Under the assumptions of the study, the equation reduces to,

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + F_x \quad (7)$$

The body force  $F_x$  consists of four components,

Thermal Buoyancy Force:

$$F_{th} = \rho_{\infty} g \beta_T (T - T_{\infty})$$

Solutal Buoyancy Force:

$$F_s = \rho_{\infty} g \beta_C (C - C_{\infty})$$

Lorentz Force (MHD effect):

$$F_{m,x} = -\sigma B_0^2 u$$

Darcy Resistance from Porous Medium:

$$\mathbf{F}^p = -\frac{\mu}{K} \mathbf{u}$$

Substitution of these body forces into the reduced  $x$ -momentum equation and dividing through by  $\rho_{\infty}$  (where  $\nu = \mu/\rho_{\infty}$ ) gives,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta_T (T - T_{\infty}) + g \beta_C (C - C_{\infty}) - \frac{\sigma B_0^2}{\rho_{\infty}} u - \frac{\nu}{K} u. \quad (8)$$

which represents the governing momentum equation.

## 2.5 Energy Equation

The energy equation describes energy conservation within the fluid system [12]. It accounts for thermal transport and viscous dissipation. For the steady two-dimensional MHD flow, the energy equation is,

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \Phi \quad (9)$$

Where  $C_p$  is the specific heat capacity at constant pressure and  $\Phi$  represents the viscous dissipation term.

The viscous dissipation term was approximated by

$$\Phi \approx \mu (\partial u / \partial y)^2$$

Thermal radiation was incorporated through the Rosseland approximation and after substitution, the energy equation became,

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{eff} \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \quad (10)$$

which is the governing energy equation and contains the thermal radiation contribution together with viscous dissipation.

## 2.6 Species Concentration Equation

The species concentration equation describes mass transport through convection and diffusion [11]. Under the assumptions adopted in the study, the concentration equation reduced to:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}. \quad (11)$$

where  $D$  denotes the mass diffusion coefficient.

## 2.7 Dimensional Governing Equations

The resultant dimensional mathematical model for the steady two-dimensional MHD buoyancy-induced heat and mass transfer flow over the stretching porous surface was therefore:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{\sigma B_0^2}{\rho_\infty} u - \frac{\nu}{K} u, \quad (12)$$

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{eff} \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2, \quad (13)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}. \quad (14)$$

## 2.8 Similarity Transformation and Non-dimensionalisation

Similarity transformations are introduced to convert the governing partial differential equations into a system of ordinary differential equations suitable for numerical solution.

The similarity variables were defined as,

$$\psi = \sqrt{a\nu} x f(\eta), \quad (15)$$

$$\eta = y \sqrt{\frac{a}{\nu}}, \quad (16)$$

$$u = \frac{\partial \psi}{\partial y} = ax f'(\eta), \quad (17)$$

$$v = -\frac{\partial \psi}{\partial x} = -\sqrt{a\nu} f(\eta), \quad (18)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad (19)$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}. \quad (20)$$

Substitution of these transformations into the dimensional governing equations (12), (13) and (14) reduced the PDE system to a nonlinear dimensionless boundary-value problem. The transformed momentum equation was given as,

$$a^2 \left[ (f')^2 - f f'' \right] = a^2 x f''' + g\beta_T (T_w - T_\infty) \theta + g\beta_C (C_w - C_\infty) \phi - a^2 x \left( \frac{\delta B_0^2}{\rho a} \right) f' - a^2 x \left( \frac{\nu}{aK} \right) f'. \quad (21)$$

Similarly, the energy equation transformed to,

$$\frac{\rho c_p \nu}{k} (f'\theta - f\theta') = \left(1 + \frac{16\delta^* T_\infty^3}{3k^*}\right) \theta'' + \frac{\rho a^2 x^2}{c_p (T_0 x)} \frac{\rho c_p \nu}{k} (f'') \quad (22)$$

