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Applying Markov Chains to the Monopoly Board Game: Identifying the Colour Set with the Highest Return on Investment

Kushagra Jain

Student, NMS, Jaipur, India

Abstract

Return on investment (ROI) in Monopoly depends on how often each square is visited, not only on property prices and rents. This study models a player's movement around the UK edition of the board as a 120-state Markov chain that tracks board position, consecutive doubles and turns spent in jail, together with the movement cards in the Chance and Community Chest decks. The steady-state distribution, found by solving the normalised linear system numerically, gives the long-run probability of landing on each square. These probabilities are combined with title-deed rents and with purchase and building costs to estimate the ROI of every colour set over 120 rolls at seven levels of development. Dark Blue yields the highest ROI up to the second house (0.629 with the full set owned and no houses), but Orange ranks first from the third house onwards and reaches the highest value recorded, 4.68, with hotels. Orange's advantage comes mainly from its high landing probability, which follows from its position shortly after the jail square. The third house gives the largest gain in rent per pound spent, although total ROI continues to rise with further building for most colour sets. The findings indicate that, once properties are developed, landing frequency rather than purchase price is the main driver of returns in Monopoly.

Keywords: Markov chains; steady-state distribution; transition matrix; stochastic processes; Monopoly; return on investment

1. Introduction

Monopoly, first released in 1935, has been played by more than a billion people around the world. Players travel around a 40-square board by rolling two dice, buying and developing properties and charging rent to opponents, with the aim of driving them into bankruptcy. Because rent is the main way players earn money from one another, the decision of which properties to buy and develop strongly influences who wins.

A widespread belief among players is that the most expensive properties produce the greatest returns, on the reasoning that a larger outlay should bring a larger reward. In practice, the profitability of a property depends on two factors: what it costs and how often opponents land on it. Landing frequency is shaped by the dice, by Chance and Community Chest cards and by the jail rules, so movement around the board is probabilistic rather than deterministic.

Over many turns, some squares are visited noticeably more often than others. Systems that change over time according to probabilities are known as stochastic processes, and one of the most widely used models for them is the Markov chain, in which the probability of the next state depends only on the

current state and not on the states before it [1]. Markov chains were introduced by the Russian mathematician Andrey Markov in the early twentieth century [1] and are now applied in economics, physics and artificial intelligence, for example in modelling stock prices and predicting text. A Markov chain is described by a transition matrix, which gives the probability of moving from each state to every other state [2]. Its steady-state distribution gives the long-run probability of occupying each state [3]. In Monopoly, a player's next position depends only on the current square and the outcome of the dice, so a Markov chain is a natural way to model movement and estimate landing probabilities. Markov chain models of Monopoly were first developed by Ash and Bishop [16], who showed that some squares are visited far more often than others. This paper applies the approach to the UK edition of the board and extends it to a comparison of return on investment across colour sets and levels of development.

Landing probabilities can be used to compare investments at the level of single properties, colour sets or whole sides of the board. Single properties offer limited returns, because houses can be built only once the full colour set is owned. Sides of the board are too coarse a unit, since each side contains colour sets with different prices and rents. Owning a complete colour set, by contrast, allows houses to be built, which raises rent considerably at a moderate cost. The colour set is therefore the most meaningful unit for assessing long-term investment, and it is the unit used in this study.

This paper addresses the following research question: **how can Markov chains be applied to Monopoly to identify the colour set that offers the highest return on investment (ROI)?** A transition matrix is first constructed from the rules of the UK edition of the game. Its steady-state distribution is then computed to obtain the probability of landing on each square. Finally, these probabilities are combined with rents and costs to calculate the ROI of every colour set at each stage of development, from unimproved properties to hotels.

2. Theoretical Background

2.1 Stochastic processes and the Markov property

A stochastic process is a family of random variables that describes how a system develops over time [4]. It is called discrete-time when changes happen at fixed steps and continuous-time when changes can happen at any instant [4]. Formally, a stochastic process is written as

$$\{X_t\}_{t\in T}$$

where X_t is the state of the system at time t and the index set T is either discrete (t = 0, 1, 2, …) or continuous ($t \geq 0$).

