

A Graph-Theoretic and Computational Framework for Robust, Energy-Efficient Distributed Localization in Wireless Sensor Networks

Ashwini Pravin Bhong¹, Dr. Vishwajeet S. Goswami²

¹Research Scholar, Department of Mathematics, Shri JYT University, Rajasthan

²Associate Professor, School of Engineering, Ajeenkya D Y Patil University, Pune

Abstract

Wireless sensor network (WSN) localization is a graph-structured estimation problem in which sensors must infer physical coordinates from inter-node measurements and a limited set of reference anchors. The attached reference manuscript establishes a vertex-centric framework in which a WSN is modeled as a weighted, time-varying graph and localization is performed through local numerical updates, graph-Laplacian consensus, rigidity information, and energy-aware communication. Building on that foundation, this paper develops a focused research framework around three objectives: (1) applying graph-theoretic properties such as connectivity, adjacency, and centrality for efficient network modeling and localization; (2) designing and validating algorithms that remain robust and adaptable under changing network conditions and topologies; and (3) evaluating the proposed approach through simulation and performance analysis against established localization strategies. The paper formulates a weighted graph model, derives a distributed nonlinear least-squares objective, introduces energy- and reliability-aware edge weights, and combines local gradient updates with consensus regularization. Connectivity is characterized using the graph Laplacian and algebraic connectivity, while rigidity and redundancy are used to assess whether geometric constraints are sufficient for unique localization. A robust Huber loss is incorporated to limit the influence of outliers. The resulting Energy-Aware Vertex-Centric Localization (EAVCL) framework uses one-hop state exchange, adaptive communication, event-triggered reactivation, and local topology updates. A synthetic evaluation protocol is specified for networks of increasing size, anchor density, communication range, noise, and packet loss. The numerical examples included in this paper are analytical illustrations rather than empirical benchmark claims; they define a reproducible methodology for future MATLAB/Python, NS-3, Cooja, or OMNeT++ validation.

Keywords: wireless sensor networks, graph theory, distributed localization, vertex-centric computing, connectivity, centrality, rigidity, graph Laplacian, consensus, energy efficiency, robustness, dynamic topology

1. Research Objectives and Contributions

The objectives supplied with the study are reformulated into a coherent graph-theoretic and computational research program. The reference manuscript similarly identifies energy-efficient distributed computation

and improved localization accuracy/scalability as central goals. (Bhong & Goswami, 2026)

1. To apply graph-theoretic properties such as connectivity, adjacency, degree, centrality, algebraic connectivity, and rigidity to construct efficient WSN models and to guide localization.
2. To design and validate a robust, adaptive, vertex-centric localization algorithm capable of operating under varying network density, anchor availability, measurement noise, packet loss, node failure, and topology changes.
3. To evaluate the proposed method through reproducible simulation and performance analysis using localization error, energy consumption, communication overhead, convergence iterations, coverage, and scalability, with comparisons to representative centralized and distributed localization techniques.

2. Introduction

A WSN consists of spatially distributed sensor nodes that sense, process, and communicate information. When the sensed phenomenon is geographically meaningful, the coordinates of sensor nodes become an essential part of the network state. Localization supports environmental monitoring, target tracking, event detection, routing, and other location-dependent services. A broad survey by Cheng et al. (2012) emphasizes that localization methods must balance accuracy with energy, scheduling, node selection, cooperation, and heterogeneous operating conditions. (Cheng et al., 2012)

A useful mathematical representation is a graph $G = (V, E)$, where V is the set of sensor vertices and E represents feasible wireless communication links. Inter-node distance measurements can be treated as edge attributes. This representation makes adjacency, connectivity, centrality, Laplacian spectra, and rigidity directly relevant to localization. The attached reference paper adopts the same interpretation and notes that sensors are vertices, wireless links are edges, measured distances are edge weights, and anchors provide coordinate references. (Bhong & Goswami, 2026)

Centralized localization aggregates measurements at a fusion point and solves a global optimization problem. Although this can be computationally convenient at the server, repeated transmission of measurements can be expensive for battery-powered nodes.

The present paper extends that direction by explicitly connecting three graph-theoretic questions with the computational process: (a) which nodes should communicate, (b) whether the resulting geometric constraints are sufficient for localization, and (c) how information should propagate through the network. The first is addressed by adjacency, degree, centrality, and energy-aware edge selection; the second by rigidity and redundancy; and the third by Laplacian consensus and spectral properties.