And finally, the transformed species equation was,

$$f'\phi - f\phi' = \frac{D}{\nu} \phi'' \quad (23)$$

From the transformed equations, the following dimensionless parameters were identified:

$$M = \frac{\sigma B_0^2}{\rho_\infty a} \quad (24)$$

$$K_p = \frac{\nu}{aK} \quad (25)$$

$$Gr = \frac{g\beta_T(T_w - T_\infty)}{a^2} \quad (26)$$

$$Gc = \frac{g\beta_C(C_w - C_\infty)}{a^2} \quad (27)$$

$$Pr = \frac{\nu}{a} = \frac{\rho c_p \nu}{k} \quad (28)$$

$$Rd = \frac{16\delta^* T_\infty^3}{3k^* k} \quad (29)$$

$$Ec = \frac{U_w^2}{c_p(T_w - T_\infty)} \quad (30)$$

$$Sc = \frac{\nu}{D} \quad (31)$$

Therefore, substituting these dimensionless parameters, the final dimensionless system becomes,

$$f'' + ff'' - (f')^2 + Gr\theta + Gc\phi - (M + K_p)f' = 0, \quad (32)$$

$$(1 + Rd)\theta'' + Pr(f\theta' - f'\theta) + PrEc(f'')^2 = 0, \quad (33)$$

$$\phi'' + Sc(f\phi' - f'\phi) = 0. \quad (34)$$

The energy equation explicitly contains both  $Rd$  and  $Ec$ . Their simultaneous variation affects the thermal field and subsequently propagates through the momentum and species equations because of the coupling among the three transport fields.

## 2.9 Boundary Conditions and Engineering Quantities

At the stretching surface the boundary conditions are,

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1.$$

Far from the surface,

$$f(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0.$$

The principal dimensionless wall quantities are

$$C_f \propto -f''(0), \quad Nu^* = -\theta'(0), \quad Sh^* = -\phi'(0).$$

## 3. Numerical Method

The third-order momentum equation and the second-order energy and species equations were converted into seven first-order ordinary differential equations by defining,

$$y_1 = f, \quad y_2 = f', \quad y_3 = f'', \quad y_4 = \theta, \quad y_5 = \theta', \quad y_6 = \phi, \quad y_7 = \phi'.$$

The corresponding first-order system is

$$\begin{aligned} y_1' &= y_2, & y_2' &= y_3, \\ y_3' &= -y_1 y_3 + y_2^2 - Gr y_4 - Gc y_6 + (M + K_p) y_2, \\ y_4' &= y_5, \\ y_5' &= \frac{Pr(y_2 y_4 - y_1 y_5) - Pr Ec y_3^2}{1 + Rd}, \\ y_6' &= y_7, & y_7' &= Sc(y_2 y_6 - y_1 y_7). \end{aligned}$$

The unknown shooting parameters are,

$$s_1 = f''(0), \quad s_2 = \theta'(0), \quad s_3 = \phi'(0).$$

While the terminal residual vector was defined by,

$$R(s) = [y_2(\eta_{\max}), y_4(\eta_{\max}), y_6(\eta_{\max})]^T,$$

with infinity norm,  $\|R\|_{\infty} = \max |R_i|$ .

The production calculation used  $\eta_{\max} = 10$ , step size  $h = 0.02$  and shooting tolerance  $10^{-9}$ . The wall quantities were evaluated from,

$$f''(0), \quad Nu^* = -\theta'(0), \quad Sh^* = -\phi'(0).$$

Table 1: Baseline dimensionless parameter values.

| Parameter | Value |
|-----------|-------|
| $Gr$      | 0.50  |
| $Gc$      | 0.20  |
| $M$       | 0.50  |
| $K_p$     | 0.10  |
| $Pr$      | 0.71  |
| $Sc$      | 0.62  |
| $Rd$      | 0.50  |
| $Ec$      | 0.10  |

The coupled parameter ranges investigated are  $0 \leq Rd \leq 1$  and  $0 \leq Ec \leq 0.20$ .

## 4. Results and Discussion

The numerical solution was used to examine the thermal, momentum, and species transport characteristics of the coupled MHD flow. Particular emphasis was placed on the simultaneous effects of thermal radiation and viscous dissipation. The baseline solution gives  $f''(0) = -0.8945281455$ ,  $Nu^* = 0.6348155626$  and  $Sh^* = 0.7738042946$ , with  $\|R\|_{\infty} = 1.5585 \times 10^{-14}$ .

The small residual norm confirmed convergence of the numerical solution.

### 4.1 Temperature Profiles at Fixed Parameter Values

At fixed  $Ec = 0.10$ , increasing  $Rd$  raises the temperature throughout the thermal boundary layer. The increase follows from the radiation contribution to the effective thermal diffusion term in the energy equation.

