The Effect of Stokes Drag on the Resonant Motion of Satellites of the Oblate Earth

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Abstract

This paper examines the effects of Stokes drag and Earth oblateness on the resonant motion of an Earth satellite. In the presence of Stokes drag, seven resonances 1:1, 2:1, 3:1, 1:2, 2:3, 3:2, and 4:3, occur and due to the oblateness of the Earth, three resonances, 1:1, 2:1, and 3:1, occur, which are not independent.

Keywords: Stokes Drag; Oblateness; Resonance.

1. Introduction

Resonant motion of heavenly bodies has great importance in solar dynamics. Moreover, dissipation mechanisms in solar dynamics are multi-dimensional. The non-gravitational dissipative force due to the gas nebula moving through solar space is known as Stokes drag, which is proportional to the velocity of particles with respect to the gas and valid for low Renold number <10. The particular case of Stokes drag is called linear drag.

Problems of resonance play an important role in solar dynamics while solving the equations of motion. Resonance may be manifested as the appearance of small divisors in the solutions of the equations of motion. Orbital resonance of Earth satellites with respect to lunisolar gravity and direct solar radiation pressure, with particular reference to those resonances, the occurrence of which is dependent only on the satellite's orbital inclination, was studied by Hughes in (1980).

Weidenschilling et al. (1985) discovered the behaviour of resonance trapping in a gas-rich scenario. Patterson (1987) extended the work of Weidenschilling to resonances of any order and showed how planetary embryos could have formed at two-body external resonances by accretion of infinitesimals caught in these orbits. Bhatnagar et al. (1986) examined the motion of a satellite by taking the gravitational forces of the Moon, Earth, and the Sun (with radiation pressure).

Ferraz-Mello (1992) studied "averaging of the elliptic asteroidal problem with Stokes drag", and, with the assistance of Beague and Ferraz-Mello (1993), studied resonance trapping and Stokes drag dissipation in the primordial solar nebula. Celletti and Stefanelli (2011) have shown that the semi-major axis decreases due to dissipation and, consequently, the collision takes place between one of the primaries and minor bodies.

Quarles et al. (2012) have studied the resonances for the coplanar Circular Restricted Three-Body Problem for the mass ratio between 0.10 and 0.15 and used the method of maximum Lyapunov exponent to locate the resonances. They showed that for a high value of mass ratio, orbital stability is ensured where a single resonance is present. Sushil et al. (2013) worked on resonance in a geocentric satellite due to Earth's equatorial ellipticity. Rosemary (2013) has given a detailed description of the perturbation theory to determine the location of resonance based on approximations to a harmonic oscillation.
Kaul et al. (2018) studied resonance in the motion of a geocentric satellite due to PR-drag. Further, they have discussed the resonance in the motion of a geocentric satellite due to PR-drag and the equatorial ellipticity of the Earth. Hassan et al. (2022) have studied the effect of Stokes drag on the resonant motion of a geocentric satellite. Presently, we have proposed to study the joint effect of the oblateness of the Earth and Stokes drag on the resonant motion of a satellite of an oblate Earth.

2. The Equations of Motion

Considering the inertial frame (E, X_0Y_0Z) with the oblate Earth E at the origin and a rotating frame (E, XYZ) relative to the inertial one, where $\overrightarrow{EX_0}$ passes through the vernal equinox γ. The oblateness of the Earth is given by $A = I_a - I_e$, where I_a is the moment of inertia of the oblate Earth about its polar axis and I_e is the moment of inertia of the oblate Earth about its equatorial axis. Let $\hat{i}_0, \hat{j}_0$ and $\hat{i}, \hat{j}$ be the unit vectors along the axes of the inertial frame and the rotating frame, with a common unit vector $\hat{k}$ along the vertical axis *EZ* (not seen in the figure).

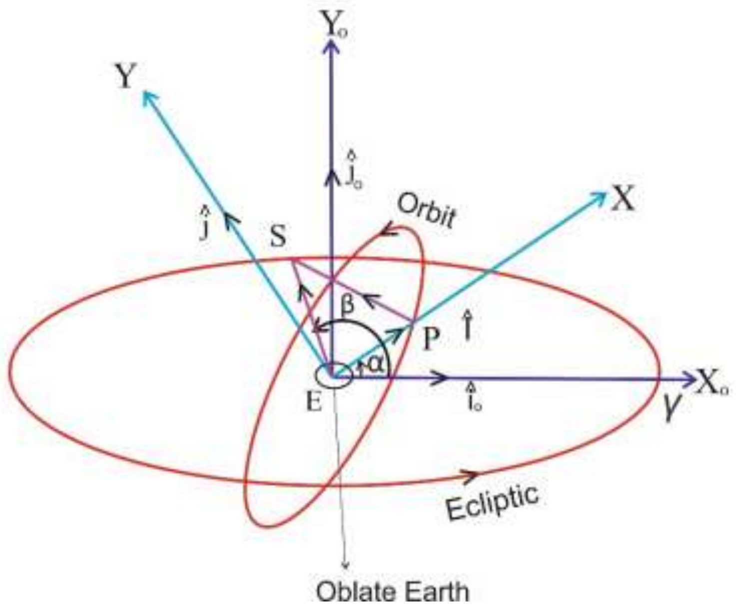

Figure 1: Representation of Satellite's Orbit around the Oblate Earth

Let $\overrightarrow{EP} = \vec{r}$ be the position vector of the satellite P, $\overrightarrow{SE} = \vec{\rho}$ be the position of the Sun S relative to the Earth E and $\overrightarrow{SP} = \vec{R}$. If M, m, μ be the masses of the Sun, Earth and the satellite respectively, then their mutual gravitational forces are given by the Earth E and $\overrightarrow{SP} = \vec{R}$. If M, m, μ be the masses of the Sun, Earth and the satellite respectively, then their mutual gravitational forces are given by the Earth E and

