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# Direct, Accelerated, and Accurate PDT Superposition Approach for Sharing of Branch Power Losses and Loads in the IEEE 9-Bus System

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**Abstract**:
Accurate allocation of individual generator contributions to specific loads and branch power losses is essential for transparent electricity pricing in deregulated power markets. Conventional allocation methodologies such as flow-tracing algorithms, Bialek's method, and Kirschen's method suffer from heavy computational burdens due to complex matrix iterations and non-invertible flow matrices. To overcome these limitations, this paper presents an innovative Power Division Theorem (PDT)-Superposition framework that eliminates iterative computations by linearizing quadratic, non-linear power dependencies into a structural current framework. Grounded in network circuit theory, the proposed framework establishes explicit nodal equations for the IEEE 9-bus system, embedding operational constraints directly into a back-solvable Mathcad solve block for direct verification and independent auditing. Experimental validation proves that while the PDT-Superposition framework provides an exact complex power benchmark, the standalone PDT formula resolves load-sharing profiles within a tight <5% error margin. This allows grid operators to strategically bypass the superposition principle entirely to minimize execution overhead. Ultimately, this approach provides a faster, direct, and highly transparent methodology for real-time power and loss tracking in modern energy management systems.

**Keywords:** Power Division Theorem (PDT), Superposition theorem, Branch power losses, Power allocation, Deregulated electricity markets, computational complexity.

## 1. Introduction

The rapid expansion and integration of open-access generation in deregulated power markets have transformed modern transmission networks into highly complex transaction hubs. Under these market structures, accurate and fair allocation of branch power losses and individual load contributions to specific generation units is no longer a trivial power flow exercise; it is an economic and operational necessity for transparent electricity pricing. When network pricing mechanisms lack transparency, market participants lose confidence, which leads to disputes and complicates independent regulatory auditing. To trace electricity and allocate costs, conventional tracking methodologies rely primarily on topological tracing schemes. Narimani et al. proposed a modified tracing framework based on Bialek's method to identify specific network lines whose power flows depend entirely on single-node generation or demand **[1]**. While this approach accelerates processing times by bypassing distribution matrices, it still struggles with dense matrix structures in highly meshed grids [1]. To address the exact equitable distribution of network costs among users, alternative models have integrated power flow analysis and modelling with the superposition theorem to isolate demand-specific impacts on branch losses [2]. This method works across both meshed and radial system topologies [2]**.** Shifting the focus to transaction frameworks, researchers often compare Power Transfer Distribution Factor (PTDF) methods—specifically evaluating the trade-offs between the computationally faster DC-PTDF models and the mathematically rigorous, complex AC-PTDF formulations [3]. To optimize transmission grids further under these configurations, advanced heuristic network reconfigurations utilizing unique bus identification schemes have been deployed to accelerate calculations over traditional quadratic allocation methods [4]. Furthermore, state-of-the-art scalable Mixed-Integer Quadratic Programming (MIQP) models have been developed to determine optimal capacity configurations and minimize active power losses by up to 94.30% [5]**.** Despite these developments, conventional configurations still carry inherent structural limitations. Most conventional methods rely on tracing algorithms, precise system modelling, and power flow analysis with acceptable approximations, using the faster DC-PTDF method to improve accuracy [1, 2, 3]**.** An analytical loss allocation technique that outperforms the traditional quadratic method still increases computational work results with less accuracy [4]**.** Meanwhile, advanced MIQP optimization frameworks are restricted to deciding the optimal physical location for loss minimization rather than tracing active transactions [5]**.** Crucially, definitive proof for the claimed results is missing across all of these conventional methods. To resolve these compounding issues, this paper introduces a direct, accelerated, and highly accurate tracking framework based on the integration of the Power Division Theorem (PDT){6] and the Superposition Theorem [7,9]. Conventional power system modelling historically rejects the application of superposition to power values because complex power (S = VI*) is inherently non-linear and quadratic. The integration of the PDT fundamentally resolves this long-standing challenge [6]. By converting quadratic, non-linear power dependencies into a linearized structural current framework, PDT provides the exact mathematical validation needed to apply superposition principles directly to complex network power sharing [7, 8, 9]. Using the standard IEEE 9-bus system, this paper demonstrates the proposed approach by establishing a base case load flow with nodal equations and constraints in a Mathcad solve block. It then illustrates how PDT evaluates power sharing based on data derived from superposition. Because the method directly relies on these two fundamental network theorems, the resulting allocation data is mathematically guaranteed to be correct, precise, and universally verifiable [6, 7, 8, 9]**.**