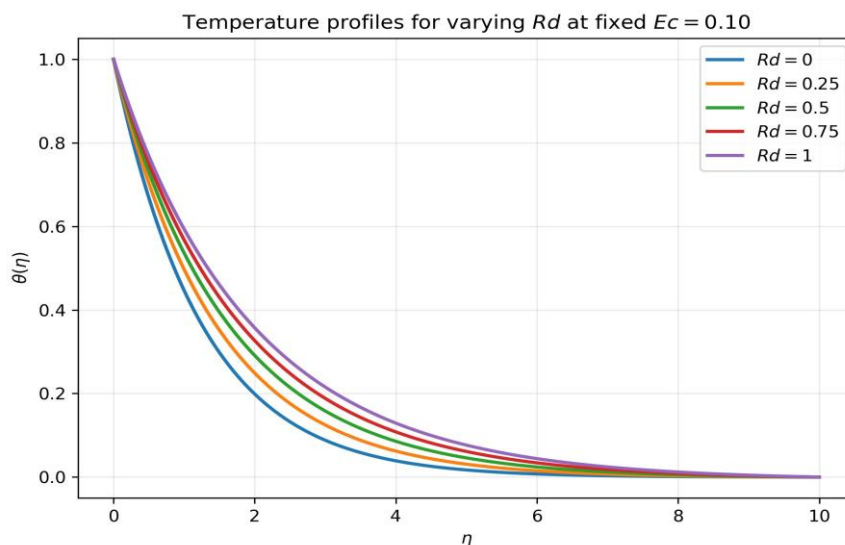
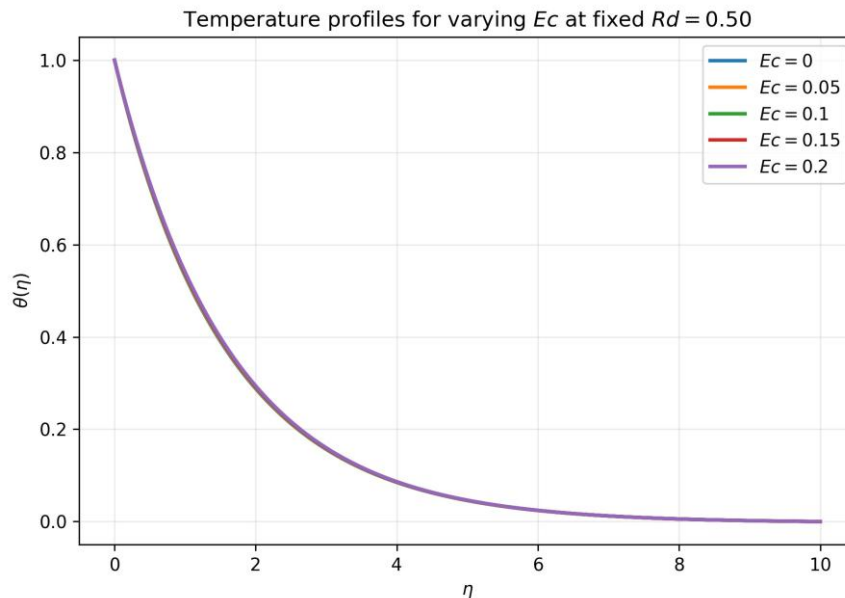


Figure 2: Temperature profiles for different  $Rd$  values at fixed  $Ec = 0.10$ .