Recent distributed localization research reinforces the importance of graph structure. Bose et al. (2022), for example, formulate WSN localization using unit-disk graphs and develop a distributed scheme based on communication wheels and interior-node structure. Their work also highlights that trilateration performance can depend strongly on the initial anchors or localized subgraph. (Bose et al., 2022)

3. Related Work and Research Gap

3.1 Localization in WSNs

Localization approaches can broadly be range-based or range-free. Range-based techniques use distance, time, angle, or related measurements, while range-free techniques exploit connectivity or other geometric information. For the present study, range-based localization is particularly appropriate because the objective is to combine edge weights with graph structure. Cheng et al. (2012) identify localization in

energy-constrained networks, cooperative localization, scheduling, and heterogeneous networks as continuing challenges. (Cheng et al., 2012)

Mao et al. (2007) provide a foundational survey of WSN localization and discuss one-hop, multi-hop, connectivity-based, and distance-based approaches. The reference manuscript uses this literature to motivate a graph-aware distributed formulation. (Bhong & Goswami, 2026)

3.2 Rigidity and Unique Localizability

Connectivity alone does not guarantee unique coordinates. A graph may be connected while still permitting multiple geometric realizations consistent with the measured edge lengths. Rigidity theory provides a natural framework for distinguishing graphs whose geometric realization is constrained from those that can flex. Eren (2016) introduced rigidity and redundancy indices for cooperative localization, with the purpose of quantifying how close a graph is to rigidity and how much redundant geometric structure it contains. (Eren, 2016)

The attached manuscript similarly stresses that global rigidity, anchor placement, and geometric constraints determine whether localization is uniquely identifiable. (Bhong & Goswami, 2026)

3.3 Distributed and Vertex-Centric Localization

Distributed localization reduces the need for global state collection. Bose et al. (2022) demonstrate a distributed localization approach based on local graph structure and prove guarantees for a class of nodes under their communication-wheel construction. (Bose et al., 2022) The present work differs in emphasis: rather than relying on a single structural construction, it combines local nonlinear optimization, consensus, energy-aware edge weighting, and adaptive communication.

3.4 Research Gap

The gap addressed here is not the absence of localization algorithms; rather, it is the need for a unified framework in which graph-theoretic structure actively controls numerical localization and communication. The desired framework should preserve accuracy while avoiding unnecessary communication, remain stable when local neighborhoods are poorly conditioned, and react locally to network changes. The attached manuscript identifies this same integration as its main methodological contribution. (Bhong & Goswami, 2026)

4. Graph-Theoretic Model of the WSN

Let the sensor network at time t be represented by a weighted graph

$$G_t = (V, E_t, W_t),$$

Where $V = \{1, 2, \dots, n\}$ is the node set, E_t is the time-dependent communication edge set, and W_t contains edge weights. Let $p_i \in \mathbb{R}^2$ denote the unknown coordinate of node i and let $A \subset V$ denote the anchor set, with known coordinates p_a for $a \in A$.

For communication radius R , a basic unit-disk rule is

$$a_{ij}(t) = 1 \text{ if } \|p_i(t) - p_j(t)\| \leq R; \quad a_{ij}(t) = 0 \text{ Otherwise.} \quad (1)$$

The binary adjacency matrix $A(t) = [a_{ij}(t)]$ identifies the one-hop neighborhood

$N_i(t) = \{j: a_{ij}(t) = 1\}$. The degree is $d_i(t) = \sum_j a_{ij}(t)$. These quantities directly determine local communication and computational load.

