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Structural and Algorithmic Analysis of Domination Parameters in Complex Graph Networks

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Abstract
The current investigation proposes a structural and algorithmic approach to analyze domination metrics in complicated graph networks. Ten benchmark graphs, namely complete graph, path graph, cycle graph, star graph, wheel graph, grid graph, Erdős–Rényi graphs, Barabási–Albert graphs and Petersen graph were studied with respect to domination form, total domination form and connected domination form. Both exact ILP methodology and greedy heuristic technique were used to test for efficiency in computation. The study revealed that domination forms were dependent on the network structure whereby denser and hub-dominated graphs had lesser dominating number of vertices in comparison to sparse graphs. The ILP produced optimal solutions at the expense of time. The greedy approach scored sub-millisecond time with an approximation ratio ranging from 1.00 to 1.33. Network density and average degree were negatively correlated to activity.

Keywords: Graph domination; Complex networks; Domination parameters; Integer Linear Programming; Greedy heuristic; Network topology; Computational optimization.

1. Introduction

1.1. Graph Theory and Network Modeling

Graph theory is one of the fundamental branches of mathematics frequently utilized for problem-solving and modeling in fields such as computer science, network analysis, biology, social sciences, and many other fields [1](Broder et al., 2000). It offers a wide range of opportunities for both theoretical exploration and practical applications, making it well-suited for multidisciplinary studies. Graph theory provides a powerful framework for modeling and analyzing relationships and structures, especially in infrastructural networks [2,3](Cooper et al., 2004; Guze, 2019).

1.2. Graph-Theoretic Representation of Complex Networks

When analyzing network structures such as infrastructural networks, these graph-theoretic concepts become essential for understanding connectivity, robustness, and vulnerability [4] (Albert et al, 2000). Graph theoretic techniques provide a convenient and effective framework for investigating such networks. It is well-known that an interconnection network can be represented by a graph where vertices correspond to the nodes or sites of the network, and edges represent the communication or connection links between these sites [5] (Barabási & Albert, 1999). Consequently, a wide range of network-related problems can be approached and solved using graph theoretical methods. While solving problems in graph theory, the graph is analyzed using certain graph-theoretic parameters and solution techniques by utilizing the relationships between vertices and edges [6] (Mej, 2010).

1.3.Domination Theory and Its Applications

Domination theory is a vital area of graph theory that revolves around determining a set of vertices that can dominate the rest of the vertices in a graph [7] (Cockayne & Hedetniemi, 1977). A dominating set is defined as the subset of vertices such that every vertex that is not in the subset is at least adjacent to one vertex in the dominating set [8] (Henning, 2009). The minimum number of vertices in the dominating set is called the domination number. In the course of time, various domination parameters and related concepts have been developed to cover various structure-related and function-related issues of networks [9] (Henning & Yeo, 2013). The given parameters are very useful as they help to study coverage, control, monitoring, and the allocation of resources needed while working with network structures. These parameters are applicable for communication networks, problems of location of facilities, sensor networks, systems of transportation, etc [10] (Haynes et al., 2020).

1.4.Structural and Algorithmic Perspectives of Domination Parameters

The study of structural characteristics of domination parameters has thus become a significant subject of contemporary studies in the field of graph theory. The fact that different graph structures may lead to markedly different domination values necessitates the analysis of structural attributes when analyzing the behavior of the parameters. At the same time, computational complexity of determination of domination parameters is high for many classes of graphs, thus creating the need for the development of efficient algorithmic approaches and structural methods. Accordingly, conducting a systematic analysis of the structural properties and domination parameters of complex graphs, as well as algorithmic techniques, provides valuable insights into complex graph networks. In this regard, the current work presents the structural and algorithmic analysis of domination parameters of complex graphs with a specific focus on their relationships with graph properties and how to analyze them.

2. Review of Literature

[12] (Geethika & Dhanasekar, 2026) researched Graph Neural Networks (GNNs) that are used for graph classification problems and proposed a method to select training nodes that could improve classifying nodes. Weighted domination with reversal centrality was established as a preprocessing method for selecting training nodes for graph classification. In contrast to standard GNNs selecting training nodes by chance, the proposed method finds a minimum-weighted dominating set subject to closeness centrality. Using this approach allowed creating two models: Weighted Domination with Closeness Centrality-based Graph Convolutional Network (WDC_GCN) and Weighted Domination with Closeness Centrality-based Graph Attention Network (WDC_GAT). Experiments showed positive results in benchmark datasets, including improved results in accuracy and F1 score.