## 2. IEEE 9 Bus System

System Base Apparent Power (S base**):** 100 MVA, System Nominal Frequency (f): 60 Hz

Voltage Bases (V base): 16.5 kV (Bus 1), 18.0 kV (Bus 2), 13.8 kV (Bus 3), and 230 kV for the transmission network (Buses 4–9). All values in the figures are in Per Unit.

**A. Base Case Load Flow**

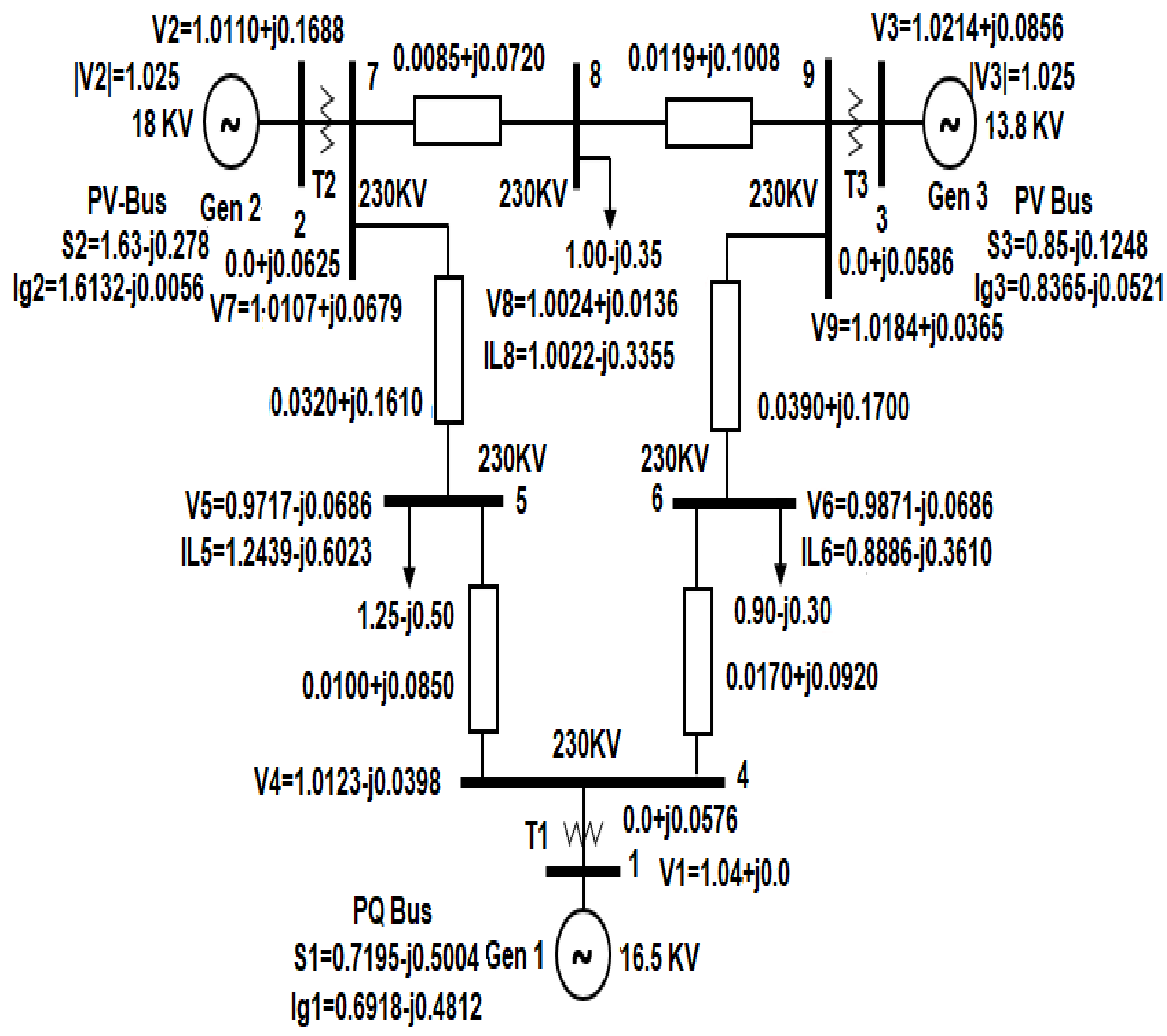


**Figure 1: IEEE 9 Bus System with Complete Details of Lines and Buses (Base Case Load Flow)**

The Half Susceptance for each line is given below:
Line 4-5 = 0.0880, Line 4-6 = 0.0790, Line 5-7 = 0.0153, Line 7-8 = 0.0745, Line 8-9 = 0.1045, Line 9-6 = 0.0179

The value of Susceptance at each node: $B44 = j(0.0880 + 0.0790) = j0.1670$,
$B55 = j(0.0880 + 0.0153) = j0.1033$, $B66 = j(0.0790 + 0.0179) = j0.0969$,
$B77 = j(0.0153 + 0.0745) = j0.0898$, $B88 = j(0.0745 + 0.1045) = j0.179$,
$B99 = j(0.1045 + 0.0179) = j0.1224$.

The dynamic load power configurations operating at the network load centers are specified as L5 = 1.25 - j0.50 p.u, L6 = 0.90 - j0.30 p.u., and L8 = 1.00 - j0.35 p.u. Each individual nodal bus susceptance operates strictly in parallel with its corresponding load parameter, all of which are fully integrated into the baseline case load flow matrices.