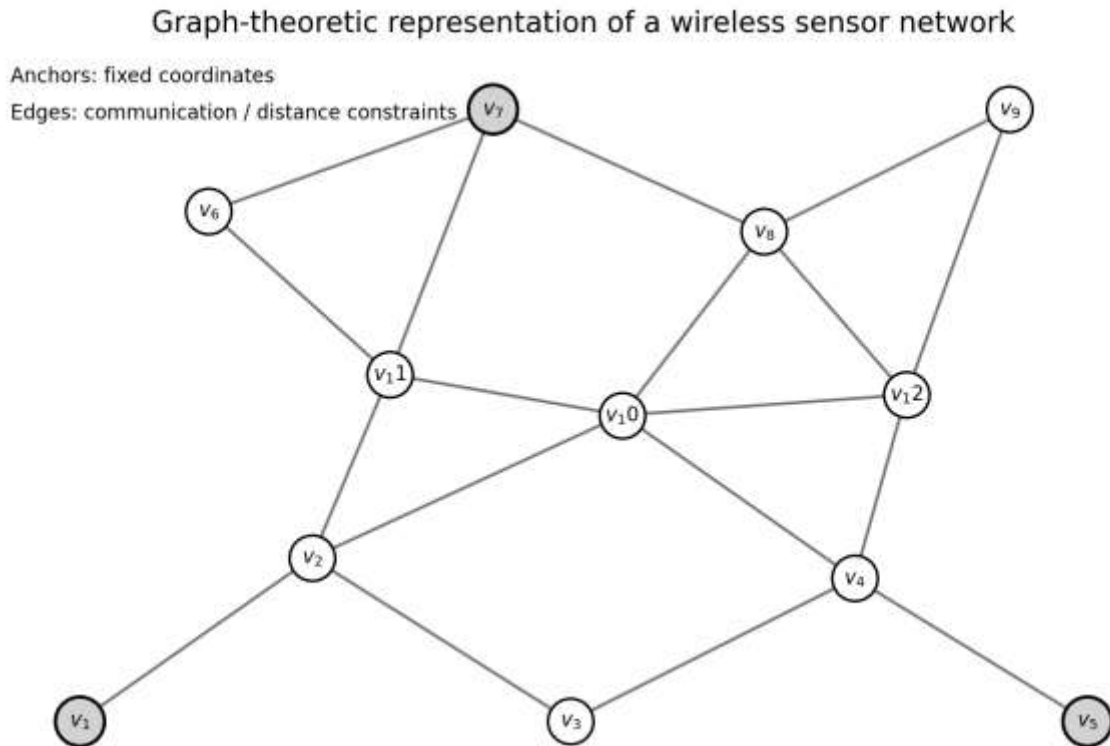


Figure 1. Graph-theoretic representation of a WSN with ordinary sensor vertices and anchor vertices.

The attached manuscript uses the same unit-disk abstraction and emphasizes that packet loss, interference, obstacles, and mobility can make the graph time varying. (Bhong & Goswami, 2026)

4.1 Connectivity and Laplacian

The degree matrix is $D = \text{diag}(d_1, \dots, d_n)$, and the unweighted graph Laplacian is

$$L = D - A. \quad (2)$$

For a connected undirected graph, L has one zero eigenvalue. The second-smallest eigenvalue $\lambda_2(L)$, called algebraic connectivity, quantifies the strength of global information coupling. Larger λ_2 generally indicates faster mixing of consensus information. The reference paper explicitly identifies algebraic connectivity as a useful indicator of network connectedness. (Bhong & Goswami, 2026)

4.2 Degree, Centrality, and Node Selection

Degree is a direct measure of neighborhood size, but it is not sufficient for selecting important localization nodes. Betweenness centrality can identify vertices that lie on many shortest paths, while closeness centrality reflects the distance of a node to the rest of the graph. For localization, these measures are used as heuristic indicators rather than as substitutes for geometric identifiability.

$$CB(i) = \sum_{s \neq i \neq t} \sigma_{st}(i) / \sigma_{st}, \quad (3)$$

where σ_{st} is the number of shortest paths from s to t and $\sigma_{st}(i)$ is the number passing through i . A high-centrality node can be useful for maintaining information flow, but selecting it must also consider residual energy and geometric contribution.

4.3 Energy- and Reliability-Aware Edge Weight

Let $e_i \in (0,1]$ denote normalized residual energy, $r_{ij} \in (0,1]$ denote measurement reliability, and d_{ij} the measured distance. An illustrative weight is

$$w_{ij} = r_{ij} \cdot ((e_i + e_j)/2) \cdot \exp(-\beta d_{ij}), \quad \beta > 0. \quad (4)$$

This weighting mechanism gives reliable, energy-rich, shorter links greater influence. The exact function can be replaced by a hardware-calibrated radio or measurement model.

5. Mathematical Formulation of Distributed Localization

The true position of an unknown node is unavailable, so each node maintains an estimate $\hat{p}_i$. For an edge (i, j) , define the distance residual

$$r_{ij}(\hat{p}) = \|\hat{p}_i - \hat{p}_j\| - d_{ij}. \quad (5)$$

A weighted nonlinear least-squares objective over all active edges is

$$JD(\hat{p}) = 1/2 \sum_{(i, j) \in E_t} w_{ij} [\|\hat{p}_i - \hat{p}_j\| - d_{ij}]^2. \quad (6)$$

Anchor information is incorporated through a penalty term

$$JA(\hat{p}) = 1/2 \sum_{(i, a) \in EA} \eta_{ia} \|\hat{p}_i - p_a\|^2, \quad (7)$$

where η_{ia} is an anchor confidence weight. A consensus regularizer can then be added to stabilize local estimates:

$$JC(\hat{p}) = (\mu/2) \sum_{(i, j) \in E_t} w^{cij} \|\hat{p}_i - \hat{p}_j\|^2. \quad (8)$$

The combined objective is

$$J(\hat{p}) = JD(\hat{p}) + JA(\hat{p}) + JC(\hat{p}). \quad (9)$$

This formulation preserves the distance-fitting objective while allowing graph-based information sharing. The reference manuscript proposes the same basic combination of local nonlinear least squares and consensus regularization. (Bhong & Goswami, 2026)