[15] (Adasme, 2026) explored the design of backbones in geometric communication networks focusing on the trade-off involving economical connection and user coverage. The authors' study focused on the comparison of two combinatorial optimization frameworks: the minimal spanning tree (MST) and the dominating tree (DT). With a view of providing a uniform framework, the authors introduced a mixed integer programming location model for optimizing the trade-off between backbone design and the expenses involved in the user assignment, by making use of the MTZ formulation, single-flow formulation as well as a cut-set formulation. The best effectiveness is shown by the developed single-flow formulation in terms of scalability. A local branching matheuristic was used for workshops involving larger networks. The theoretical analysis addressed the issues of the computational complexity, the structure of the model being considered, and the dominancy issues.

[11] (Tuncel, 2025) established the paired disjunctive dominance number . The paired disjunctive domination number has been studied in the more general setting of graph and network sensitivity and vulnerability. Based on this idea, the authors studied the paired disjunctive bondage number (PDBN), which is the smallest number of edges that must be deleted to enhance the paired disjunctive domination number of a graph or network. They computed PDBN for certain well-known classes of networks and studied some families of trees, such as double star, comet, double comet, (E_tp) and binomial trees. Furthermore, they proposed a method for calculating PDBN and discussed the difficulty of the technique, as well as its application in practice with a case study on a real-life network. The findings provide fundamental insights on graph vulnerability and the design of robust networks.

[13] (Dayap et al., 2025) carried out an exhaustive bibliometric examination of the literature on domination found in graph theory research during the years 1961-2024 by utilizing Scopus indexes. The research employed an investigation of publication history, key players, cooperative behaviour, citation impact, and emerging themes. An important finding of the research was the steady increase in the production of this scientific field, mainly in the last several decades, with the names of Henning, Hedetniemi, and Haynes being named among the most cited scientists. Co-authorship analysis showed that great international collaboration is observed, with the United States as a major research centre. Also, keyword analysis showed several significant clusters including graph algorithms, theoretical background, domination issues, and binary graph operations. Finally, the authors drew an important conclusion regarding the shift from theoretical thinking to computational and applied research.

[19] (Meena et al., 2025) considered the double Roman domination number - a certain graph-theoretical parameter which is calculated using a double Roman dominating function that assigns numbers from a set = {0, 1, 2, 3} to the vertices of the graph according to certain neighborhood conditions. In order to calculate the parameter, total weight is minimized in the graph. The parameter has been studied for various families of graphs relevant to chemical graph theory, including hexagonal chains, Pyrene graphs, and chemical graphs describing saturated and unsaturated hydrocarbons. They also provided Python pseudocode for calculating the proposed parameter for hexagonal chain and Pyrene graphs. The results suggest the possible usage of the parameter in network design since it provides efficient resource allocation, improved resilience of the systems, and enhanced defense strategy in the network which is modeled with the help of molecules.

[14] (Meybodi et al., 2024) considered the application of Graph Neural Networks (GNNs) in the context of graph type learning atypical problem of inadequate long-distance message propagation. The authors introduced a new framework for hierarchical message passing, which allows dividing the original graph into smaller graph using the theory of domination. After that, a Graph Attention Network (GAT) was applied to bring the learned parameters together and transfer them from node to node in the graph. The testing of the suggested model on the standard node classification datasets has confirmed that the offered architecture has demonstrated results equivalent to traditional GNNs, including improved performance on the graphs with gaps in connections. The results of the study are indicative of the fact that the application of graph theory domination combined with hierarchical message passing may be effectively applied in cases of GNNs.

[20] (Hu et al., 2023) suggested an enhanced fuzzy graph clustering method FCAN-MOPSO to find the clusters accurately in complicated networks considering network topology and node properties. The suggested method applies instance-frequency-weighted regularization upon the existing FCAN algorithm to tackle imbalanced fuzzy membership distributions. FCAN-MOPSO further decomposes the optimization problem into numerous sub-problems and utilizes multi objective particle swarm optimization to reach consensus with a compromise between global exploration and local exploitation. A theoretical analysis of its global convergence was presented. Extensive studies on five real-world complex networks showed that FCAN-MOPSO is more accurate and converges faster than state-of-the-art clustering algorithms. The results show that the suggested approach is an effective and efficient framework to find exactly clusters in complicated networks.