$\overrightarrow{SP}=\vec{R}$. If M, m, μ be the masses of the Sun, Earth and the satellite respectively, then their mutual gravitational forces are given by

$$\vec{F}_{SP}=-\frac{GM\mu}{R^3}\vec{R},\quad \vec{F}_{EP}=-\frac{Gm\mu}{r^3}\vec{r}-\frac{3GA}{2r^5}\vec{r},\quad \vec{F}_{SE}=-\frac{GMm}{\rho^3}\vec{\rho}-\frac{3GA}{2\rho^5}\vec{\rho}. \qquad \text{[McCuskey 1967]} \tag{1}$$

The force of stokes drag (non-gravitational dissipative force) due to the collision of the satellite P with the molecules in the frame of proto planetary nebulas is given by $\vec{S}_D=-c\mu\left[-\dot{\vec{R}}+(1-\lambda\Omega)\vec{R}\times\hat{k}\right]$, where $c\in[0,1)$ is the dissipative constant, depending on several physical parameters [Beauge and Ferraz-Mello 1993] like the viscosity of the gas, the radius of the satellite and the mass of the satellite, where $\Omega=\Omega(R)=R^{-3/2}$ is the Keplerian angular velocity, $\lambda\in$[o1) is the ratio between gas and Keplerian velocities [Murray 1994].

Let at any t, $\vec{\omega}$ be the angular velocity of the rotating frame relative to the inertial frame with the Earth at the origin; then, the operator's relation between the two systems is given by

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\vec{\omega}\times \tag{2}$$

$$\Rightarrow \frac{d\vec{r}}{dt}=\frac{\partial\vec{r}}{\partial t}+\vec{\omega}\times\vec{r},$$

$$\Rightarrow \frac{d^2\vec{r}}{dt^2}=\frac{d}{dt}\left(\frac{\partial\vec{r}}{\partial t}+\vec{\omega}\times\vec{r}\right)=\left(\frac{\partial}{\partial t}+\vec{\omega}\times\right)\left(\frac{\partial\vec{r}}{\partial t}+\vec{\omega}\times\vec{r}\right),$$

$$\ddot{\vec{r}}=\frac{\partial^2\vec{r}}{\partial t^2}+\vec{\omega}\times\frac{\partial\vec{r}}{\partial t}+\frac{\partial\vec{\omega}}{\partial t}\times\vec{r}+\vec{\omega}\times\frac{\partial\vec{r}}{\partial t}+\vec{\omega}\times(\vec{\omega}\times\vec{r}). \tag{3}$$

If $\hat{i}$ be the unit vector along the direction of the satellite, then $\vec{r}=r\hat{i}$ and Equation (3) is reduced to

$$\Rightarrow \ddot{\vec{r}}=\frac{\partial^2 r}{\partial t^2}\hat{i}+2\frac{\partial r}{\partial t}(\omega\times\hat{i})+r\left(\frac{\partial\vec{\omega}}{\partial t}\times\hat{i}\right)+r\vec{\omega}\times(\vec{\omega}\times\hat{i}). \tag{4}$$

Let α be the angle of direction of the satellite with the direction of vernal equinox, then $\vec{\omega}=\dot{\alpha}\vec{k}$ where $\dot{\alpha}$ is the angular velocity of the satellite relative to the Earth. Thus, Equation (4) reduces to

$$\ddot{\vec{r}}=\hat{i}\left(\frac{\partial^2 r}{\partial t^2}\right)+2\frac{\partial r}{\partial t}(\dot{\alpha}\hat{k}\times\hat{i})+r\left(\frac{\partial\dot{\alpha}}{\partial t}\hat{k}\times\hat{i}\right)+r\dot{\alpha}\hat{k}\times(\dot{\alpha}\hat{k}\times\hat{i})=\left(\frac{\partial^2 r}{\partial t^2}\right)\hat{i}+2\frac{\partial r}{\partial t}(\dot{\alpha}\hat{j})+r(\ddot{\alpha}\hat{j})+r\dot{\alpha}^2\left[\hat{k}\times(\hat{k}\times\hat{i})\right],$$

$$\Rightarrow \ddot{\vec{r}}=\left(\frac{\partial^2 r}{\partial t^2}-r\dot{\alpha}^2\right)\hat{i}+\left(2\dot{\alpha}\frac{\partial r}{\partial t}+r\ddot{\alpha}\right)\hat{j}. \tag{5}$$

In the triangle EPS of Figure 1,

$$\overrightarrow{SE}+\overrightarrow{EP}=\overrightarrow{SP},$$

$$\Rightarrow \vec{\rho}+\vec{r}=\vec{R}\Rightarrow\vec{r}=\vec{R}-\vec{\rho},$$

$$\Rightarrow \ddot{\vec{r}}=\ddot{\vec{\rho}}-\ddot{\vec{R}},$$

$$\Rightarrow \ddot{\vec{r}}=\frac{\vec{F}_{SE}}{\mu}-\frac{\vec{F}_{SP}+\vec{F}_{EP}+\vec{S}_D}{\mu},$$

$$\Rightarrow \ddot{\vec{r}}=-\frac{GM\mu}{\mu R^3}\vec{R}+\frac{Gm\mu}{\mu r^3}\vec{r}+\frac{3GA}{2\mu r^5}\vec{r}+\frac{3GA}{2\rho^5 m}\vec{\rho}-\frac{c\mu\left[\vec{R}-(1-\lambda\Omega)\vec{R}\times\hat{k}\right]}{\mu}+\frac{GMm}{\rho^3 m}\vec{\rho},$$

$$\Rightarrow \ddot{\vec{r}}=-\left(\frac{GM}{R^3}\right)\vec{R}+\frac{Gm}{r^3}\vec{r}+\frac{3GA}{2\mu r^5}\vec{r}+\frac{3GA}{2\rho^5 m}\vec{\rho}-c\left[\vec{R}-(1-\lambda\Omega)\vec{R}\times\hat{k}\right]+\frac{GMm}{\rho^3}\vec{\rho}. \tag{6}$$