5.1 Local Gradient Update

For a nonzero estimated distance $q_{ij} = \|\hat{p}_i - \hat{p}_j\|$, the gradient of the squared distance residual with respect to $\hat{p}_i$ is

$$\nabla_i (1/2 r_{ij}^2) = r_{ij} \cdot (\hat{p}_i - \hat{p}_j)/q_{ij}. \quad (10)$$

Thus node i can calculate a local gradient using only its one-hop neighbor states. Including the anchor and consensus terms gives a generic update

$$\hat{p}_i^{(k+1)} = \hat{p}_i^{(k)} - \alpha_k [g_i^D + g_i^A + \mu g_i^C], \quad (11)$$

where α_k is a step size. This avoids construction of a global Jacobian and is consistent with the vertex-centric philosophy of the reference manuscript. (Bhong & Goswami, 2026)

5.2 Robust Huber Loss

Distance measurements can contain outliers because of multipath, non-line-of-sight propagation, interference, or faulty ranging. A robust alternative replaces the quadratic loss with a Huber function:

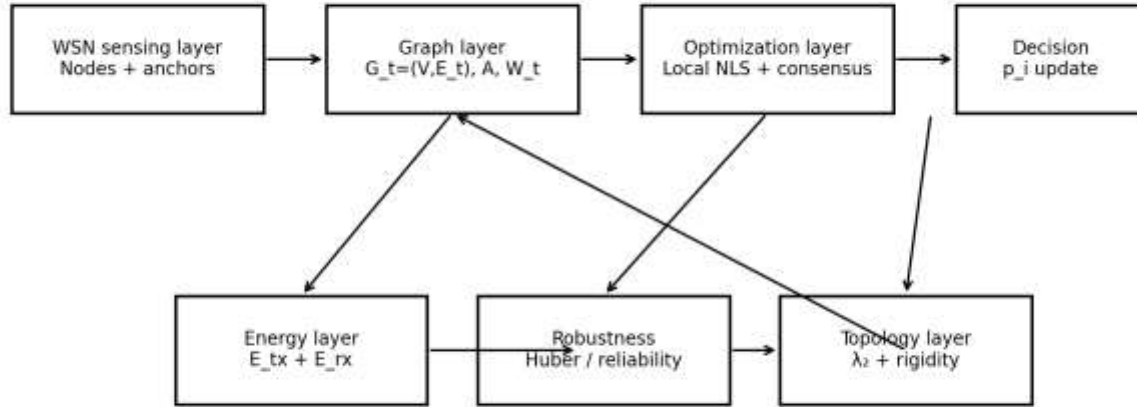
$$\rho_\delta(r) = \begin{cases} 1/2 r^2, & |r| \leq \delta; \\ \delta(|r| - 1/2\delta), & |r| > \delta. \end{cases} \quad (12)$$

The robust objective becomes $JR = \sum w_{ij} \rho_\delta(r_{ij})$. This limits the influence of large residuals while retaining a quadratic response near the optimum. The reference manuscript recommends robust M-estimation as a future extension; the present paper incorporates it into the proposed research design. (Bhong & Goswami, 2026)

6. Proposed Energy-Aware Vertex-Centric Localization Framework

The proposed framework, termed EAVCL, is organized into six interacting layers: graph construction, neighborhood discovery, energy/reliability assessment, local numerical optimization, convergence monitoring, and adaptive communication. The architecture is shown in Figure 2.

Proposed graph-theoretic and computational localization framework



Vertex-centric feedback loop: only local states and selected neighbor information are exchanged

Figure 2. Proposed graph-theoretic and computational architecture for distributed localization.

6.1 Node State

Each node stores a compact state

$$s_i = (\hat{p}_i, E_i, c_i, a_i), \quad (13)$$

where $\hat{p}_i$ is the current coordinate estimate, E_i is residual energy, c_i is local confidence, and a_i is an activity/convergence indicator. This avoids storing the complete network distance matrix.

6.2 Adaptive Communication

A node that has reached a stable estimate should not continue transmitting its full state at every iteration. Define a local activity indicator

$$\chi_i(k) = 1 \text{ if } \|\hat{p}_i^k - \hat{p}_i^{(k-1)}\| > \varepsilon_p \text{ or } |r_i^k - r_i^{(k-1)}| > \varepsilon_r; 0 \text{ otherwise.} \quad (14)$$

Active nodes exchange full state vectors. Inactive nodes may send occasional heartbeat messages. The reference manuscript proposes the same principle of reducing communication after local convergence. (Bhong & Goswami, 2026)

6.3 Energy-Aware Edge Selection

Let $S_i(k) \subseteq N_i(k)$ be the selected communication neighborhood. The selection problem can be written as a local constrained optimization:

$$\min \{S_i \subseteq N_i \mid \sum_{j \in S_i} C_{ij} \text{ subject to } \lambda_2(L_i(S_i)) \geq \lambda_{\min}. \quad (15)$$

Here C_{ij} is a local communication cost and $\lambda_{\min}$ is a minimum desired connectivity threshold. In practice, a greedy rule can rank candidate neighbors by a combined score involving reliability, residual energy, distance, centrality, and geometric contribution.