[16] (Mandyam & Sridhar, 2022) proposed the concept of power centrality measures, known as Dominance Centrality, suitable for complicated networks that have weighted or directed edges and weighted nodes. DONEX is the main measure of Dominance Centrality that is formulated by virtue of being Pareto-optimal and allows giving the values to nodes based on expected intensity of interaction indicated by weights of edges and probabilities of links according to the formulated welfare maximization model. The analysis reveals that DONEX allows capturing the flows that are different in their intensities, which shows the extent to which a particular node influences its environment. In addition, the authors present the NetProp algorithm that allows specifying the spread of dominant nodes by defining the boundaries of maximum and minimum influence of the network. Moreover, DONEX can also be improved by direct extension to multiple hops in the neighbourhoods, taking into account the effects that may be non-local.

[17] (Zenil et al., 2018) reviewed information-theoretic measures of complexity of graphs and complex networks. They reviewed and compared Shannon's entropy, lossless compressibility and algorithmic complexity, pointing out their benefits and limits for the identification of structural features of networks. The paper considered constraints of computable measures and compared them with the invariant features of algorithmic measures. The authors also contend that the present methods for algorithmic complexity may be subject to similar restrictions as classic statistical methods such as Shannon entropy. They also considered the existing definitions of algorithmic complexity for labeled and unlabeled graphs. Overall, the research found several opportunities for creating improved information-theoretic measurements beyond standard methodologies to characterize and understand complex network architectures.

[18] (Tang et al., 2012) developed a novel way for characterizing graph topologies by employing complicated network models. First, they proved that structure graphs may be described as small-world complex networks and proposed Complex Network properties Representation of a Graph (CNCRG), which considers both topological and dynamic properties. Based on these features, this work explored graph classification and clustering from various object perspectives and characteristic-view recognition for individual objects. Experiments were performed on one synthetic picture collection and two real-world image datasets. Results showed the increased object classification and clustering performance of CNCRG. In addition, its representations provided good structured view spaces using multidimensional scaling (MDS) and principal component analysis (PCA), notably for graphs derived from 2D views of 3D objects.

Table 1 Comparative analysis of recent studies on graph domination, centrality, and network-based learning

Authors (Year)	Research Focus	Method / Approach	Key Findings / Contribution
Geethika & Dhanasekar (2026)	Training-node selection for graph classification using GNNs	Weighted domination with closeness centrality; WDC-GCN and WDC-GAT	Improved classification accuracy and F1 scores on benchmark datasets.
Adasme (2026)	Backbone design in geometric communication networks	Comparison of Minimum Spanning Tree (MST) and Dominating Tree (DT); MIP formulations and local branching matheuristic	Single-flow formulation showed strong scalability and addressed cost–coverage trade-offs in network design.
Tuncel (2025)	Graph vulnerability and paired disjunctive domination	Paired disjunctive bondage number (PDBN); analysis of network and tree families	Derived PDBN results for several graph classes and provided insights into robust and vulnerable network structures.
Dayap et al. (2025)	Evolution of domination research in graph theory	Scopus-based bibliometric analysis (1961–2024)	Identified major researchers, collaboration patterns, research clusters, and the shift toward computational and applied domination studies.
Meena et al. (2025)	Double Roman domination in chemical graphs	Double Roman dominating functions; computational/pseudocode-based analysis	Demonstrated applications to chemical graph families and potential for resource allocation and network resilience.
Meybodi et al. (2024)	Long-range message propagation in GNNs	Domination-based hierarchical graph partitioning combined with GAT	Improved/maintained GNN performance, particularly for graphs with connectivity gaps, demonstrating the utility of domination theory in GNNs.
Hu et al. (2023)	Clustering of complex networks	FCAN-MOPSO integrating frequency-weighted regularization and multi-objective PSO	Achieved improved clustering accuracy and convergence compared with state-of-the-art methods.
Mandyam & Sridhar (2022)	Centrality and influence in complex networks	DONEX dominance centrality and NetProp algorithm	Captured weighted and probabilistic node influence and enabled analysis of multi-hop network propagation.
Zenil et al.	Complexity	Shannon entropy, compressibility,	Compared information-

(2018)	measurement of graphs and networks	and algorithmic complexity	theoretic approaches and identified opportunities for improved characterization of network complexity.
Tang et al. (2012)	Graph topology representation and classification	Complex Network properties Representation of a Graph (CNCRG); MDS and PCA	Improved graph-based object classification and clustering by incorporating topological and dynamic properties.