If $\hat{\rho}$ be the unit vector along $\overrightarrow{ES}=\vec{\rho}$ and β be the angle between the direction of the Sun and the direction of the vernal equinox γ then

$$\hat{\rho}=\cos\beta\vec{i}_0+\sin\beta\vec{j}_0 \text{ and } \dot{\vec{\rho}}=\vec{\rho}\times\dot{\vec{\beta}}$$

$$\ddot{\vec{\rho}}=\dot{\vec{\rho}}\times\dot{\vec{\beta}}+\vec{\rho}\times\ddot{\vec{\beta}}=\dot{\vec{\rho}}\times\dot{\vec{\beta}}, \qquad (as\ \ddot{\vec{\beta}}=0)$$

$$=(\vec{\rho}\times\dot{\vec{\beta}})\times\dot{\vec{\beta}}=(\vec{\rho}\cdot\dot{\vec{\beta}})\dot{\vec{\beta}}-(\dot{\vec{\beta}}\cdot\dot{\vec{\beta}})\vec{\rho},$$

$$\Rightarrow \ddot{\vec{\rho}}=\left[\dot{\beta}\hat{k}\cdot\left(\cos\beta\hat{i}+\sin\beta\hat{j}\right)\right]\dot{\vec{\beta}}-\dot{\beta}^2\vec{\rho},$$

$$\Rightarrow -\frac{GMm}{\rho^3 m}\vec{\rho}=0\dot{\vec{\beta}}-\dot{\beta}^2\vec{\rho},$$

$$\Rightarrow \frac{GM}{\rho^3}\vec{\rho}=\dot{\beta}^2\vec{\rho},$$

$$\Rightarrow \dot{\beta}^2=\frac{GM}{\rho^3}.$$

Now, Equation (6) becomes

$$\ddot{\vec{r}}=-\left(\frac{GM}{R^3}\right)\vec{R}+\frac{Gm}{r^3}\vec{r}+\frac{3GA}{2\mu r^5}\vec{r}-c\left[\dot{\vec{R}}-(1-\lambda\Omega)\vec{R}\times\hat{k}\right]+\dot{\beta}^2\vec{\rho}+\frac{3GA}{2\rho^5 m}\vec{\rho},$$

$$\Rightarrow \ddot{\vec{r}}=\left(\frac{Gm}{r^3}+\frac{3GA}{2\mu r^5}\right)\vec{r}+\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}\right)\vec{\rho}-c\left[\dot{\vec{R}}-(1-\lambda\Omega)\vec{R}\times\hat{k}\right]-\frac{GM}{R^3}\vec{R},$$

$$\Rightarrow \ddot{\vec{r}}=\left(\frac{Gm}{r^3}+\frac{3GA}{2\mu r^5}\right)r\hat{i}+\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}\right)\rho(\cos\beta\hat{i}+\sin\beta\hat{j})-c\left[\dot{\vec{R}}-(1-\lambda\Omega)\vec{R}\times\hat{k}\right]$$
$$-\frac{GM}{R^3}(r\hat{i}+\rho\cos\beta\hat{i}+\rho\sin\beta\hat{j}). \tag{7}$$

Scalar product of $\hat{i}$ with Equation (5) and Equation (7), and comparing the results, one can find the equations of motion of the satellite in polar form as

$$\frac{\partial^2 r}{\partial t^2}-r\dot{\alpha}^2=\left(\frac{Gm}{r^3}+\frac{3GA}{2\mu r^5}\right)r+\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}\right)\rho\cos(\alpha-\beta)-c\left[\dot{\vec{R}}\cdot\hat{i}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\hat{i}\right]-\frac{GM}{R^3}(r+\rho\cos(\alpha-\beta)),$$

$$\frac{\partial^2 r}{\partial t^2}-\dot{\alpha}^2 r-\left(\frac{Gm}{r^2}+\frac{3GA}{2\mu r^4}\right)=\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}-\frac{GM}{R^3}\right)\rho\cos(\alpha-\beta)-c\left[\dot{\vec{R}}\cdot\hat{i}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\hat{i}\right]-\frac{GMr}{R^3}. \tag{8}$$

Scalar product of $\hat{j}$ with Equation (5) and Equation (7), and comparing the results, one can find the equations of motion of the satellite in polar form

$$\Rightarrow 2\frac{\partial r}{\partial t}(\dot{\alpha})+r\left(\frac{\partial\dot{\alpha}}{\partial t}\right)=\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}\right)\rho\left[-\sin(\alpha-\beta)\right]-c\left[\dot{\vec{R}}\cdot\hat{j}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\hat{j}\right]+\frac{GM\rho}{R^3}\sin(\alpha-\beta),$$

$$\Rightarrow 2r\frac{\partial r}{\partial t}(\dot{\alpha})+r^2\frac{\partial\dot{\alpha}}{\partial t}=-\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}\right)\rho r\sin(\alpha-\beta)-cr\left[\dot{\vec{R}}\cdot\hat{j}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\hat{j}\right]+\frac{GMr\rho}{R^3}\sin(\alpha-\beta),$$

$$\Rightarrow \frac{\partial}{\partial t}\left(r^2\dot{\alpha}\right)=-\left(\dot{\beta}^2+\frac{3GA}{2\rho^5 m}\right)\rho r\sin(\alpha-\beta)-cr\left[\dot{\vec{R}}\cdot\hat{j}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\hat{j}\right]+\frac{GMr\rho}{R^3}\sin(\alpha-\beta). \tag{9}$$