$$TOL = 10^{-5}$$

Node 1 $\frac{V1 - V4}{Z14} \approx I1$ Node 2 $\frac{V2 - V7}{Z27} \approx I2$ Node 3 $\frac{V3 - V9}{Z39} \approx I3$

Node 4 $\frac{V4 - V6}{Z46} + \frac{V4 - V5}{Z45} + B44 \cdot V4 \approx I1$

Node 5 $\frac{V5 - V4}{Z45} + \frac{V5 - V7}{Z57} + \frac{L5}{\overline{V5}} + B55 \cdot V5 \approx 0$

Node 6 $\frac{V6 - V9}{Z69} + \frac{V6 - V4}{Z46} + \frac{L6}{\overline{V6}} + B66 \cdot V6 \approx 0$

Node 7 $\frac{V7 - V8}{Z78} + \frac{V7 - V5}{Z57} + B77 \cdot V7 \approx I2$

Node 8 $\frac{V8 - V9}{Z89} + \frac{V8 - V7}{Z78} + \frac{L8}{\overline{V8}} + B88 \cdot V8 \approx 0$

Node 9 $\frac{V9 - V8}{Z89} + \frac{V9 - V6}{Z69} + B99 \cdot V9 \approx I3$

Constraint Equations

$S2 \approx \overline{V2} \cdot I2$
$S3 \approx \overline{V3} \cdot I3$
$Re(S2) \approx 1.63$
$Re(S3) \approx 0.85$
$|V2| \approx 1.025$
$|V3| \approx 1.025$

Figure 2: Load Flow Equations with Constraints; Mathcad Solve Block: Tolerance Set at $10^{-5}$