6.4 Dynamic Topology Handling

Let $\Delta N_i(k) = N_i(k) \Delta N_i(k-1)$ be the symmetric difference between consecutive neighborhoods. A node need not restart localization after a small change. Instead, it uses the previous estimate as a warm start and recomputes local quantities only when the topology or residual changes beyond a threshold. The

reference paper specifically proposes warm starts and local topology updates for dynamic networks. (Bhong & Goswami, 2026)

7. Algorithmic Procedure

The EAVCL procedure is summarized below.

Energy-Aware Vertex-Centric Localization (EAVCL) workflow

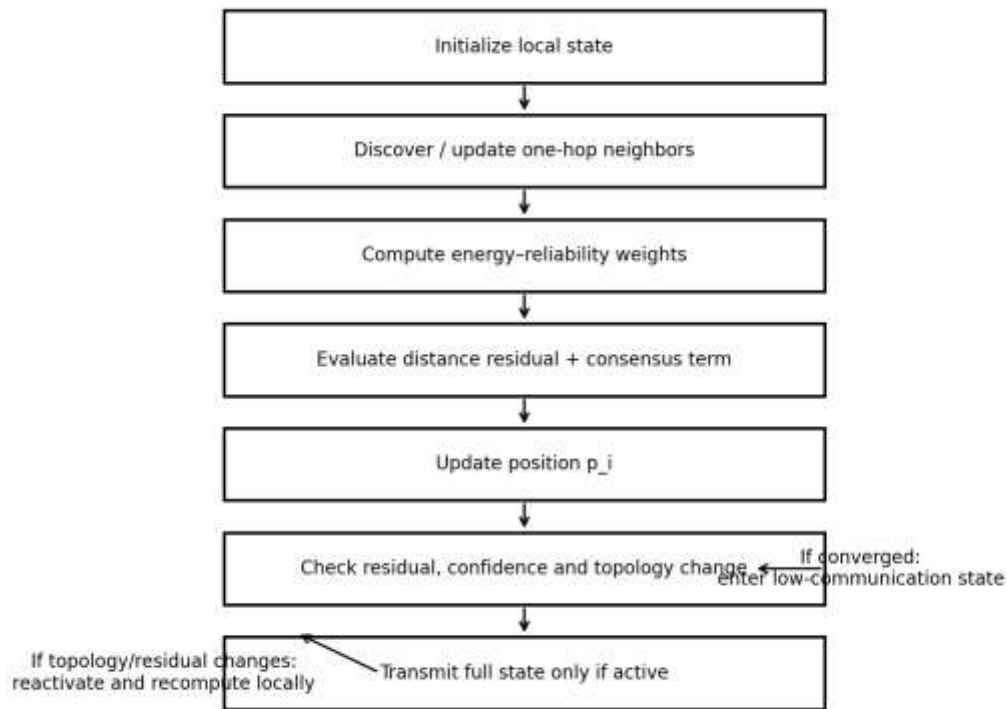


Figure 3. Local update and adaptive communication cycle of EAVCL.

Algorithm 1. EAVCL

1. Input: graph G_t , neighbor sets $N_i(t)$, anchor coordinates, measured distances, residual energy, reliability estimates, and algorithmic tolerances.
2. Initialize $\hat{p}_i$ using an anchor-based estimate, centroid estimate, or previous position in a dynamic network.
3. Construct local energy/reliability weights w_{ij} and identify a selected neighborhood S_i .
4. Exchange only the compact state required by the local update.
5. Compute distance, anchor, consensus, and—when enabled—robust-loss gradients.
6. Update $\hat{p}_i$ using Equation (11).
7. Calculate local residual, confidence, and movement of the estimate.
8. If the node satisfies the convergence thresholds, reduce communication activity.
9. If a topology, energy, reliability, or residual change exceeds its trigger, reactivate the node.
10. Terminate when network-level error and activity criteria are satisfied or the maximum number of iterations is reached.