3. Problem Statement

Complex graph networks demonstrate a variety of structural features, which makes studying domination parameters complex since different types of graphs can yield completely different values and properties for domination. While domination theory has been widely developed and its different parameters have been introduced to solve coverage, control, monitoring, vulnerability, and resource allocation issues in networks, already existing studies have tended to address either a certain domination parameter, specific classes of graphs, or practical-oriented problems rather than approach the matter systematically from the structural and algorithmic point of view. Besides, calculating domination parameters for complex graphs can turn to be computationally complex, which makes conventional methods inefficient and unscalable. In recent studies, domination concepts have been increasingly applied to graph neural networks, network vulnerability, optimization and computational approaches; however, the relationship between the structural properties of complex graphs, the behavior of domination parameters and the development of efficient algorithmic approaches is still not sufficiently integrated. This leads to the necessity of a comprehensive study on the effect of graph structural properties on domination parameters and on the computational and algorithmic issues involved in determining them. In this work, we tackle this topic by conducting a structural and algorithmic analysis of domination parameters in complex graph networks, aiming to understand their interactions with graph features and to propose appropriate methodologies for their analysis.

4. Research Methodology

The methodologies applied in comprehending the structural and algorithmic behavior of domination parameters over complex graph networks are laid out in this section, which is broken down into six steps. The process of this research includes: dataset construction, formal definition, algorithm creation, implementation of algorithms, experiments, and statistical analysis as displayed in Figure 1

4.1. Research Design

A computational and comparative research approach is implemented in this study for the purpose of analysis of how domination parameters behave both structurally and algorithmically in the context of complex graph networks. The methodology is based on the creation of graph datasets, formalization of domination parameters, design of algorithms, computational implementation, and experimentation with statistical analysis. A variety of benchmark graphs have been taken into account with a view to representing deterministic, regular, structured, and stochastic types of networks. Both ILP and greedy approaches are employed to evaluate quality of solutions and computational efficiency. The investigation of the structural characteristics (density and average degree of the graph) is also carried out with the regard to normalized domination values. The entire research process is available in figure 1.

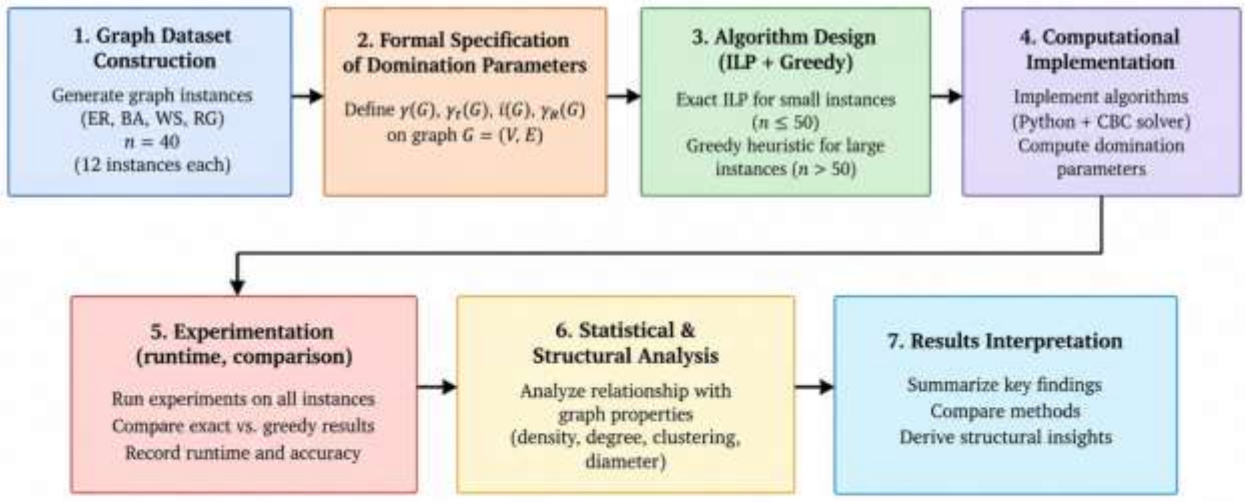

Figure 1 Research workflow adopted for the structural and algorithmic analysis of domination parameters.