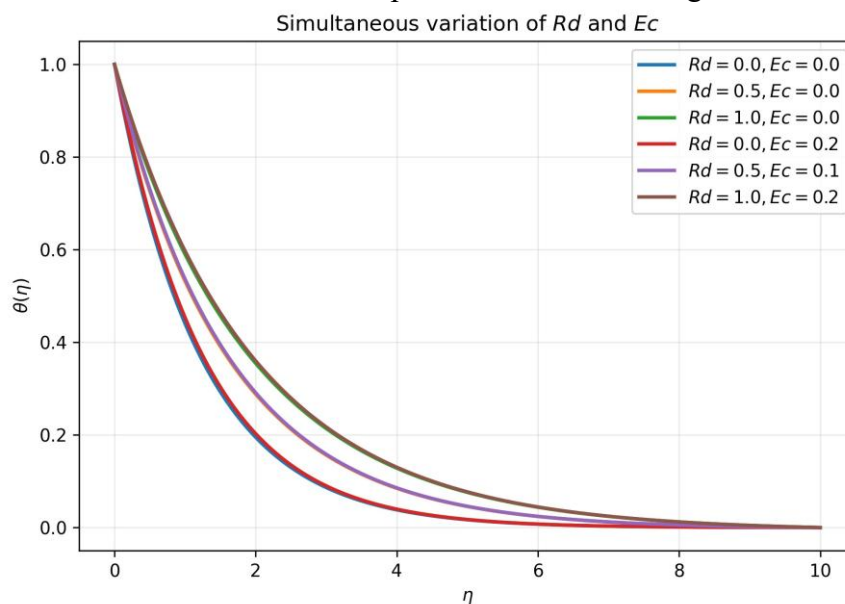
At fixed  $Rd = 0.50$ , increasing  $Ec$  also produces a monotonic increase in temperature because the viscous dissipation contribution,  $PrEc(f'')^2$  supplies internal heat to the fluid.



**Figure 3: Temperature profiles for different  $Ec$  values at fixed  $Rd = 0.50$ .**

## 4.2 Simultaneous Variation of $Rd$ and $Ec$

Figure (4) shows the temperature response when  $Rd$  and  $Ec$  vary simultaneously. The profiles confirm the combined thermal influence of radiative transport and viscous heating.

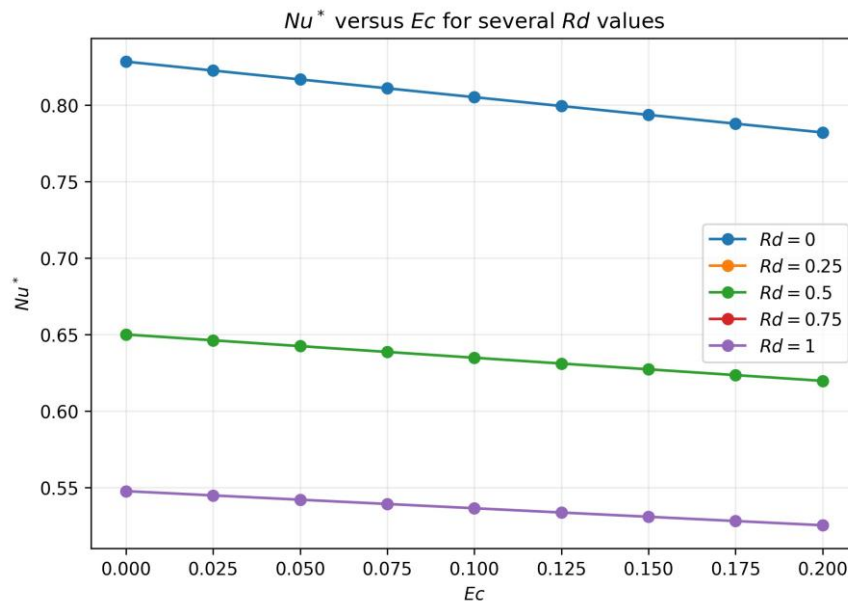


**Figure 4: Temperature profiles under simultaneous variation of  $Rd$  and  $Ec$ .**

The simultaneous variation of  $Rd$  and  $Ec$  changes the thermal field through radiative transport and viscous heating. The resulting temperature changes affect the buoyancy term  $Gr\theta$  and propagate to the velocity and concentration fields through the coupled equations.