1. **Variables to be found:** I1, I2, I3, S1, S2, V2, V3, V4, V5, V6, V7, V8, V9, Voltage at bus 1 kept at 1.04+j0.0.The other variables are guessed initially. Results are marked in the Figure 1.
2. Susceptance Current **IB** = (B44 × V4 + B55 × V5 + B66 × V6 + B77 × V7 + B88 × V8 + B99 × V9) = 0.0069+j0.7599
3. Total Generator current **IG** = I1 + I2 + I3 = 3.1415-j0.5389
4. Total Load Current= **IL**= IL5 + IL6 + IL8 = 3.1346-j1.2988
5. **Verifying Current balance**: IL + IB = 3.1346 - j1.2988 + 0.0069 + j0.7599 = IG = 3.1415-j0.5389
6. **Load Impedances**: ZL5 = V5 / IL5 = 0.6544 - j0.2618, ZL6 = V6 / IL6 = 0.9784 - j0.3261, ZL8 = V8 / I L8 = 0.8952 - j0.3133
7. **Actual Line/Branch Loss Calculation (TBL):**
8. Line 1-4 = TB14 = $|(V1-V4)|^2/Z14$, Line 2-7 = TB27 = $|(V2-V7)|^2/Z27$, Line 3-9 = TB39 = $|(V3-V9)|^2/Z39$, Line 5-7 = TB57 = $|(V7-V5)|^2 / Z57$, Line 4-5 = TB45 = $|(V4-V5)|^2 / Z45$, Line 4-6 = TB46 = $|(V4-V6)|^2 / Z46$, Line 6-9 = TB69 = $|(V9-V6)|^2 / Z69$, Line 8-9 = TB89 = $|(V9-V8)|^2 / Z89$, Line7-8 = TB78 = $|(V7-V8)|^2 / Z78$, Total Line/Branch Loss =
9. TBL = TB14+TB27+TB39+TB57+TB45+TB46+TB69+TB89+TB78 = **0.0495 - j0.5166**
10. **Effect of Susceptances (EB):**
11. Bus 4 = EB4 = (B44) × $|V4|^2$, Bus 5 = EB5 = (B55) × $|V5|^2$, Bus 6 = EB6 = (B66) × $|V6|^2$, Bus 7 = EB7 = (B77) × $|V7|^2$, Bus 8 = EB8 = (B88) × $|V8|^2$, Bus 9 = EB9 = (B99) × $|V9|^2$
12. Total = EB4+EB5+EB6+EB7+EB8+EB9 = j0.7633
13. **Verifying Power Balance:** Total Generation **TG** = S1 + S2 + S3 = 3.1995-j0.9032:
14. Total Load **TL** = L5 + L6 + L8 = 3.15 - j1.15: Total Loss **TO = TG - TL** = 0.0495 + j0.2468 (Effect of B included), Actual total loss TBL = TO - EB = 0.0495 + j0.2468 - j0.7633 = **0.0495 - j0.5166**

**B: Applying Superposition Principle:**

$$TOL = 10^{-5}$$

Node 4 $$\frac{V4 - V6}{Z46} + \frac{V4 - V5}{Z45} + B44 \cdot V4 \approx 0.6918 - j0.4812$$

Node 5 $$\frac{V5 - V4}{Z45} + \frac{V5 - V7}{Z57} + \frac{V5}{ZL5} + B55 \cdot V5 \approx 0$$

Node 6 $$\frac{V6 - V9}{Z69} + \frac{V6 - V4}{Z46} + \frac{V6}{ZL6} + B66 \cdot V6 \approx 0$$

Node 7 $$\frac{V7 - V8}{Z78} + \frac{V7 - V5}{Z57} + B77 \cdot V7 \approx 1.6132 - j0.0056$$

Node 8 $$\frac{V8 - V9}{Z89} + \frac{V8 - V7}{Z78} + \frac{V8}{ZL8} + B88 \cdot V8 \approx 0$$

Node 9 $$\frac{V9 - V8}{Z89} + \frac{V9 - V6}{Z69} + B99 \cdot V9 \approx 0.8365 - j0.0521$$

**Figure 3: Equations for Superposition Principle: Mathcad Solve Block**

In the load bus equations (5, 6, 8) in Figure 2, the load currents are replaced as given below in Figure 3: IL5 = V5/ZL5, IL6 = V6/ZL6, IL8 = V8/ZL8. Nodes 1, 2, and 3, along with their constraint equations, are removed in Figure 3, because the values of I1, I2, and I3 were already determined by including these constraints in the solve block of Figure 2.

1. **Input:** Generator Currents: I1, I2, and I3
2. **Variables to be found:** V4, V5, V6, V7, V8, and V9: The solution of voltages V4 to V9 resulted in
3. **Calculation of Line/Branch Currents:**