4.2. Graph Dataset

The ten benchmark graphs comprising deterministic, structured, and stochastic models utilized are as follows: Petersen graph, extremal/regular features (complete, cycle, path, star, wheel); planar lattice (grid graph); and stochastic models (Erdős–Rényi random graphs, Barabási–Albert scale-free network) deriving their features and properties from real infrastructural networks. Structural properties of these graphs are stated in the table 1.

Table 2 Structural properties of the graph dataset

Graph	Type	|V|	|E|	Avg. deg.	Density
Petersen graph	Deterministic	10	15	3.0	0.333
Complete graph K8	Extremal	8	28	7.0	1.0
Cycle C12	Regular	12	12	2.0	0.182
Path P12	Tree	12	11	1.83	0.167
Star S12	Tree (extremal)	13	12	1.85	0.154
Wheel W12	Structured	13	24	3.69	0.308
Grid 4x4	Planar lattice	16	24	3.0	0.2
ER random G(20,0.15)	Stochastic (sparse)	20	37	3.7	0.195
ER random G(20,0.30)	Stochastic (dense)	20	67	6.7	0.353
Barabasi-Albert BA(20,2)	Scale-free	20	36	3.6	0.189

4.3. Domination Parameters

In the instance of connected graph G = (V,E), a collection of vertices D ⊆ V qualifies as a dominating set if every point not belonging to D has at least one point in D in proximity. Five domination characteristics were described and assessed:

$\gamma(G) = min\{ |D| : D \subseteq V, \forall v \in V \backslash D, \exists u \in D \; with \; uv \in E \}$ (1)

Domination number -- size of a smallest dominating set.

$\gamma_t(G) = min\{ |D| : D \subseteq V, \forall v \in V, \exists u \in D \; with \; uv \in E \}$ (2)

Total domination number — every vertex, including those in D, must have a neighbour in D.

$\gamma c(G) = min\{ |D| : D \; dominates \; G \; and \; G[D] \; is \; connected \}$ (3)

Connected domination number — the induced subgraph on D must be connected.

$i(G) = min\{ |D| : D \; dominates \; G \; and \; D \; is \; independent \}$ (4)

Independent domination number — combines domination with independence.

$\gamma^{x2}(G) = min\{ |D| : D \subseteq V, \forall v \in V \backslash D, |N(v) \cap D| \geq 2 \}$ (5)

Double domination number — every outside vertex needs two neighbours in D, for redundancy.

4.4. Algorithmic Framework

Two methods that complement each other have been employed: an integer linear programming formulation (ILP) that is close to optimal and a polynomial-time greedy heuristic that will be useful for scaling the process.

4.4.1. Exact ILP Formulation

Domination set calculations can be framed as the integer programming problem of the set-covering type. In this case, $x_v \in \{0,1\}$ denotes selection of vertex v, and N[v] denotes its closed neighborhood:

$minimise \; \Sigma_v \in v \; x_v \; s.t. \; \Sigma_u \in N[_v] \; x_u \geq 1 \; \forall v \in V, x_v \in \{0,1\}$ (6)

The global optimal solution is obtained through the SciPy framework by the branch-and-bound MILP. The concepts of total, connected, and double domination are derived from the changes made in the coverage constraint.

4.4.2. Greedy Heuristic

The procedure of maximum coverage in the greedy way has been carried out in the following way: (i) set $= \emptyset, U = V$, (ii) choose v* such that $|N[v *] \cap U|$ is maximized, (iii) include v* in D and remove all vertices from $N[v *]$ from U, (iv) repeat until $U = \emptyset$. The time complexity of this approach is $O(n^2)$ and the approximation ratio is equal to $1 + ln(\Delta + 1)$ where U is the set of Not Chosen Vertices. The disconnected components in the connected problem of domination are linked using shortest path.

4.5. Computational Environment

The development of the algorithms has been undertaken in Python 3.11, using the NetworkX library as well as SciPy MILP solver. The theoretical complexity of the employed methods can be seen in Table 2.

Table 3 Time complexity of the algorithmic approaches.