These equations are not integrable, so we replace r and $\dot{\alpha}$ by their steady-state value r_0 and $\dot{\alpha}_0$ by the perturbation technique, which can be introduced in Equations (8) and (9)

$$\frac{d^2r}{dt^2}-\dot{\alpha}^2r-\frac{Gm}{r^2}-\frac{3GA}{2\mu r^4}=\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)\rho\cos\left(\dot{\alpha}-\dot{\beta}\right)t-c\left[\dot{\vec{R}}\cdot\hat{i}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\hat{i}\right]-\frac{GMr_0}{R^3}. \tag{10}$$

$$\frac{d}{dt}\left(\dot{r}^2\alpha\right)=-\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}\right)\rho r_0\sin\left(\dot{\alpha}_0-\dot{\beta}_0\right)t-cr_0\left[\dot{\vec{R}}\cdot\hat{j}-(1-\lambda\Omega)\left(\vec{R}\times\hat{k}\right)\cdot\vec{j}\right]+\frac{GMr_0}{R}\sin\left(\dot{\alpha}_0-\dot{\beta}_0\right). \tag{11}$$

Also,

$$\dot{\vec{R}}=\frac{d\vec{R}}{dt}=\frac{d}{dt}(\vec{r}+\vec{\rho})=\frac{d\vec{r}}{dt}+\frac{d\vec{\rho}}{dt}=\frac{\partial\vec{r}}{\partial t}+\vec{\omega}\times\vec{r}+\frac{d}{dt}\rho\hat{\rho}=\frac{\partial\vec{r}}{\partial t}+\dot{\alpha}\hat{k}\times r\hat{i}+\rho\frac{d\hat{\rho}}{dt},$$

$$=\frac{\partial r}{\partial t}\hat{i}+\dot{\alpha}r\hat{k}\times\hat{i}+\rho\frac{d}{dt}\left(\cos\beta_0\hat{i}+\sin\beta_0\hat{j}_0\right)=\frac{\partial r}{\partial t}\hat{i}+\dot{\alpha}r\hat{j}+\rho\dot{\beta}\left(-\sin\beta\hat{i}_0+\cos\beta\hat{j}_0\right),$$

$$=\frac{\partial r}{\partial t}\hat{i}+\dot{\alpha}r\hat{j}+\rho\dot{\beta}\left[-\sin\beta\left(\cos\alpha\hat{i}-\sin\alpha\hat{j}\right)+\cos\beta\left(\sin\alpha\hat{i}-\cos\alpha\hat{j}\right)\right],$$

$$=\frac{\partial r}{\partial t}\hat{i}+\dot{\alpha}r\hat{j}+\rho\dot{\beta}\left[(\sin\alpha\cos\beta-\sin\beta\cos\alpha)\hat{i}+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)\hat{j}\right],$$

$$=\frac{\partial r}{\partial t}+\dot{\alpha}r\hat{j}+\rho\dot{\beta}\left[\sin(\alpha-\beta)\hat{i}+\cos(\alpha-\beta)\hat{j}\right],$$

$$=0\hat{i}+\dot{\alpha}_0r\vec{j}+\rho\dot{\beta}\left[\sin(\dot{\alpha}_0-\dot{\beta}_0)t\hat{i}+\cos(\dot{\alpha}_0-\dot{\beta}_0)\right].$$

$$\therefore\dot{\vec{R}}=\rho\dot{\beta}\sin(\dot{\alpha}_0-\dot{\beta}_0)t\hat{i}+\rho\dot{\beta}\cos(\dot{\alpha}_0-\dot{\beta}_0)t\hat{j}+\dot{\alpha}_0r_0\hat{j}. \tag{12}$$

As $AM\geq GM$, so

$$\Rightarrow\frac{\rho^2+r^2}{2}\geq\sqrt{\rho^2r^2},$$

$$\Rightarrow\rho^2+r^2\geq 2\rho r\geq 2\rho r\cos(\alpha-\beta),\quad \text{if}\quad \alpha\neq\beta$$

$$=\rho^{-\frac{3}{2}}\left(1+\frac{r^2}{\rho^2}\right)^{-\frac{3}{4}}\left[1-\frac{2r\rho\cos(\alpha-\beta)}{\rho^2+r^2}\right]^{-\frac{3}{4}},$$

$$=\rho^{-\frac{3}{4}}\left(1+\frac{r^2}{\rho^2}\right)^{-\frac{3}{4}}\left[1-\frac{2r\rho\cos(\alpha-\beta)}{\rho^2\left(1+\frac{r^2}{\rho^2}\right)}\right]^{-\frac{3}{4}},$$

$$=\rho^{-\frac{3}{2}}\left[1-\frac{3r^2}{4\rho^2}\right]\left[1-\frac{2r}{\rho}\left(1+\frac{r^2}{\rho^2}\right)^{-}\cos(\alpha-\beta)\right]^{-\frac{3}{4}},$$

$$=\rho^{-\frac{3}{2}}\left[1-\frac{3r^2}{4\rho^2}\right]\left[1-\frac{2r}{\rho}\left(1-\frac{r^2}{\rho^2}\right)\cos(\alpha-\beta)\right]^{-\frac{3}{4}},$$

$$=\rho^{-\frac{3}{2}}\left[1-\frac{3r^2}{4\rho^2}\right]\left[1+\frac{3}{4}\cdot\frac{2r}{\rho}\left(1-\frac{r^2}{\rho^2}\right)\cos(\alpha-\beta)\right]+\frac{-\frac{3}{4}\left(-\frac{7}{4}\right)}{2}\left\{-\frac{2r}{\rho}\left(1-\frac{r^2}{\rho^2}\right)\cos(\alpha-\beta)\right\}^2+\ldots,$$

$$=\rho^{-\frac{3}{2}}\left[1-\frac{3r^2}{4\rho^2}\right]\left[1+\frac{3}{2}\cdot\frac{r}{\rho}\cos(\alpha-\beta)+\frac{21}{32}\cdot\frac{4r^2}{\rho^2}\left(1-\frac{2r^2}{\rho^2}+\frac{r^4}{\rho^4}\right)\cos^2(\alpha-\beta)\right].$$