**Figure 2 are identical to the solution of the equations in Figure 3**

L1-4 = (V1-V4) / Z14    L2-7 = (V2-V7) / Z27    L3-9 = (V3-V9) / Z39    L5-7 = (V7-V5) / Z57

L4-5 = (V4-V5) / Z45    L4-6 = (V4-V6) / Z46    L6-9 = (V9-V6) / Z69    L6-9 = (V9-V6) / Z69

L7-8 = (V7-V8) / Z78    IL5 = V5 / ZL5    IL6 = V6 / ZL6    IL8 = V8 / ZL8

| | | |
|---|---|---|
| L1-4 = 0.6918 - j0.4812 | L2-7 = 1.6132 - j0.0056 | L3-9 = 0.8365 - j0.0521 |
| L5-7 = 0.8620 - j0.0705 | L4-5 = 0.3889 - j0.4315 | L4-6 = 0.2963 - j0.2187 |
| L6-9 = 0.5985 - j0.0465 | L8-9 = 0.2425 - j0.1302 | L7-8 = 0.7573 - j0.0258 |
| IL5 = 1.2438 - j0.6024 | IL6 = 0.8886 - j0.3609 | IL8 = 1.0022 - j0.3355 |

**Figure 4: Line, Load Currents: Source Currents I1, I2, I3 Present**

**Source I1 Only**    **Source I2 only**    **Source I3 only**    **All Sources Present**

| I1 = 0.6918-j0.4812<br>I2 = 0.0; I3 = 0.0 | I2 = 1.6132-j0.0056<br>I1 = 0.0; I3 = 0.0 | I3 = 0.8365-j0.0521<br>I1 = 0.0; I2 = 0.0 | I1, I2, I3 Present<br>Total Value |
|---|---|---|---|
| L1-4=0.6918-j4812 | 0.0-j0.0 | 0.0-j0.0 | L1-4=0.6918-j4812 |
| 0.0-j0.0 | L2-7=1.6132-j0.0056 | 0.0-j0.0 | L2-7=1.6132-j0.0056 |
| 0.0-j0.0 | 0.0-j0.0 | L3-9=0.8365-j0.0521 | L3-9=0.8365-j0.0521 |
| L5-7=-0.1073+j0.0614 | L5-7=0.7804-j0.0701 | L5-7=0.1890-j0.0618 | L5-7=0.8621-j0.0705 |
| L4-5=0.3687-j0.3111 | L4-5=-0.1208-j0.0715 | L4-6=0.1410-j0.0489 | L4-5=0.3889-j0.4315 |
| L4-6=0.3088-j0.2144 | L4-6=0.1271-j0.0103 | L4-6=-0.1396+j0.0060 | L4-6=0.2963-j0.2187 |
| L6-9=-0.1110+j0.0624 | L6-9=0.3263-j0.0678 | L6-9=0.3832-j0.0411 | L6-9=0.5985-j0.0465 |
| L8-9=0.0954-j0.0900 | L8-9=-0.3166+j0.0053 | L8-9=0.4637-j0.0455 | L8-9=0.2425-j0.1302 |
| L7-8=0.0957-j0.0813 | L7-8=0.8483+j0.0177 | L7-8=-0.1867+J0.0378 | L7-8=0.7573-j0.0258 |
| IL5=0.2496-j0.2740 | IL5=0.6646-j0.1917 | IL5=0.3296-j0.1367 | IL5=1.2438-j0.6024 |
| IL6=0.1872-j0.1753 | IL6=0.4558-j0.1253 | IL6=0.2456-j0.0603 | IL6=0.8886-j0.3609 |
| IL8=0.1669-j0.2098 | IL8=0.5515-j0.0693 | IL8=0.2839-j0.0564 | IL8=1.0022-j0.3355 |

**Figure 5: Superposition Principle**
**Line/Branch and Load Currents at Each Source and Their Total Values**

The line/branch and load currents calculated using the voltages in Figure 4 are identical to those obtained using the superposition principle in Figure 5

**C. PDT Calculation for Power Sharing using Superposition Principle Data:**

Isource1 × $\overline{I_{total}}$ × Z Isource2 × $\overline{I_{total}}$ × Z Isource3 × $\overline{I_{total}}$ × Z $(Itotal)^2$ × Z