A Markov chain is a discrete-time stochastic process in which the probability of moving to the next state depends only on the present state. This is the Markov property:

$$P(X_n = s_n \mid X_{n-1} = s_{n-1}, X_{n-2} = s_{n-2}, \dots, X_0 = s_0) = P(X_n = s_n \mid X_{n-1} = s_{n-1})$$

for every sequence of states s_0, s_1, …, s_n. In words, once the state at step n − 1 is known, the earlier history gives no further information about the state at step n [5].

2.2 Transition matrices

A Markov chain is defined by a set of states S = $\{s_1, s_2, \dots, s_N\}$ and a transition probability matrix P whose entry P_{ij} is the probability of moving from state s_i to state s_j:

$$P_{ij} = P\big(X_{n+1} = s_j \mid X_n = s_i\big)$$

Rows correspond to starting states and columns to destination states, so each row is a probability distribution [6]. Every row must therefore sum to one:

$$\sum_{j=1}^{N} P_{ij} = 1 \quad \text{for all } i$$

For instance, a three-state chain might have

$$P = \begin{pmatrix} 0.3 & 0.6 & 0.1 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.4 & 0.2 \end{pmatrix}$$

From state s_1, the system stays in s_1 with probability 0.3, moves to s_2 with probability 0.6 and moves to s_3 with probability 0.1. Because the rows and columns refer to the same ordered list of states, P is always square.

2.3 Steady-state distribution

The aim in Monopoly is to find which squares are landed on most often over many turns, not just on the next move. This raises two questions about the long-run behaviour of the chain:

- Does the chain settle into a stable long-run probability distribution over its states?
- Is that distribution independent of the starting state?

Both questions are answered by the steady-state distribution, which exists and is unique when the chain is ergodic. A chain is ergodic when it is:

- **Irreducible:** every state can be reached from every other state in a finite number of steps.
- **Aperiodic:** returns to a state are not restricted to a fixed cycle length [7].

The Monopoly chain is irreducible, because any square can eventually be reached with enough rolls. It is aperiodic, because the variable step sizes produced by the dice prevent movement from following a fixed cycle. A well-defined steady-state distribution therefore exists.

Starting from an initial distribution π_0, the distribution after n transitions is

$$\pi_n = \pi_0 P^n$$

As n → ∞, π_n converges to the steady-state distribution π, which satisfies

$$\pi P = \pi \qquad (1)$$

Once the chain reaches π, a further transition leaves it unchanged, so π gives the long-run probability of occupying each state.

2.4 Solving for the steady-state distribution

Since πI = π, where I is the identity matrix (Appendix A), Equation (1) can be rewritten as πP − πI = 0, that is π(P − I) = 0. Each row of P describes probability flowing out of a state, whereas the steady state is determined by probability flowing into each state. Transposing gives a homogeneous linear system in the unknown column vector π^T [8]:

$$(P^T - I)\,\pi^T = 0$$

For a three-state chain this system is

$$\begin{pmatrix} p_{11}-1 & p_{21} & p_{31} \\ p_{12} & p_{22}-1 & p_{32} \\ p_{13} & p_{23} & p_{33}-1 \end{pmatrix} \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

which corresponds to the equations

$$\begin{aligned} (p_{11}-1)\pi_1 + p_{21}\pi_2 + p_{31}\pi_3 &= 0 \\ p_{12}\pi_1 + (p_{22}-1)\pi_2 + p_{32}\pi_3 &= 0 \\ p_{13}\pi_1 + p_{23}\pi_2 + (p_{33}-1)\pi_3 &= 0 \end{aligned}$$

This system does not have a unique solution. Because the columns of P^T − I each sum to zero, one equation is a linear combination of the others and the matrix has rank n − 1 [9]. The system is therefore

under-determined, with infinitely many solutions. A unique probability distribution is obtained by adding the normalisation condition [10]:

$$\sum_{i=1}^{n} \pi_i = 1 \qquad (2)$$

In practice, the last row of $P^T - I$ is replaced by a row of ones, giving the coefficient matrix A and right-hand side b:

$$A = \begin{pmatrix} p_{11}-1 & p_{21} & p_{31} & \cdots & p_{n1} \\ p_{12} & p_{22}-1 & p_{32} & \cdots & p_{n2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$$

The steady state is then the solution of the standard linear system

$$A\,\pi^T = b \qquad (3)$$

which can be solved with Gaussian elimination, LU decomposition, or by finding the eigenvector of P^T for eigenvalue 1.