As $r<<\rho.$ so neglecting $(r/\rho)^3$ and higher terms

$$=\rho^{-\frac{3}{2}}\left[1-\frac{3r^2}{4\rho^2}\right]\left[1+\frac{3}{2}\cdot\frac{r}{\rho}\cos(\alpha-\beta)+\frac{21}{8}\frac{r^2}{\rho^2}\cos^2(\alpha-\beta\right],$$

$$=\rho^{-\frac{3}{2}}\left[1+\frac{3}{2}\cdot\frac{r}{\rho}\cos(\alpha-\beta)+\frac{21}{8}\frac{r^2}{\rho^2}\cos^2(\alpha-\beta)-\frac{3r^2}{4\rho^2}\right],$$

$$\Omega=R^{-\frac{3}{2}}=\left[\rho^2+r^2-2\rho r\cos(\alpha-\beta)\right]^{-\frac{3}{4}}=\left(\rho^2+r^2\right)^{-\frac{3}{4}}\left[1-\frac{2r\rho\cos(\alpha-\beta)}{\rho^2+r^2}\right]^{-\frac{3}{4}},\quad \left[\because A.M(\rho^2,r^2)\geq GM(\rho^2,r^2)\right]$$

$$\Omega=\rho^{-\frac{3}{2}}\left[1+\frac{3}{2}\frac{r}{\rho}\cos(\dot{\alpha}_0-\dot{\beta}_0)t+\frac{21r^2}{16\rho^2}\cos(2\dot{\alpha}_0-2\dot{\beta}_0)t\right]. \tag{13}$$

Since we know that $r^2\dot{\alpha}=\text{constant}=h$ (say) and $r=1/u$ hence

$$\frac{dr}{dt}=-r^2\frac{du}{d\alpha}\cdot\frac{h}{r^2}=-h\frac{du}{d\alpha},\qquad \Rightarrow\frac{d^2r}{dt^2}=-h\frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right)\cdot\frac{d\alpha}{dt},$$

$$\Rightarrow\frac{d^2r}{dt^2}=-h\dot{\alpha}\frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right),\qquad \Rightarrow\frac{d^2r}{dt^2}=-r^2\dot{\alpha}^2\frac{d^2u}{d\alpha^2},$$

$$\therefore\frac{d^2r}{dt^2}=-\frac{\dot{\alpha}^2}{u^2}\frac{d^2u}{d\alpha^2}.$$

Now, putting the value of $\frac{d^2r}{dt^2},\dot{\vec{R}}$ and $\vec{R}$ in Equation (10), we get

$$\begin{aligned}-\frac{\dot{\alpha}^2}{u^2}\frac{d^2u}{d\alpha^2}-\frac{\dot{\alpha}^2}{u}=&-\frac{Gm}{r^2}-\frac{3GA}{2\mu r^4}+\rho\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)\cdot\cos(\dot{\alpha}_0-\dot{\beta})t\\&-c\Big[\left\{\rho\dot{\beta}\sin(\dot{\alpha}_0-\dot{\beta})t\hat{i}+\rho\dot{\beta}\cos(\dot{\alpha}_0-\dot{\beta})t\hat{j}+\dot{\alpha}_0r_0\hat{j}\right\}\cdot\hat{i}\\&-\frac{GMr_0}{R^3}-(1-\lambda\Omega)\left\{(\vec{\rho}+\vec{r})\times\hat{k}\right\}\cdot\hat{i}\Big],\end{aligned}$$

$$\begin{aligned}\Rightarrow-\frac{\dot{\alpha}^2}{u^2}\left[\frac{d^2u}{d\alpha^2}+u\right]=&\frac{Gm}{r^2}+\frac{3GA}{2\mu r^4}+\rho\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)\cos(\dot{\alpha}_0-\dot{\beta})t-\frac{GMr_0}{R^3}\\&-c\left[\left\{\rho\dot{\beta}\sin(\dot{\alpha}_0-\dot{\beta})t\right\}-(1-\lambda\Omega)\left\{(\vec{\rho}+\vec{r})\times\hat{k}\right\}\cdot\hat{i}\right],\end{aligned}$$

$$\begin{aligned}\Rightarrow\frac{d^2u}{d\alpha^2}+u=&-\frac{Gm}{r^4\dot{\alpha}^2}-\frac{3GA}{2\mu r^6\dot{\alpha}^2}+\frac{GMr_0u^2}{R^3\dot{\alpha}^2}-\frac{\rho}{\dot{\alpha}^2}u^2\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)\times\cos(\dot{\alpha}_0-\dot{\beta})t\\&+\frac{c}{\dot{\alpha}^2}u^2\left[\dot{\beta}+1-\lambda\Omega\right]\rho\sin(\dot{\alpha}_0-\dot{\beta})t.\end{aligned} \tag{14}$$

$$\left[\begin{aligned}\because\left\{(\vec{\rho}+\vec{r})\times\hat{k}\right\}\cdot\hat{i}&=\left\{\left(-\rho\cos\beta\hat{i}_0+\rho\sin\beta\hat{j}_0+r\hat{i}\right)\times\hat{k}\right\}=\left(-\rho\cos\beta\hat{j}_0+\rho\sin\beta\hat{i}_0-r\hat{j}\right)\cdot\hat{i}\\&=\rho\left(-\cos\beta\sin\alpha+\sin\beta\cos\alpha\right)=-\rho\sin(\alpha-\beta)=-\rho\sin(\dot{\alpha}_0-\dot{\beta})t\end{aligned}\right]$$