This procedure preserves the central idea of the reference manuscript while making the roles of graph connectivity, centrality, robustness, and adaptive topology explicit. The original EAVCL algorithm likewise uses selected-neighbor exchange, local gradients, local residuals, and low-communication states. (Bhong & Goswami, 2026)

8. Graph-Theoretic Analysis and Convergence

8.1 Consensus Component

Consider the linear consensus update $x(k+1) = (I - \gamma L)x(k)$. For a connected undirected graph with Laplacian eigenvalues $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_n$, convergence of disagreement requires

$$0 < \gamma < 2/\lambda_n. \quad (16)$$

The disagreement contraction factor is

$$\rho = \max\{k \geq 2\} |1 - \gamma \lambda_k|. \quad (17)$$

When $\rho < 1$, disagreement decays geometrically. The attached manuscript provides the same spectral convergence argument and identifies λ_2 as an indicator of information mixing. (Bhong & Goswami, 2026)

8.2 Rigidity and Redundancy

For localization, connectivity is necessary but not sufficient. In two-dimensional range-based localization, the rigidity matrix captures the infinitesimal geometric constraints induced by the edges. If the graph lacks sufficient rigidity, different configurations can satisfy the same local distances. Eren (2016) shows how rigidity and redundancy indices can be used to quantify graph structure relevant to unique localizability. (Eren, 2016)

Accordingly, EAVCL should not select edges solely according to shortest distance or largest degree. A useful communication subgraph should preserve connectivity and sufficient geometric diversity. This is particularly important in nearly collinear neighborhoods, where distance constraints may be poorly conditioned.

8.3 Localization Error Metrics

For M non-anchor nodes, root-mean-square localization error is

$$RMSE = \sqrt{(1/M) \sum_{i \notin A} \|\hat{p}_i - p_i^* \|^2}. \quad (18)$$

Additional metrics should include mean absolute error, 95th-percentile error, localization coverage, convergence iterations, packet count, total energy, and per-node energy variance. The reference manuscript explicitly recommends evaluating localization jointly through RMSE, energy, and communication rather than accuracy alone. (Bhong & Goswami, 2026)

8.4 Computational Complexity

Each local gradient uses only incident edges. If $\bar{d}$ is the average degree, one iteration has approximately $O(|E|) = O(n \bar{d})$ edge-local operations. For K iterations, the aggregate work is $O(K|E|)$. Memory per node is approximately $O(d_i)$, because node i stores only its local measurements and selected neighbor states. This degree-dependent scaling is a central advantage of the vertex-centric design. (Bhong & Goswami, 2026)

9. Energy and Communication Model

A first-order radio model can represent transmission energy for a k -bit packet over distance d as

$$E_{tx}(k, d) = E_{elec} k + E_{amp} k d^\alpha, \quad (19)$$

with reception energy

$$E_{rx}(k) = E_{elec} k. \quad (20)$$

The path-loss exponent α depends on the propagation environment. Heinzelman et al. (2000) established the importance of communication organization for reducing energy dissipation in wireless microsensor networks. The reference manuscript adopts a comparable communication-energy perspective. (Bhong & Goswami, 2026)

For edge (i, j) , the cumulative network communication cost can be represented by

$$E_{comm} = \sum_{t \in E} \sum_{(i,j) \in E} t [b_{ij}(t)(E_{elec} + E_{amp} d_{ij}^{\alpha})], \quad (21)$$

where $b_{ij}(t)$ is the number of transmitted bits. The central design trade-off is therefore to accept modest local computation in exchange for fewer and smaller network transmissions.

10. Simulation and Validation Methodology

Because the attached manuscript describes its numerical case as synthetic rather than a measured benchmark, the present paper treats simulation as a research protocol rather than reporting fabricated experimental results. The proposed experiments should be implemented in MATLAB or Python, with optional network-level validation in NS-3, Cooja, or OMNeT++. The reference manuscript itself recommends these environments and proposes varying network size, anchor percentage, communication radius, noise, and packet loss. (Bhong & Goswami, 2026)

10.1 Network Generation

- Network sizes: $n \in \{100, 250, 500, 1000\}$.
- Deployment: uniform random positions in a square region, with optional clustered deployments.
- Communication graph: unit-disk or probabilistic link model.
- Anchor ratios: 5%, 10%, 15%, and 20%.
- Measurement noise: Gaussian range noise with controlled standard deviation, plus an outlier condition.
- Packet loss: 0%, 5%, 10%, and 20%.
- Dynamic topology: controlled node mobility, random node failure, and time-varying link reliability.

10.2 Comparison Algorithms

The evaluation should compare EAVCL against representative baselines. The precise implementation must be fixed before experimentation so that all algorithms receive the same node locations, anchor set, measurements, and random seeds.