3. Model Assumptions

The model simplifies the game so that it can be handled as a Markov chain, while keeping the rules that govern movement.

1. **No mortgaging, selling or trading.** Once bought, a property stays with its owner for the rest of the game. These actions depend on player choices, which a purely random model cannot represent.
2. **No auctions.** A player who lands on an unowned property buys it if they can afford it; otherwise it stays unowned. This removes bidding strategy, which is hard to express in probabilistic terms.
3. **Simplified jail exit.** In the full rules a player may pay a fine or use a "Get Out of Jail Free" card at any point. The model assumes players always try to roll doubles first and leave by paying only when forced to, so that time spent in jail follows a fixed probabilistic pattern without decisions.
4. **Random card reshuffling.** Chance and Community Chest cards are assumed to be reshuffled after every draw rather than returned to the bottom of the deck. In the real game, drawing in a fixed order makes card effects partly predictable; reshuffling makes the chain time-homogeneous, which the steady-state calculation requires.

4. Construction of the Monopoly Transition Matrix

The squares of the UK board are numbered 0 to 39, starting at Go and moving in the direction of play (Table 1).

No.	Square	No.	Square	No.	Square	No.	Square
0	Go	10	Jail / Just Visiting	20	Free Parking	30	Go to Jail
1	Old Kent Road	11	Pall Mall	21	Strand	31	Regent Street
2	Community Chest	12	Electric Company	22	Chance	32	Oxford Street
3	Whitechapel Road	13	Whitehall	23	Fleet Street	33	Community Chest
4	Income Tax	14	Northumberland Avenue	24	Trafalgar Square	34	Bond Street

No.	Square	No.	Square	No.	Square	No.	Square
5	King's Cross Station	15	Marylebone Station	25	Fenchurch St. Station	35	Liverpool St. Station
6	The Angel Islington	16	Bow Street	26	Leicester Square	36	Chance
7	Chance	17	Community Chest	27	Coventry Street	37	Park Lane
8	Euston Road	18	Marlborough Street	28	Water Works	38	Super Tax
9	Pentonville Road	19	Vine Street	29	Piccadilly	39	Mayfair

Table 1. Numbering of the squares on the UK Monopoly board.

4.1 State space

A first approach is a 40-state chain with one state per square. Two features of the board require changes to this:

- **Go to Jail (square 30)** is never occupied at the end of a move, because landing there sends the player straight to jail.
- **Square 10** combines two different situations, Just Visiting and In Jail, which must be separated. In the model, state 10 represents Just Visiting and state 30 represents In Jail.

Even with these changes, a 40-state chain does not satisfy the Markov property, because it ignores two rules whose outcome depends on more than the current square: three consecutive doubles send a player to jail, and leaving jail depends on how many turns have already been spent there. The state therefore records three quantities:

- the board position p, with $p \in \{0, 1, \ldots, 39\}$;
- the number of consecutive doubles d, with $d \in \{0, 1, 2\}$, reset to 0 on entering jail;
- the number of turns spent in jail t, with $t \in \{0, 1, 2\}$, reset to 0 on leaving jail.

Each state is written as (d, p, t).

4.1.1 Consecutive doubles

A double gives the player another roll, but a third consecutive double sends the player to jail instead of moving them. So from a state with $d = 2$, the probability of moving from p to $p + k$ on a further double is zero. Tracking three values of d at each of the 40 positions gives $3 \times 40 = 120$ states. Once a player is in jail, d is reset to 0 and doubles stop being counted, while the time spent in jail is tracked separately.

4.1.2 Time spent in jail

The rules allow four ways out of jail:

- paying the fine before the third turn (excluded from the model);
- rolling a double within the first three turns;
- paying the fine after failing to roll a double in three turns;
- using a "Get Out of Jail Free" card (excluded from the model).