Approach	Time complexity	Solution quality
Brute-force enumeration	$O(2^n)$	Exact
Exact ILP (branch-and-bound)	Exponential worst case	Exact
Greedy heuristic	$O(n^2)$–$O(nm)$	$\leq(1+\ln(\Delta+1))\cdot\gamma(G)$
Connected domination (greedy+connect)	$O(n^3)$	Heuristic, near-optimal

4.6. Evaluation Metrics

The evaluation of the performance of both exact and heuristic methodologies has been carried out using a total of four different metrics which complement each other. Firstly, the calculation of the domination number γ(G), total domination number γ_t(G), and connected domination number γc(G) was carried out in order to examine the structural requirements of different benchmark graphs. Secondly, approximate ratio was determined to evaluate the performance of the greedy heuristic as against that of the exact ILP solution:

$$R = \frac{\gamma greedy(G)}{\gamma exact(G)}$$

When R=1, it is an indicator of the optimality of the solution, but the larger the R value, the farther the solution deviates from the true solution. The study also reports running time (in milliseconds) of exact ILP and greedy techniques for increasing sizes of graph in order to test the computational scalability. Finally, normalized domination number was calculated as:

$$\gamma norm(G) = \frac{\gamma(G)}{n}$$

Where, $n = |V|$, Pearson's correlation coefficient (r) was then used to examine the relationship between normalized domination and structural properties, particularly graph density and average degree.

5. Results and Discussion

The computational results of the proposed technique are presented in this section. Structural behavior of domination parameters, computational behavior of precise and heuristic approaches, and the interaction between network structure and normalized domination are addressed with attention.

5.1. Domination Numbers across Graph Classes

Table 3 outlines the values of domination number $\gamma(G)$, total domination number $\gamma_t(G)$ and connected domination number $\gamma_c(G)$ for each of the ten benchmark classes of graphs. The findings reveal a considerable level of difference in domination characteristics depending on the characteristics of the graph.

The theoretical hierarchy

$$\gamma(G) \leq \gamma t(G) \leq \gamma c(G)$$

is satisfied across all benchmark graphs. Complete graph, star graph and wheel graph have the lowest possible domination number, $\gamma(G) = 1$ because in these types of graphs each hub vertex can dominate the other vertices quite effectively. However, path graphs, cycle graphs and grid graphs have to use bigger dominating sets due to their lower neighborhood coverage ability.

The structural influence is particularly high for the connected domination parameter. For both C_{12} and P_{12}, $\gamma c(G) = 10$, compared to $\gamma(G) = 4$.This means that the connectivity requirement on the dominating vertices might drastically increase the number of selected vertices in sparse network topologies. Similarly, the 4×4 grid has $\gamma(G) = 4$, whereas its connected domination number increases to 9.

Table 4 Domination, total domination, and connected domination numbers.

Graph	n	m	γ(G)	γ_t(G)	γc(G)	γ_t/γ
Petersen graph	10	15	3	4	4	1.33
Complete graph K8	8	28	1	2	1	2.0
Cycle C12	12	12	4	6	10	1.5
Path P12	12	11	4	6	10	1.5
Star S12	13	12	1	2	1	2.0

Wheel W12	13	24	1	2	1	2.0
Grid 4x4	16	24	4	6	9	1.5
ER random G(20,0.15)	20	37	5	7	9	1.4
ER random G(20,0.30)	20	67	3	3	4	1.0
Barabasi-Albert BA(20,2)	20	36	4	5	5	1.25