Method	Core principle	Communication profile	Expected strength
Centralized NLS	Global nonlinear least squares	High raw-data movement	Strong global optimization
Trilateration	Sequential 3-anchor geometry	Local but propagation-sensitive	Simple and fast when geometry is favorable
Consensus-assisted localization	Laplacian information sharing	Iterative neighbor exchange	Good distributed information mixing
EAVCL	Weighted local NLS + consensus + adaptive communication	Selected one-hop exchange	Joint energy, robustness and topology awareness

10.3 Performance Metrics

- Localization accuracy: RMSE, mean error, median error, and 95th-percentile error.
- Energy: total network energy, mean per-node energy, and energy variance.

- Communication: packets, transmitted bits, and active-neighbor exchanges.
- Convergence: iterations to threshold and percentage of nodes converged.
- Scalability: runtime and memory growth as n increases.
- Robustness: performance degradation under noise, packet loss, and node failure.
- Coverage: fraction of non-anchor nodes whose error is below a specified tolerance.

10.4 Statistical Validation

For each configuration, at least 20 independent topology realizations should be generated, as suggested in the reference manuscript. Report mean, standard deviation, confidence intervals, and—where appropriate—paired statistical tests across identical realizations. Random seeds should be recorded to make the results reproducible. (Bhong & Goswami, 2026)

11. Analytical Synthetic Case Study

To illustrate the computational scale without claiming empirical performance, consider $n=100$ nodes, average degree $\bar{d}=6$, 10 anchors, and $K=20$ local iterations. The expected number of undirected edges is

$$|E| \approx n \bar{d} / 2 = (100 \times 6) / 2 = 300. \quad (22)$$

Thus, if every undirected edge is processed once per iteration, approximately 300 edge updates occur per iteration and 6,000 edge updates occur over 20 iterations. This reproduces the scale calculation in the attached reference manuscript. (Bhong & Goswami, 2026)

Parameter	Value	Interpretation
Nodes	100	Moderately large synthetic WSN
Average degree	6	Local neighborhood size
Undirected edges	300	$n \bar{d} / 2$
Anchors	10	Known coordinate references
Iterations	20	Local numerical iterations
Edge updates	6,000	300×20
Node state	(x,y,confidence)	Compact message representation

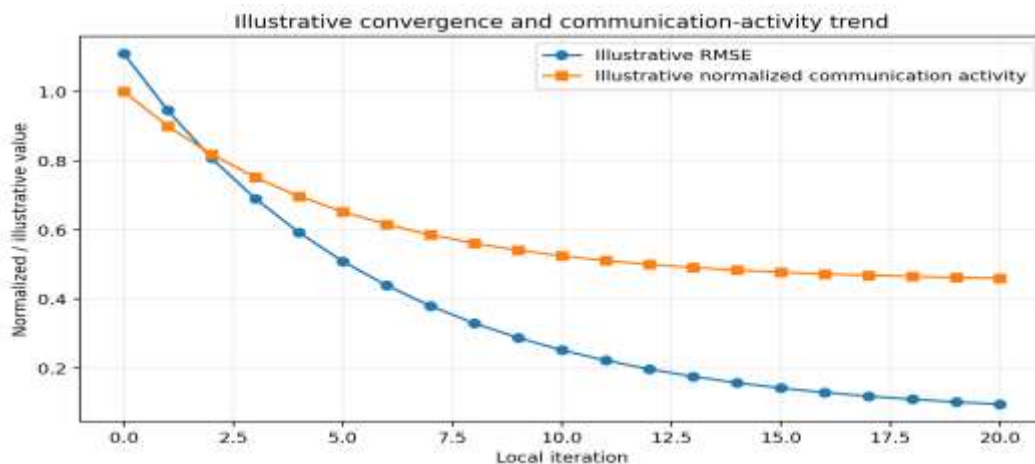


Figure 4. Illustrative convergence/communication trend. Values are synthetic and are included only to demonstrate how results can be visualized; they are not experimental measurements.

12. Expected Results and Hypotheses

The research should test the following hypotheses rather than assume that the proposed method is superior in every condition.

1. H1: Incorporating adjacency, degree, and connectivity into edge selection will reduce unnecessary communication while maintaining a connected localization subgraph.
2. H2: Combining distance residuals with consensus regularization will improve stability in poorly conditioned local neighborhoods.
3. H3: Energy- and reliability-aware edge weights will reduce the energy cost per successfully localized node compared with indiscriminate neighbor exchange.
4. H4: Robust loss functions will reduce degradation in RMSE under range-measurement outliers.
5. H5: Local topology updates and warm starts will reduce recovery cost after small node failures or mobility-induced changes compared with global recomputation.
6. H6: The advantage of the vertex-centric approach will become more pronounced as network size increases when the average degree remains bounded.