Now,

$$\begin{aligned}\left(\dot{\beta}+1-\lambda\Omega\right)\sin(\dot{\alpha}_0-\dot{\beta})t&=\left(\dot{\beta}+1\right)\sin(\dot{\alpha}_0-\dot{\beta})t-\lambda\Omega\sin(\dot{\alpha}_0-\dot{\beta})t\\&=\left(\dot{\beta}+1\right)\sin(\dot{\alpha}_0-\dot{\beta})t-\lambda\rho^{-\frac{3}{2}}\left[1+\frac{3r}{2\rho}\cos(\dot{\alpha}_0-\dot{\beta})t+\frac{9r^2}{16\rho^2}+\frac{21r^2}{16\rho^2}\cos(2\dot{\alpha}_0-2\dot{\beta})t\right]\sin(\dot{\alpha}_0-\dot{\beta})t,\end{aligned}$$

$$=\left(\dot{\beta}+1-\lambda\rho^{-\frac{3}{2}}\right)\sin(\dot{\alpha}_0-\dot{\beta})t-\frac{3\lambda r}{4\rho^{\frac{5}{2}}}\sin(2\dot{\alpha}_0-2\dot{\beta})t-\frac{9\lambda r^2}{16\rho^{\frac{7}{2}}}\sin(\dot{\alpha}_0-\dot{\beta})t-\frac{21\lambda r^2}{32\rho^{\frac{7}{2}}}\left[\sin(3\dot{\alpha}_0-3\dot{\beta})t-\sin(\dot{\alpha}_0-\dot{\beta})t\right],$$

$$=\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)\sin(\dot\alpha_0-\dot\beta)t-\left(\frac{3\lambda r}{4\rho^{\frac{5}{2}}}\right)\sin(2\dot\alpha_0-2\dot\beta)t+\left(-\frac{9\lambda r^2}{16\rho^{\frac{7}{2}}}+\frac{21\lambda r^2}{32\rho^{\frac{7}{2}}}\right)\cdot\sin(\dot\alpha_0-\dot\beta)t-\frac{21\lambda r^2}{32\rho^{\frac{7}{2}}}\sin(3\dot\alpha_0-3\dot\beta)t,$$

Thus,

$$\begin{aligned}\left(\dot\beta+1-\lambda\Omega\right)\sin(\dot\alpha_0-\dot\beta)t=\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)\sin(\dot\alpha_0-\dot\beta)t-\frac{3\lambda r}{4\rho^{\frac{5}{2}}}\sin(2\dot\alpha_0-2\dot\beta)+\frac{3\lambda r^2}{32\rho^{\frac{7}{2}}}\sin(\dot\alpha_0-\dot\beta)t\\ -\frac{21\lambda r^2}{32\rho^{\frac{7}{2}}}\sin(3\dot\alpha_0-3\dot\beta)t.\end{aligned}\tag{15}$$

Collaboration of Equations (14) and (15) yields the single equation of motion of the satellite as

$$\begin{aligned}\frac{d^2u}{d\alpha^2}+u=-\frac{Gm}{r^4\dot\alpha_0{}^2}-\frac{3GAu^6}{2\mu\dot\alpha_0{}^2}+\frac{GMr_0u^2}{\dot\alpha_0{}^2R^3}-\frac{\rho}{\dot\alpha_0{}^2}\left(\dot\beta^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)\times u^2\cos(\dot\alpha_o-\dot\beta)t\\ +\frac{c\rho u^2}{\dot\alpha_0{}^2}\left[\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)\sin(\dot\alpha_0-\dot\beta)t-\frac{3\lambda r}{4\rho^{\frac{5}{2}}}\sin(2\dot\alpha_0-2\dot\beta)t+\frac{3\lambda r^2}{32\rho^{\frac{7}{2}}}\sin(\dot\alpha_0-\dot\beta)t-\frac{21\lambda r^2}{32\rho^{\frac{7}{2}}}\sin(3\dot\alpha_0-3\dot\beta)t\right],\end{aligned}$$

$$\begin{aligned}\therefore\frac{d^2u}{d\alpha^2}+u=-\frac{Gm}{r^4\dot\alpha_0{}^2}-\frac{3GAu^6}{2\mu\dot\alpha_0{}^2}+\frac{GMr_0}{\dot\alpha_0{}^2R^3}u^2-\frac{\rho}{\dot\alpha^2}\left(\dot\beta^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)u^2\cos(\dot\alpha_0-\dot\beta)t\\ +\frac{c\rho}{\dot\alpha_0{}^2}\left[\begin{aligned}&\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)u^2\sin(\dot\alpha_0-\dot\beta)t-\frac{3\lambda u}{4\rho^{\frac{5}{2}}}\sin(2\dot\alpha_0-2\dot\beta)t\\ &+\frac{3\lambda}{32\rho^{\frac{7}{2}}}\sin(\dot\alpha_0-\dot\beta)t-\frac{21\lambda}{32\rho^{\frac{7}{2}}}\sin(3\dot\alpha_0-3\dot\beta)t\end{aligned}\right].\end{aligned}\tag{16}$$

3. Resonance in the Motion of Satellite

The complete solution of the unperturbed equation of motion $\frac{d^2u}{d\alpha^2}+u=-\frac{Gm}{r_0{}^4\dot\alpha_0^2}$ is given by

$$\frac{l}{r}=1+e\cos(\alpha-\psi),$$

where $l=a(1-e^2)$ and e,ψ are constants.

$$\therefore u=\frac{1+e\cos(\alpha-\psi)}{a(1-e^2)}.$$

Let us consider $\alpha-\psi=\dot\alpha_0t=nt(\text{say})$ where n is the frequency of the satellite. Since eccentricity $e<1$ so, $(1+e\cos nt)^p\approx1+pe\cos nt$.