Under Assumption 3, players always try to roll a double before paying, which keeps the transitions random. Jail states are indexed by $t = 0, 1, 2$; no state for $t = 3$ is needed, because the player must leave on the third turn. The jail transitions are:

- on entering jail: $(d, p, 0) \rightarrow (0, 30, 0)$, the first turn in jail;
- with no double: $(0, 30, 0) \rightarrow (0, 30, 1)$, the second turn;

- with no double again: (0, 30, 1) → (0, 30, 2), the third turn;
- on the next turn, the player moves out from (0, 30, 2) in the normal way.

With these states the chain captures ordinary movement, the doubles rule and the conditions for leaving jail.

4.2 Structure of the transition matrix

The model therefore uses a 120 × 120 transition matrix whose entries

$$P_{(d_1,p_1,t_1),(d_2,p_2,t_2)}$$

give the probability of moving from state (d_1, p_1, t_1) to state (d_2, p_2, t_2) [11]. These probabilities depend on the dice total (2 to 12), on whether a double is rolled, and on movement cards from the Chance and Community Chest decks.

4.3 Dice probabilities

With two six-sided dice, the probability of a total k is the number of ways of rolling k divided by 36 (Table 2).

Total k	2	3	4	5	6	7	8	9	10	11	12
P(k)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Table 2. Probability of each total with two six-sided dice.

4.4 Movement probabilities

Movement depends on whether the roll is a double. On a non-double, the player moves k squares and the doubles counter returns to 0:

$$P_{(d,p,0),(0,p+k,0)} = \begin{cases} P(k) - \dfrac{1}{36}, & k \text{ even} \\ P(k), & k \text{ odd} \end{cases}$$

For even k, 1/36 is subtracted because one of the ways of making k is a double, for example (1, 1) for 2 or (2, 2) for 4. Odd totals cannot be doubles, so no adjustment is needed.

On a double with d ≠ 2, the player moves k squares and the doubles counter increases by one:

$$P_{(d,p,0),(d+1,p+k,0)} = P_{\text{double}}(k) = \frac{1}{36}$$

If the player has already rolled two doubles (d = 2) and rolls a third, they go directly to jail. There are six possible doubles, (1, 1) to (6, 6), so

$$P_{\text{jail}} = P_{(2,p,0),(0,30,0)} = 6 \times \frac{1}{36} = \frac{1}{6}$$

4.5 Jail transitions

While the player is in jail:

- for t < 2, a double releases the player, who moves k squares from square 10:

$$P_{(0,30,t),(0,10+k,0)} = P_{\text{double}}(k) = \frac{1}{36}$$

- otherwise the player stays in jail:

$$P_{(0,30,t),(0,30,t+1)} = 1 - P_{\text{doubles}}$$

- on the third turn (t = 2), the player must leave, paying the fine, and moves normally:

$$P_{(0,30,2),(0,10+k,0)} = \begin{cases} P(k) - \dfrac{1}{36}, & k \text{ even} \\ P(k), & k \text{ odd} \end{cases} \quad \text{(non-double)}, \qquad P_{(0,30,2),(1,10+k,0)} = \frac{1}{36} \quad \text{(double)}$$

4.6 Chance and Community Chest cards

Money is not tracked in the model, so only cards that move the player affect the transition matrix. In the Chance deck, 10 of the 16 cards move the player from a Chance square p_C (7, 22 or 36):

- six send the player to a named square: Go, Trafalgar Square, Mayfair, Pall Mall, Jail or King's Cross Station;
- two send the player to the nearest station;
- one sends the player to the nearest utility;
- one sends the player back three squares.