These are testable hypotheses. They should be accepted or rejected only after controlled simulation and statistical analysis.

13. Discussion

The proposed framework makes graph theory operational rather than purely descriptive. Adjacency determines the information available to a node; degree measures local computational and communication burden; centrality can support node selection; λ_2 measures connectivity strength; and rigidity assesses whether the available geometric constraints are sufficient for localization. This creates a direct pipeline from graph structure to numerical computation.

The principal benefit of vertex-centric localization is locality. A node does not require the full $n \times n$ distance matrix. Instead, it needs its own state, local measurements, and selected neighbor states. For sparse graphs, the aggregate work per iteration is therefore proportional to the number of edges rather than the square of the number of nodes. The reference manuscript identifies degree-dependent scalability as a major feature of the approach. (Bhong & Goswami, 2026)

The framework also recognizes a fundamental limitation: no algorithm can recover a unique configuration when the network does not contain sufficient geometric information. A connected graph can remain non-rigid, anchors can be poorly placed, and severe noise can make otherwise valid constraints unreliable. This is why connectivity and rigidity must be evaluated jointly.

Another limitation is that energy-aware edge selection creates a trade-off. Aggressive pruning can reduce communication but may remove geometrically important edges and lower rigidity. Consequently, the selection threshold should preserve a minimum level of connectivity and, where possible, redundancy. Eren (2016) provides a theoretical basis for viewing redundancy as a robustness resource in localization graphs. (Eren, 2016)

Finally, the proposed adaptive communication strategy is particularly relevant to dynamic networks. When a node has converged, continuing to transmit full state vectors is wasteful. Conversely, when a neighbor fails or the local residual increases, the node should reactivate. This event-triggered philosophy is consistent with the reference manuscript's recommendation for adaptive communication and future provable guarantees. (Bhong & Goswami, 2026)

14. Centralized Versus Distributed Graph-Theoretic Localization

Criterion	Centralized NLS	Conventional local methods	Proposed EAVCL
Data movement	High; measurements forwarded to fusion point	Mostly local	Selected one-hop state
Optimization	Global	Local/sequential	Local weighted NLS + consensus
Graph information	Often represented globally	May use local connectivity	Connectivity + centrality + rigidity + Laplacian
Energy awareness	Usually external to solver	Varies	Built into weights and communication
Dynamic topology	May require global update	Local in some methods	Local change detection + warm start
Outliers	Depends on objective	Often sensitive	Huber robustification
Scalability	Global matrix can become costly	Good local scaling	Degree-dependent local scaling

15. Conclusion

This paper has developed a graph-theoretic and computational framework for distributed WSN localization around three explicit research objectives: graph-based network modeling and localization, robust/adaptive algorithm design, and simulation-based comparative evaluation. The framework represents a WSN as a weighted time-varying graph, uses adjacency and neighborhood structure for local computation, incorporates centrality and energy-aware edge selection, uses Laplacian consensus for information diffusion, and employs rigidity concepts to assess geometric sufficiency.

The proposed EAVCL algorithm transforms localization from a single global optimization problem into a collection of coordinated vertex-level numerical problems. Its local objective combines distance fitting, anchor constraints, and consensus stabilization, while robust loss functions address measurement outliers. Adaptive communication reduces unnecessary state exchange after convergence, and topology triggers permit local recovery after mobility or link changes.

The work deliberately distinguishes theoretical formulation from empirical evidence. The numerical case study is an analytical illustration; actual claims of accuracy, energy reduction, or superiority should be made only after implementation of the proposed simulation protocol. This distinction is important for a rigorous research paper and provides a clear pathway to experimental validation.

The framework can be extended toward distributed Gauss–Newton or Levenberg–Marquardt methods, joint rigidity/connectivity edge selection, event-triggered convergence guarantees, three-dimensional localization, heterogeneous sensing, NLOS-aware models, and physical test-bed validation. These directions are consistent with the future-work agenda of the attached manuscript. (Bhong & Goswami, 2026)

16. Future Research Directions

- Develop a distributed Gauss–Newton/Levenberg–Marquardt solver using local Jacobian and Hessian

- approximations.
- Define a composite graph score combining algebraic connectivity, rigidity index, redundancy, centrality, residual energy, and measurement reliability.
- Develop event-triggered communication rules with formal bounds on communication reduction and localization error.
- Extend the framework to 3-D WSNs and heterogeneous range/angle/time measurements.
- Integrate NLOS detection and robust M-estimation with adaptive measurement weighting.
- Validate the method on MATLAB/Python simulations and then on NS-3, Cooja, OMNeT++, or a physical sensor platform.
- Study adversarial or faulty-node conditions and develop trust-aware graph pruning.

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