| Power from Source 1 | Power from Source 2 | Power from Source 3 | Total Power Loss in the Line |
|---|---|---|---|
| $(\|I1\|)^2 \times Z14$ S14 = j0.0409 | 0.0+j0.0 | 0.0+j0.0 | S14 = j0.0409 |
| 0.0+j0.0 | $(\|I2\|)^2 \times Z27$ S27 = j0.1627 | 0.0+j0.0 | S27 = j0.1627 |
| 0.0+j0.0 | 0.0+j0.0 | $(\|I3\|)^2 \times Z39$ S39 = j0.0412 | S39 = j0.0412 |
| S571 -0.0104-j0.0141 | S572 0.0226+j0.1089 | S573 0.0118+j0.0257 | S57 0.0240+j0.1205 |
| S451 -0.0005+j0.0240 | S452 0.0066-j0,0022 | S453 -0.0028+j0.0069 | S45 0.0033+j0.0287 |
| S461 0.0020+j0.0128 | S462 -0.0016+j0.0041 | S463 0.0019-j0.0044 | S46 0.0023+j0.0125 |
| S691 -0.0082-j0.0105 | S692 0.0121+j0.0327 | S693 0.0102+j0.0390 | S69 0.0141+j0.0612 |
| S891 0.0014+j0.0034 | S892 0.0031-j0.0083 | S893 -0.0036+j0.0125 | S89 0.0009+j0.0076 |
| S781 0.0049+j0.0049 | S782 0.0029+j0.0465 | S783 -0.0029-j0.0100 | S78 0.0049+j0.0414 |
| SL51 0.3610-j0.0001 | SL52 0.5741+j0.3526 | SL53 0.3147+j0.1475 | SL5 1.2498+j0.5 |
| SL61 0.2533-j0.0115 | SL62 0.4230+j0.1987 | SL63 0.2234+j0.1126 | SL6 0.8997+j0.2998 |
| SL81 0.2610-j0.0636 | SL82 0.4794+j0.2839 | SL83 0.2595+j0.1297 | SL8 1.0000+j0.3500 |

**Figure 6: Line loss and load power allocation determined by the PDT formula using the superposition data from Figure 5.**

Model Calculation: Line5-7: S57 = S571 + S572 + S573

S571 = L5-7 (Source1) × $\overline{I_{total}}$ × Z5-7 = (-0.1073+j0.0614) × (0.8620+j0.0705) × (0.0320+j1610)

S572 = L5-7 (Source2) × $\overline{I_{total}}$ × Z5-7 = (0.7804-j0.0701) × (0.8620+j0.0705) × (0.0320+j1610)

S573 = L5-7 (Source3) × $\overline{I_{total}}$ × Z5-7 = (0.1890-j0.0618) × (0.8620+j0.0705) × (0.0320+j1610)

Similarly Each Source Power Sharing on all the Lines and Loads are calculated and given in Figure 6.

Note that reactive powers have positive values for line impedances and loads. This is because the calculation of complex power loss is given by $S = I^2 \times Z = V \times I^* = P + JQ$..However, if $V^2 / Z$ is used, it results in P - jQ when the current `I ‘ through the impedance is lagging the voltage ` V ‘ across it. That is why the total line loss calculated in Section A, Step 7 was initially written as 0.0495 - j0.5166, whereas summing each individual line loss in the last column of Figure 6 yields the correct total line loss of 0.0495 + j0.5166

**D. Direct calculation of Load Sharing using PDT formula :**

Sharing of $j^{th}$ load by $i^{th}$ th Source = $j^{th}$ Load Power × $i^{th}$ Source current/Total of Source Currents

| | Load 5<br>1.25 - j0.50 | Load 6<br>0.90 - j0.30 | Load 8<br>1.00 - j0.35 |
|---|---|---|---|
| Source 1 | 0.2433 - j0.2598 | 0.1819 - j0.1727 | 0.2002 - j0.1959 |
| Source 2 | 0.6658 - j0.1448 | 0.4744 - j0.0743 | 0.5285 - j0.0909 |
| Source 3 | 0.3409 - j0.0954 | 0.2438 - j0.0530 | 0.2713 - j0.0632 |

**Figure 7: Source Sharing of Loads by using PDT Formula**

Model Calculation: Load 5= 1.25 – j 0.50

Source 1 to Load 5 = (1.25 – j0.50) × (0.6918 – j0.4812) / (3.1346 – j0.2988) = 0.2433 – j0.2598

Source 2 to Load 5 = (1.25 – j0.50) × (1.6132 – j0.0056) / (3.1346 – j0.2988) = 0.6658 – j0.1448

Source 3 to Load 5 = (1.25 – j0.50) × (0.8365 – j0.0521) / (3.1346 – j0.2988) = 0.3409 – j0.0954