Current state	New state	Card	Probability
(d, p_C, 0)	(d, 0, 0)	Go	1/16
(d, p_C, 0)	(d, 5, 0)	King's Cross Statio	n 1/16
(d, p_C, 0)	(0, 30, 0)	Jail	1/16
(d, p_C, 0)	(d, 11, 0)	Pall Mall	1/16
(d, p_C, 0)	(d, 24, 0)	Trafalgar Square	1/16
(d, p_C, 0)	(d, 39, 0)	Mayfair	1/16
(d, 7, 0)	(d, 15, 0)	Nearest station	2/16
(d, 22, 0)	(d, 25, 0)	Nearest station	2/16
(d, 36, 0)	(d, 5, 0)	Nearest station	2/16
(d, 7, 0)	(d, 12, 0)	Nearest utility	1/16
(d, 22, 0)	(d, 28, 0)	Nearest utility	1/16
(d, 36, 0)	(d, 12, 0)	Nearest utility	1/16
(d, p_C, 0)	(d, $p_C - 3$	, 0) Back three squar	es 1/16

Table 3. Transition probabilities from Chance movement cards.

In the Community Chest deck, 2 of the 16 cards move the player from a Community Chest square p_{CC} (2, 17 or 33), one to Go and one to Jail (Table 4).

Current state	New state	Card	Probability
(d, p_{CC}, 0)	(d, 0, 0)	Go	1/16
(d, p_{CC}, 0)	(0, 30, 0)	Jail	1/16

Table 4. Transition probabilities from Community Chest movement cards.

5. Steady-State Landing Probabilities

5.1 Numerical solution

For Monopoly, P is the 120 × 120 matrix described in Section 4 (the full matrix is available from the author on request). Applying Equation (3) gives a system of 120 linear equations in 120 unknowns:

$$\begin{pmatrix} p_{0,0}-1 & p_{1,0} & p_{2,0} & \cdots & p_{119,0} \\ p_{0,1} & p_{1,1}-1 & p_{2,1} & \cdots & p_{119,1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \pi_0 \\ \pi_1 \\ \vdots \\ \pi_{119} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \qquad (4)$$

A system of this size has no practical closed-form solution and is solved numerically. Since it has the form Ax = b, it can be solved efficiently by LU decomposition with partial pivoting [12], which factorises A as

$$PA = LU$$

where L is lower triangular with ones on its diagonal, U is upper triangular, and P here denotes a permutation matrix that reorders rows for numerical stability (the method is outlined in Appendix B). The NumPy function np.linalg.solve(A, b) applies this factorisation internally. The complete script, which reads the transition matrix from an Excel file, is:

```
import numpy as np   # matrix operations
import pandas as pd  # reading Excel files
def find_steady_state(P):
    assert P.shape[0] == P.shape[1], "Transition matrix must be square"
    n = P.shape[0]          # number of states
    A = P.T - np.eye(n)     # P^T – I
    A[-1] = np.ones(n)      # replace last row with ones (normalisation)
    b = np.zeros(n)
    b[-1] = 1               # right-hand side for the normalisation equation
    return np.linalg.solve(A, b)   # solve A * pi^T = b
file_path = "MonopolyTransitionMatrix.xlsx"
P = pd.read_excel(file_path, header=None).to_numpy()
steady_state = find_steady_state(P)
print("Steady-State Distribution:\n", steady_state)
```

Listing 1. Python code used to compute the steady-state distribution.

5.2 Results

The probability of landing on each square is obtained by adding the steady-state probabilities of the three states (d = 0, 1, 2) at that position. Table 5 lists the results, which form the basis of the ROI calculations in Section 6.

Square (0–19)	Probability (%)	Square (20–39)	Probability (%)
Go	2.92	Free Parking	2.82
Old Kent Road	2.01	Strand	2.61
Community Chest	1.78	Chance	1.04
Whitechapel Road	2.04	Fleet Street	2.57
Income Tax	2.20	Trafalgar Square	2.99
King’s Cross Station	2.80	Fenchurch St. Station	2.89
The Angel Islington	2.13	Leicester Square	2.54
Chance	0.82	Coventry Street	2.52
Euston Road	2.19	Water Works	2.65
Pentonville Road	2.17	Piccadilly	2.44
Just Visiting	2.14	In Jail	9.41
Pall Mall	2.56	Regent Street	2.52
Electric Company	2.62	Oxford Street	2.48
Whitehall	2.18	Community Chest	2.23
Northumberland Avenue	2.43	Bond Street	2.36
Marylebone Station	2.64	Liverpool St. Station	2.29
Bow Street	2.68	Chance	0.82

Square (0–19)	Probability (%)	Square (20–39)	Probability (%)
Community Chest	2.30	Park Lane	2.06
Marlborough Street	2.82	Super Tax	2.05
Vine Street	2.81	Mayfair	2.49

Table 5. Steady-state probability of each board location.