Also,

$$\frac{du}{dt}=\frac{du}{d\alpha}\cdot\frac{d\alpha}{dt}=\frac{du}{d\alpha}\cdot\dot\alpha_0\qquad\Rightarrow\frac{d^2u}{dt^2}=\frac{d}{dt}\left(\frac{du}{dt}\right)=\frac{d}{dt}\left(\frac{du}{d\alpha}\cdot\dot\alpha_0\right)=\dot\alpha_0\frac{d}{d\alpha}\left(\frac{du}{d\alpha}\right)\cdot\frac{d\alpha}{dt}=\dot\alpha_0^2\frac{d^2u}{d\alpha^2}.$$

After substituting the value of u and $\frac{d^2u}{dt^2}$ in Equation (16), we get

$$\Rightarrow \frac{d^2u}{dt^2}+n^2u=\left(-\frac{Gm}{r_0^{\ 4}}-\frac{3GA}{2\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0}{R^3a^2\left(1-e^2\right)}\right)+\frac{18GAe}{\mu a^6\left(1-e^2\right)^6}\cos nt-\frac{\rho\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2}\cos(n-\dot{\beta})t$$
$$-\frac{\rho e\left(\dot{\beta}^2+\frac{3GA}{2\rho^5m}-\frac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2}\left\{\cos(2n-\dot{\beta})+\cos\dot{\beta}t\right\}+\frac{c\rho\left(\dot{\beta}+1-\lambda\rho^{-\frac{3}{2}}\right)}{a^2\left(1-\rho^2\right)^2}\sin(n-\dot{\beta})t$$
$$+\frac{c\rho\left(\dot{\beta}+1-\lambda\rho^{-\frac{3}{2}}\right)e}{a^2\left(1-e^2\right)^2}\left\{\sin(2n-\dot{\beta})t-\sin\dot{\beta}t\right\}-\frac{3\lambda c}{4\rho^{\frac{3}{2}}a\left(1-e^2\right)}\sin(2n-2\dot{\beta})t$$
$$-\frac{3\lambda ce}{8\rho^{\frac{3}{2}}a\left(1-e^2\right)}\left\{\sin(3n-2\dot{\beta})t+\sin(n-2\ddot{\beta})t\right\}+\frac{3\lambda c}{32\rho^{\frac{5}{2}}}\sin(n-\dot{\beta})t-\frac{21c\lambda}{32\rho^{\frac{5}{2}}}\sin(3n-3\dot{\beta})t$$
$$+\frac{GMr_0e}{R^3a^2\left(1-e^2\right)^2}\cos nt,$$

$$\Rightarrow \frac{d^2u}{dt^2}+n^2u=-\frac{Gm}{r_0^{\ 4}}-\frac{3GA}{2\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0}{R^3a^2\left(1-e^2\right)}+\left[\frac{18GAe}{\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0e}{R^3a^2\left(1-e^2\right)^2}\right]\cos nt$$
$$+\frac{\rho e\left(-\dot{\beta}^2-\frac{3GA}{2\rho^5m}+\frac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2}\cos\dot{\beta}t-\frac{c\rho\left(\dot{\beta}+1-\lambda\rho^{-\frac{3}{2}}\right)e}{a^2\left(1-e^2\right)^2}\sin\dot{\beta}t+\frac{\rho\left(-\dot{\beta}^2-\frac{3GA}{2\rho^5m}+\frac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2}\cos(n-\dot{\beta})t$$
$$+\left[\frac{c\rho\left(\dot{\beta}+1-\lambda\rho^{-\frac{3}{2}}\right)}{a^2\left(1-e^2\right)^2}+\frac{3c\lambda}{32\rho^{\frac{5}{2}}}\right]\sin(n-\dot{\beta})t+\frac{\rho e\left(-\dot{\beta}^2-\frac{3GA}{2\rho^5m}+\frac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2}\cos(2n-\dot{\beta})t$$
$$+\frac{c\rho\left(\dot{\beta}+1-\lambda\rho^{-\frac{3}{2}}\right)e}{a^2\left(1-e^2\right)^2}\sin(2n-\dot{\beta})t-\frac{3\lambda ce}{8\rho^{\frac{3}{2}}a\left(1-e^2\right)}\sin(n-2\dot{\beta})t-\frac{3\lambda e}{4\rho^{\frac{3}{2}}a\left(1-e^2\right)}\sin(2n-2\dot{\beta})t$$
$$-\frac{3\lambda ce}{8\rho^{\frac{3}{2}}a\left(1-e^2\right)}\sin(3n-2\dot{\beta})t-\frac{21c\lambda}{32\rho^{\frac{5}{2}}}\sin(3n-3\dot{\beta})t,$$

$$\Rightarrow \frac{d^2u}{dt^2}+n^2u=\gamma_1+\gamma_2\cos nt+\gamma_3\cos\dot{\beta}t+\gamma_4\sin\dot{\beta}t+\gamma_5\cos(n-\dot{\beta})t+\gamma_6\sin(n-\dot{\beta})t$$
$$+\gamma_7\cos(2n-\dot{\beta})t+\gamma_8\sin(2n-\dot{\beta})t+\gamma_9\sin(n-2\dot{\beta})t+\gamma_{10}\sin(2n-2\dot{\beta})t \qquad (17)$$
$$+\gamma_{11}\sin(3n-2\dot{\beta})t+\gamma_{12}\sin(3n-3\dot{\beta})t.$$

where,

$$\gamma_1=\left[-\frac{Gm}{r_0^{\,4}}-\frac{3GA}{2\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0}{R^3a^2\left(1-e^2\right)}\right],\ \gamma_2=\left[\frac{18GAe}{\mu a^6\left(1-e^2\right)^6}+\frac{GMr_0e}{R^3a^2\left(1-e^2\right)^2}\right],\ \gamma_3=\frac{\rho e\left(-\dot\beta^2-\dfrac{3GA}{2\rho^5m}+\dfrac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2},$$

$$\gamma_4=\frac{-c\rho\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)e}{a^2\left(1-e^2\right)^2},\qquad \gamma_5=\frac{\rho\left(-\dot\beta^2-\dfrac{3GA}{2\rho^5m}+\dfrac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2},\qquad \gamma_6=\left[\frac{c\rho\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)}{a^2\left(1-e^2\right)^2}+\frac{3c\lambda}{32\rho^{\frac{5}{2}}}\right],$$