**E. Complex Power comparision of PDT and PDT-Superposition Theorem Frame Work:**

| | Load 5<br>1.2498 - j0.5000 | Load 6<br>0.8999 - j0.2999 | Load 8<br>1.0000 - j0.3500 |
|---|---|---|---|
| Source 1 | 0.3610 + j0.0001 | 0.2533 + j0.0115 | 0.2610 + j0.0636 |
| Source 2 | 0.5741 - j0.3526 | 0.4232 - j0.1988 | 0.4794 - j0.2839 |
| Source 3 | 0.3147 - j0.1475 | 0.2234 - j0.1126 | 0.2595 - j0.1297 |

**Figure 8: Source Sharing of Loads using PDT-Superposition Framework**

Note that the load sharing allocation yields distinct results between the two methodologies**.** A comparison between Figure 7 (PDT Formula) and Figure 8 (PDT-Superposition Framework) reveals shifting shares across all three sources for Loads 5, 6, and 8, particularly regarding the sign and magnitude of the reactive power components.

**F. Apparent Power comparision of PDT and PDT-Superposition Theorem Frame Work:**

| | Load 5<br>1.3463 | Load 6<br>0.9487 | Load 8<br>1.0595 |
|---|---|---|---|
| Source 1 | 0.3559 | 0.2508 | 0.2801 |
| Source 2 | 0.6814 | 0.4802 | 0.5362 |
| Source 3 | 0.3540 | 0.2495 | 0.2786 |

**Figure 9: Source Sharing of Load Apparent Power Magnitude using PDT**

| | Load 5<br>1.3463 | Load 6<br>0.9487 | Load 8<br>1.0595 |
|---|---|---|---|
| Source 1 | 0.3610 | 0.2536 | 0.2687 |
| Source 2 | 0.6738 | 0.4676 | 0.5572 |
| Source 3 | 0.3476 | 0.2581 | 0.2981 |

**Figure 10: Source Sharing of Load Apparent Power Magnitude using PDT-Superposition Frame work**

| | Load 5<br>1.3463 | Load 6<br>0.9487 | Load 8<br>1.0595 |
|---|---|---|---|
| Source 1 | 1.41% | 1.1% | -4.26% |
| Source 2 | -1.13% | -2.69% | 3.76% |
| Source 3 | -1.05% | 0.27% | 3.98% |

**Figure 11: Difference of Apparent Power Magnitude by PDT with PDT-Superposition Frame work**

The results indicate that the percentage difference in apparent power magnitude sharing remains strictly under 5% when evaluated with **respect to the** PDT-Superposition Framework (Figure 10)**.** Given this narrow margin, the PDT-Superposition Framework serves as the only definitive approach when high-precision complex power sharing is mandatory.

However, if a 5% error margin relative to this baseline is acceptable for the apparent power magnitude, the standard PDT method becomes highly preferred; it completely bypasses the superposition principle, thereby significantly minimizing computational complexity and execution time.

## 3. Discussions and Conclusions:

This paper successfully introduces a novel, direct, and non-iterative mathematical framework combining the Power Division Theorem (PDT) and the Superposition Theorem to solve branch power loss and load sharing allocations in deregulated power markets. Tested on the standard IEEE 9-bus system, the proposed framework completely bypasses the heavy computational iterations and structural tracking complexities inherent to conventional flow-tracing methodologies like Bialek's or Kirschen's methods. By linearizing non-linear power dependencies into a structural current framework within a Mathcad environment, the method guarantees exactly auditable, transparent, and back-solvable allocation profiles.

Comparative analysis reveals key operating insights based on the required engineering precision:

1. **For High-Precision Complex Power Sharing:** The **PDT-Superposition Framework** serves as the definitive analytical benchmark. It isolates shifting shares and precisely captures both the sign and magnitude of individual generator contributions to reactive power components.
2. **For Accelerated Computational Execution:** The **standard PDT method** provides an excellent alternative when an apparent power magnitude error margin of under 5% is acceptable. By bypassing the superposition principle entirely, it drastically minimizes processing times and system overhead.

Ultimately, this dual-framework approach empowers grid operators to easily balance computational speed against mathematical precision. It provides a fair, transparent, and universally verifiable mechanism for transmission cost and loss allocation in modern, active distribution networks.

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