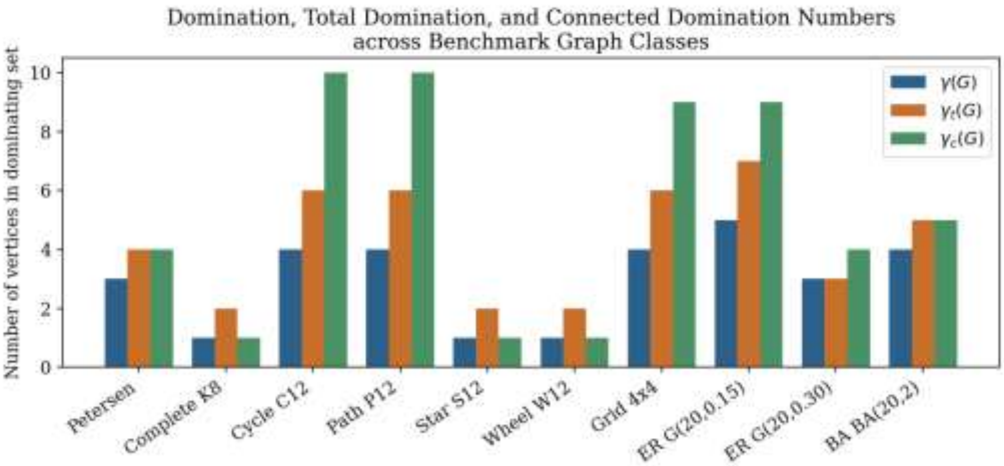

Figure 2 Comparison of $\gamma(G)$, $\gamma_t(G)$, and $\gamma c(G)$ across the benchmark graph classes.

Results confirm the bound $\gamma(G) \leq \gamma_t(G) \leq \gamma c(G)$. Hub-dominated structures (Star, Wheel, Complete K8) achieve the minimum $\gamma = 1$ via a single high-degree vertex, while sparse, low-degree structures (path, cycle, grid) show markedly larger domination numbers, with connected domination approaching n in the absence of shortcuts.

5.2. Exact ILP vs. Greedy Heuristic

The exact ILP solver and greedy heuristic were evaluated using Erdős–Rényi graphs with increasing numbers of vertices, n=9–80, while maintaining p=0.2. Table 4 compares the resulting domination values, computation times, and approximation ratios.

The exact ILP method obtained the optimal domination value for every tested instance. The greedy heuristic also achieved the optimal solution for n=9, 20, and 70, producing an approximation ratio of 1.0. For the remaining instances, the heuristic produced near-optimal solutions, with the approximation ratio ranging from 1.143 to 1.333.

A pronounced difference was observed in computational time. The exact ILP runtime increased from 3.69 ms at n=9 to 705.1 ms at n=80, whereas the greedy heuristic remained below 1 ms for all tested graph sizes. At n=80, the greedy method required only 0.681 ms compared with 705.1 ms for the exact method.

The approximation ratio ranged from 1.00 to 1.33, with a mean of approximately 1.12. These findings demonstrate that the greedy heuristic provides a substantial computational advantage while maintaining close agreement with the optimal ILP solutions

Table 5 Runtime and solution-quality comparison: exact ILP vs. greedy heuristic

n	m	γ exact	γ greedy	t exact (ms)	t greedy (ms)	Ratio
9	10	3	3	3.69	0.039	1.0
20	38	5	5	7.26	0.126	1.0
30	86	6	7	21.08	0.24	1.167
40	149	6	8	16.25	0.291	1.333
50	227	6	7	83.13	0.287	1.167
60	335	7	8	217.92	0.421	1.143
70	467	7	7	469.74	0.501	1.0
80	624	7	8	705.1	0.681	1.143

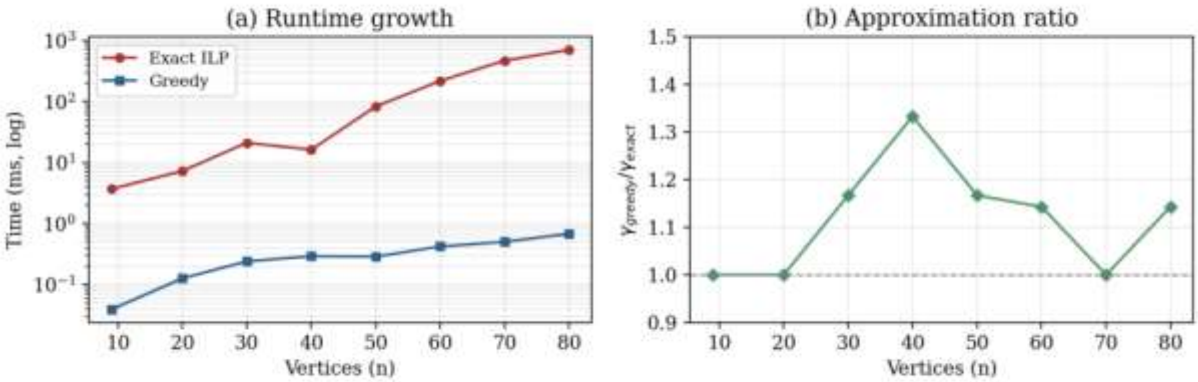

Figure 3 (a) Computation-time growth (log scale) and (b) approximation ratio of the greedy heuristic vs. graph size.