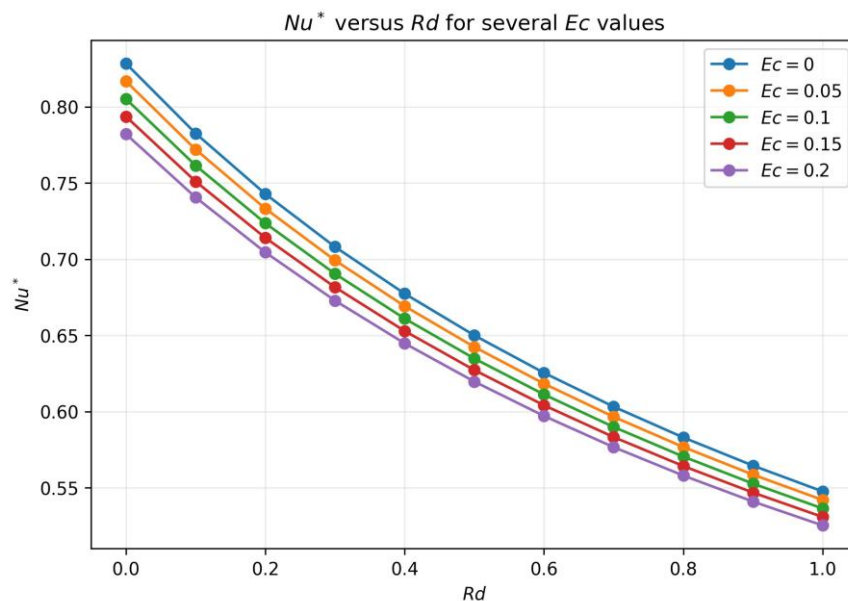
### 4.3 Effect of $Rd$ and $Ec$ on the Nusselt Number

For every tested radiation level,  $Nu^*$  decreases as  $Ec$  increases. Likewise,  $Nu^*$  decreases with  $Rd$  for all tested  $Ec$  values. The trends agree with earlier MHD stretching-flow studies reporting strong thermal sensitivity to radiation and viscous dissipation [5,7,8].



**Figure 5: Variation of  $Nu^*$  with  $Ec$  for several  $Rd$  values.**

Similarly,  $Nu^*$  decreases monotonically with  $Rd$  for all tested values of  $Ec$  as shown in figure (6) below.

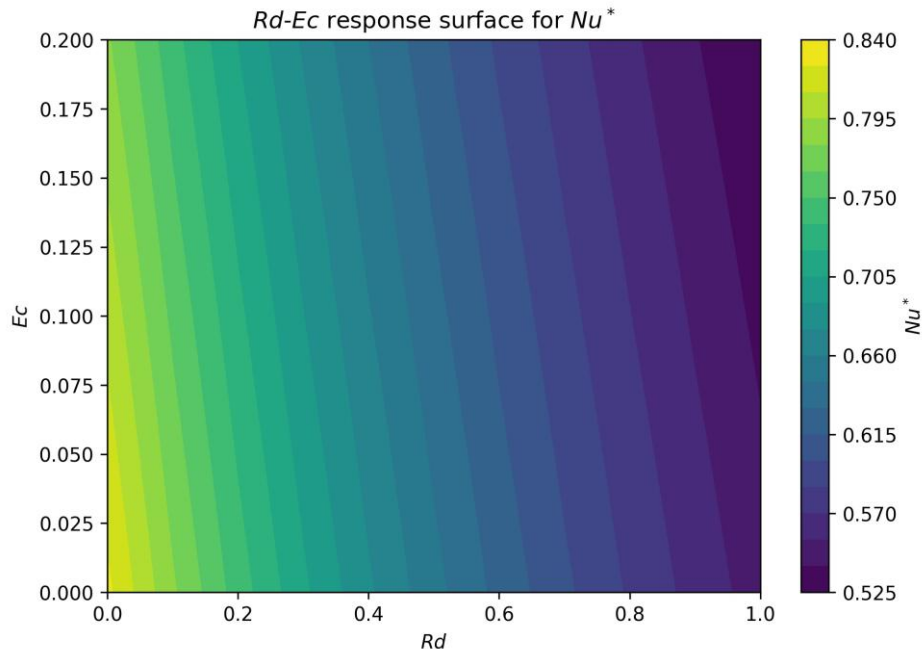


**Figure 6: Variation of  $Nu^*$  with  $Rd$  for several  $Ec$  values.**

The reduction in the dimensionless Nusselt number,  $Nu^*$  indicates a reduction in the dimensionless surface heat-transfer rate as either thermal radiation or viscous dissipation increases.

#### 4.4 Coupled $Rd$ – $Ec$ Response Surface

The combined effect of  $Rd$  and  $Ec$  on the dimensionless Nusselt number was represented using a two-parameter response surface.