$$\gamma_7=\frac{\rho e\left(-\dot\beta^2-\dfrac{3GA}{2\rho^5m}+\dfrac{GM}{R^3}\right)}{a^2\left(1-e^2\right)^2}=\gamma_3,\qquad \gamma_8=\frac{c\rho\left(\dot\beta+1-\lambda\rho^{-\frac{3}{2}}\right)e}{a^2\left(1-e^2\right)^2}=-\gamma_4,\qquad \gamma_9=\frac{-3\lambda ce}{8\rho^{\frac{3}{2}}a\left(1-e^2\right)},$$

$$\gamma_{10}=\frac{-3\lambda c}{4\rho^{\frac{3}{2}}a\left(1-e^2\right)},\qquad \gamma_{11}=\frac{-3\lambda ce}{8\rho^{\frac{3}{2}}a\left(1-e^2\right)}=\gamma_9,\qquad \gamma_{12}=\frac{-21c\lambda}{32\rho^{\frac{5}{2}}}.$$

The auxiliary equation of Equation (17) is

$\omega^2+n^2=0\Rightarrow\omega=\pm ni,$ hence

$CF=H_1\cos nt+H_2\sin nt=k\left(\cos\in_1\cos nt+\sin\in_1\sin nt\right)$

where $H_1=k\cos\in_1,\quad H_2=k\sin\in_1$.

Thus, $C.F=K\cos(nt-\in_1)$ where $K,\in_1$ are constants.

$$P.I=\frac{1}{D^2+n^2}\left[\begin{array}{l}\gamma_1+\gamma_2\cos nt+\gamma_3\cos\dot\beta t+\gamma_4\sin\dot\beta t+\gamma_5\cos(n-\dot\beta)t+\gamma_6\sin(n-\dot\beta)t+\gamma_7\cos(2n-\dot\beta)t\\+\gamma_8\sin(2n-\dot\beta)t+\gamma_9\sin(n-2\dot\beta)t+\gamma_{10}\sin(2n-2\dot\beta)t+\gamma_{11}\sin(3n-2\dot\beta)t+\gamma_{12}\sin(3n-3\dot\beta)t\end{array}\right],$$

$$=\frac{\gamma_1}{n^2}+\frac{\gamma_2t\sin t}{2n}+\frac{\gamma_3\cos\dot\beta t}{n^2-\dot\beta^2}+\frac{\gamma_4\sin\dot\beta t}{n^2-\dot\beta^2}+\frac{\gamma_5\cos(n-\dot\beta)t}{n^2-\left(n-\dot\beta\right)^2}+\frac{\gamma_6\sin(n-\dot\beta)t}{n^2-\left(n-\dot\beta\right)^2}+\frac{\gamma_7\cos(2n-\dot\beta)}{n^2-(2n-\dot\beta)^2}+\frac{\gamma_8\sin(2n-\dot\beta)t}{n^2-(2n-\dot\beta)^2}$$

$$+\frac{\gamma_9\sin(n-2\dot\alpha)t}{n^2-(n-2\dot\beta)^2}+\frac{\gamma_{10}\sin(2n-2\dot\beta)t}{n^2-(2n-\dot\beta)^2}+\frac{\gamma_{11}\sin(3n-2\dot\beta)t}{n^2-(3n-2\dot\beta)^2}+\frac{\gamma_{12}\sin(3n-3\dot\beta)t}{n^2-(3n-3\dot\beta)^2},$$

$$\begin{aligned}\Rightarrow u=k\cos(nt-\in_1)+\frac{\gamma_1}{n^2}+\frac{\gamma_2t\sin nt}{2n}+\frac{\gamma_3\cos\dot\beta t}{(n+\dot\beta)(n-\dot\beta)}+\frac{\gamma_4\sin\dot\beta t}{(n+\dot\beta)(n-\dot\beta)}+\frac{\gamma_5\cos(n-\dot\beta)t}{\dot\beta(2n-\dot\beta)}+\frac{\gamma_6\sin(n-\dot\beta)t}{\dot\beta(2n-\dot\beta)}\\-\frac{\gamma_7\cos(2n-\dot\beta)}{(n-\dot\beta)(3n-\dot\beta)}-\frac{\gamma_8\sin(2n-\dot\beta)t}{(n-\dot\beta)(3n-\dot\beta)}+\frac{\gamma_9\sin(n-2\dot\beta)t}{4\dot\beta(n-\dot\beta)}-\frac{\gamma_{10}\sin(2n-2\dot\beta)t}{(n-2\dot\beta)(3n-2\dot\beta)}-\frac{\gamma_{11}\sin(3n-2\dot\beta)t}{4(n-\dot\beta)(2n-\dot\beta)}\\-\frac{\gamma_{12}\sin(3n-3\dot\beta)t}{(2n-3\dot\beta)(4n-3\dot\beta)}.\end{aligned}\tag{18}$$

On vanishing the denominator of any term in Equation (18) to zero, we get some points at which motion becomes indeterminate, and hence resonance occurs at these points. Thus, the resonances occur at the points where $n=\dot\beta,\ 2n=\dot\beta,\ 3n=\dot\beta,\ n=2\dot\beta,\ 2n=3\dot\beta,\ 3n=2\dot\beta$ and $4n=3\dot\beta.$ The seven resonances of the problem under consideration are 1:1, 2:1, 3:1, 1:2, 2:3, 3:2 and 4:3. These resonances occur due to Stokes drag, but 1:1, 2:1 and 3:1 resonances occur due to Earth's oblateness only. Thus, the resonances 1:1, 2:1 and 3:1 due to the oblateness of the Earth are not independent of Stokes drag.

4. Conclusion

When the additional effect of the Earth's oblateness is considered alongwith Stokes drag, the joint effects of both are not independent. The resonances 1:1, 2:1 and 3:1 occur due to both Stokes drag and oblateness of the Earth.

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