5.3. Effect of Structure on Domination Number

The normalized domination number γ(G)/n for Erdős–Rényi graphs (n = 25) was calculated for increasing edge probability and associated with density and average degree (Table 5, Figure 4).

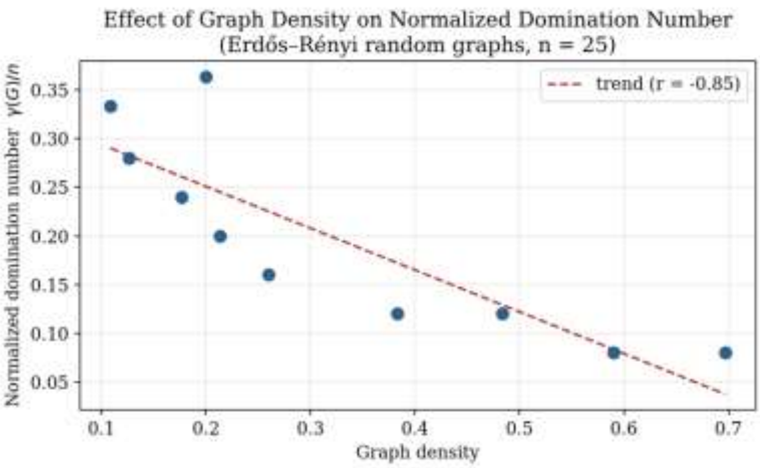

Figure 4 Graph density vs. normalised domination number γ(G)/n.

Table 6 Correlation between structural properties and γ(G)/n

Property	Pearson r	p-value	Interpretation
Graph density	-0.854	0.0017	Strong negative
Average degree	-0.909	0.0003	Strong negative

The two features are highly and negatively correlated with the ratio $\gamma(G)/n$; that is, dense networks require relatively few dominating vertices, because a vertex covers a larger area, while sparse networks require more of such vertices for complete coverage.

5.4. Discussion

Three patterns appear. First, the conservation $\gamma(G) \leq \gamma_{\mathrm{t}}(G) \leq \gamma\mathrm{c}(G)$ holds true for every class. In addition, the difference increases for sparse structures that resemble trees because of the extra cost incurred by the needs for connectivity or total coverage. The second observation is about the fact that the efficient run time of the greedy algorithm and its low approximation ration make it very suitable for use in large-scale networks, where solving by ILP is not possible. The third observation confirms that the great negative correlation between density and domination means that the level of connectivity density determines the efficiency of monitoring and robustness much better than the size of the structure. Nevertheless, the current findings add on what was said in Section 2 and prove that the methods based on domination analysis can be applied to the analysis of resources distribution, monitoring methods, and examining the robustness of structures. The only limitation is in the exact ILP-solving methods that should work for structures with less than several hundred vertices.

6. Conclusion & Future Scope

This research conducted an evaluation of domination parameters in complicated graphs, involving various properties of graph theory and computational optimization methods. The results coming from ten classes of graphs have shown that the domination parameters depend strongly on the structural characteristics of the networks being studied. The graphs, which show dominant subgraphs such as complete, star, and wheel graphs have minimum domination parameters. Meanwhile, thin graphs such as paths, cycles, or grids need bigger dominating sets, especially if taking into account connectivity conditions. The analysis further confirmed the hierarchy $\gamma(G) \leq \gamma t(G) \leq \gamma c(G)$ across the investigated graph classes. The comparison of exact ILP with greedy methods showed that while ILP always achieved optimum solutions, it was greatly affected by computational time as the size of the graph increased, whereas the greedy heuristic was able to maintain computing times of less than a millisecond along with nearly optimum solutions in terms of approximation ratios of 1.00 to 1.33. In addition, the density of the graph and its average degree were found to be strongly negatively correlated with normalized domination number, indicating that denser networks can achieve effective coverage with fewer dominating vertices. Future inquiry could incorporate the suggested framework into bigger real-world networks that are weighted, directed, dynamic and temporal from where regular ILP approaches might become computationally limiting. Further research might bring in advanced optimization techniques including metaheuristics and parallel or learning based optimization methods, as well as systematically examining individual and double parameters introduced in our study. These wider applications could improve usage of domination-based methodology for large-scale network monitoring, resource distribution, evaluation of vulnerabilities of network and its design.

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