**Figure 7: Coupled  $Rd$ – $Ec$  response of  $Nu^*$ .**

Across the investigated domain,  $Nu^*$  ranges from 0.828429 at  $(Rd, Ec) = (0, 0)$  to 0.525282 at  $(Rd, Ec) = (1, 0.20)$ , giving a 36.6% reduction. This response shows that simultaneous increases in radiation and viscous dissipation substantially alter the predicted surface heat-transfer rate.

$$\frac{828429 - 0.525282}{0.828429} \times 100 = 36.6\%$$

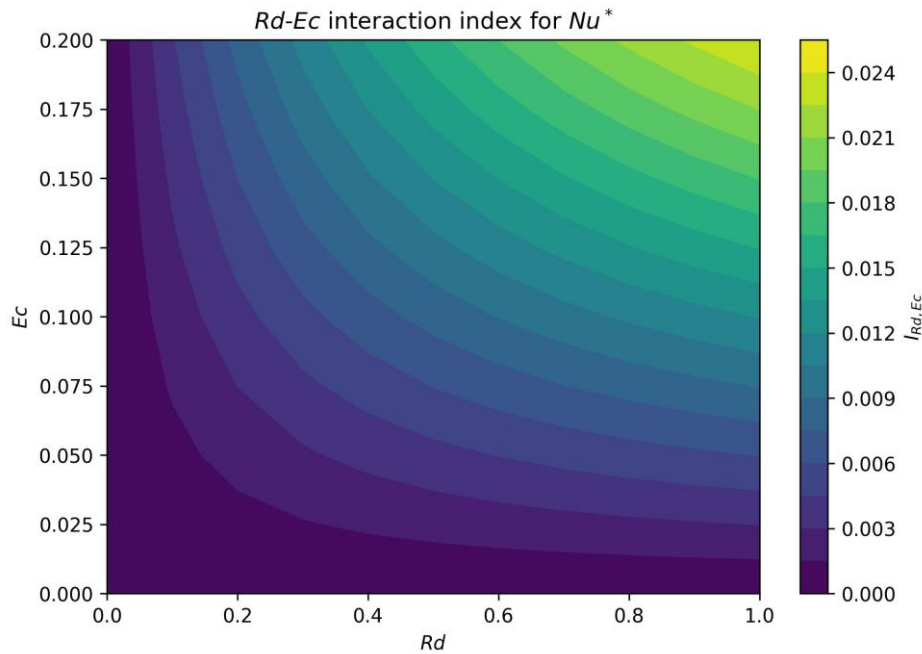
Thus, simultaneous variation of thermal radiation and viscous dissipation produces a substantial reduction in the dimensionless Nusselt number across the investigated parameter domain.

#### 4.5 Quantification of the $Rd$ – $Ec$ Interaction

To quantify the combined effect of thermal radiation and viscous dissipation, the interaction index adopted in the thesis was defined as,

$$I_{Rd, Ec} = Nu^*(Rd, Ec) - Nu^*(Rd, 0) - Nu^*(0, Ec) + Nu^*(0, 0).$$

Under this definition, an interaction index of zero represents an additive response. A positive value indicates the presence of interaction between  $Rd$  and  $Ec$ .



**Figure 8: Interaction index for the coupled  $Rd$ – $Ec$  response.**

The interaction index remains positive throughout the investigated parameter region. At  $(Rd, Ec) = (0.50, 0.10)$ , the interaction index is  $I_{Rd, Ec} = 0.008044$ , and at  $(Rd, Ec) = (1.00, 0.20)$ , the interaction index increases to,  $I_{Rd, Ec} = 0.024011$ . Thus, the positive values demonstrate that the combined thermal response differs from the simple sum of the isolated effects of thermal radiation and viscous dissipation.

## 4.6 Wall Transport Quantities

Table (2) presents representative wall quantities obtained from the numerical results.

Table 2: Representative wall quantities from the thesis numerical results.

| $Rd$ | $Ec$ | $f''(0)$ | $Nu^*$   | $Sh^*$   |
|------|------|----------|----------|----------|
| 0.00 | 0.00 | 0.922368 | 0.828429 | 0.759230 |
| 0.00 | 0.10 | 0.920551 | 0.805177 | 0.759904 |
| 0.00 | 0.20 | 0.918746 | 0.782100 | 0.760573 |
| 0.50 | 0.00 | 0.895834 | 0.650024 | 0.773310 |
| 0.50 | 0.10 | 0.894528 | 0.634816 | 0.773804 |
| 0.50 | 0.20 | 0.893228 | 0.619696 | 0.774295 |
| 1.00 | 0.00 | 0.878209 | 0.547600 | 0.782823 |
| 1.00 | 0.10 | 0.877194 | 0.536415 | 0.783206 |
| 1.00 | 0.20 | 0.876184 | 0.525282 | 0.783586 |

Table (2) shows a pronounced variation in  $Nu^*$  across the tested  $Rd$ - $Ec$  combinations, while  $Sh^*$  changes only slightly. This difference follows from the governing equations:  $Rd$  and  $Ec$  enter the energy equation directly, whereas the species equation contains neither parameter explicitly. The resulting concentration response therefore arises through coupling with the temperature and velocity fields.

## 6. Conclusion

This study investigated the coupled thermal radiation–viscous dissipation interaction in MHD buoyancy-induced heat and mass transfer flow over a Stretching Porous Surface. The governing continuity, momentum, energy, and species concentration equations were formulated and transformed into nonlinear dimensionless boundary-value problem using similarity transformations. The resultant equations were solved using the shooting technique and the fourth-order Runge-Kutta method implemented in Python. Results were graphed and tabulated with major results confirming that, thermal radiation and viscous dissipation both reduce the dimensionless Nusselt number in the investigated MHD porous stretching-surface flow. The Nusselt number decreases from 0.828429 at  $(Rd, Ec) = (0,0)$  to 0.525282 at  $(Rd, Ec) = (1,0.20)$ , representing approximately 36.6% reduction. The interaction index remains positive, increasing from 0.008044 at  $(0.50,0.10)$  to 0.024011 at  $(1.00,0.20)$ . Under the adopted definition of the interaction index the positive values indicate a non-additive combined thermal response. The Sherwood number changes more -weakly than the Nusselt number because the two thermal parameters, thermal radiation and viscous dissipation do not occur explicitly in the species equation.

The results demonstrated that the coupled  $Rd - Ec$  interaction plays an important role in the thermal behavior of the investigated MHD porous stretching-surface flow.

## 10. References

- [1] Nield, D. A., & Bejan, A. (2013). Convection in porous media. Springer.
- [2] Zhang, X., & Liu, S. (2020). Numerical study on MHD flow over a stretching porous surface with thermal radiation effects. Thermal Science and Engineering Progress, 19, 100-115.
- [3] Sharma, R., Singh, S., & Singh, U. (2018). The influence of viscous dissipation on MHD boundary layer flow over a stretching surface. International Journal of Engineering Science and Technology, 10, 12-22.
- [4] Kumar et al. (2019). A numerical study on the impact of viscous dissipation on MHD flow over a stretching sheet with porous medium.
- [5] Chamkha, A. J., & Rashad, A. M. (2018). MHD heat and mass transfer flow over a stretching porous surface with thermal radiation and viscous dissipation effects. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 40(11), 521.
- [6] Wing et al. (2019). Numerical simulation of MHD flow over a stretching porous sheet considering the influence of thermal radiation.
- [7] Hayat, T., Ali, S., & Shehzad, S. (2011). The radiation effects on MHD flow of Maxwell fluid in a channel with porous medium. Nonlinear Analysis: Real World Applications, 12, 1281-1292.
- [8] Turkyilmazoglu, M. (2011). Thermal radiation effects on the time-dependent MHD permeable flow having variable viscosity. International Journal of Heat and Mass Transfer, 54, 2733-2740.

- [9] Howell, J. R., Siegel, R., & Menguc, M. P. (2010). Thermal radiation heat transfer. CRC Press. CRC Press.
- [10] Crane, L. J. (1970). Flow past a stretching plate. Similarity Transformations. Zeitschrift für Angewandte Mathematik und Physik, 21, 645-647.
- [11] Makinde, O. D. (2010). On combined heat and mass transfer with MHD flow and thermal radiation. International Journal of Heat and Mass Transfer.
- [12] Incropera, F. P., DeWitt, D. P., Bergman, T. L., & Lavine, A. S. (2017). Fundamentals of heat and mass transfer. Wiley.
- [13] Penton, R. L. (2006). Incompressible flow. John Wiley & Sons.
- [14] White, F. M. (2006). Viscous fluid flow (3rd ed.). McGraw-Hill.