The most striking feature of Table 5 is the high probability of being in jail. This is expected, since several rules send players there: three doubles in a row, a "Go to Jail" card and the Go to Jail square [13]. As a consequence, the colour sets that follow the jail square, Orange and Red, have higher landing probabilities than most other sets, although still far below that of jail itself. Trafalgar Square is the most frequently visited property (2.99%), consistent with earlier Markov analyses of the board [16]. The Chance squares, by contrast, have the lowest probabilities on the board (0.82% to 1.04%), because ten of the sixteen Chance cards move the player to another square.

6. Return on Investment by Colour Set

6.1 Definition of ROI

The return on investment of a property or colour set is defined as

$$\text{ROI} = \frac{\text{Expected Rent Income (ERI)}}{\text{Total Investment Cost (TIC)}} \qquad (5)$$

The expected rent income combines the steady-state landing probabilities with the rent charged on each property:

$$\text{ERI} = \sum_i \pi_i \times \text{Rent}_i \qquad (6)$$

where π_i is the long-run probability of landing on square i in a single roll. The total investment cost is the price of the properties plus the cost of any houses or hotels built on them:

$$\text{TIC} = \text{Property cost} + \text{Building cost} \qquad (7)$$

For a colour set, the expected rent income of all its properties is added together and divided by the total cost of the set.

6.2 Worked example: Orange set with three houses

Table 6 gives the data for the Orange set with three houses on each property, taken from the title-deed cards [14]. Houses on Orange properties cost £100 each.

Property	Price (£)	Rent with 3 houses (£)	Landing probability	Expected rent income (£)
Bow Street (16)	180	550	π_{16} = 0.0268	550 × 0.0268
Marlborough Street (18)	180	550	π_{18} = 0.0282	550 × 0.0282
Vine Street (19)	200	600	π_{19} = 0.0281	600 × 0.0281

Table 6. Data for the Orange set with three houses on each property.

$$\text{TIC}_{\text{Orange}} = (180 + 180 + 200) + 3 \times 100 \times 3 = 1460$$

$$\text{ERI}_{\text{Orange}} = \sum_{i \in \{16,18,19\}} \pi_i \times \text{Rent}_i$$

$$\text{ROI}_{\text{Orange}} = \frac{(550 \times 0.0268) + (550 \times 0.0282) + (600 \times 0.0281)}{1460} = \frac{47.1}{1460} = 0.0323$$

The same calculation was carried out for all eight colour sets at seven levels of development: base rent without the full set, base rent with the full set, one to four houses, and a hotel.

6.3 Scaling to a typical game

A typical game of Monopoly lasts about 30 rolls per player and is usually played by four players [15], giving roughly 120 rolls in total. ROI is therefore reported over 120 rolls as a measure of returns over a full game:

$$\text{ROI}_{120} = \text{ROI}_1 \times 120$$

For the Orange set with three houses, ROI_{120} = 0.03227 × 120 = 3.87.

6.4 Results

Table 7 gives ROI over 120 rolls for every colour set at each level of development. The three highest values at each level are ranked in brackets.

Colour set	Base rent, set incomplete	Base rent, full set	1 house	2 houses	3 houses	4 houses	Hotel
Brown	0.122	0.244	0.332	0.685	1.57	2.25	2.75
Light Blue	0.162	0.325	0.553	1.17	2.83	3.53 (2=)	4.12 (2)
Pink	0.209	0.418	0.621	1.33	3.00	3.41	3.55
Orange	0.261 (3)	0.523 (3)	0.851 (2)	1.78 (3)	3.87 (1)	4.35 (1)	4.68 (1)
Red	0.270 (2)	0.540 (2)	0.813	1.66	3.47 (3)	3.53 (2=)	3.57 (3)
Yellow	0.255	0.510	0.815 (3)	1.80 (2)	3.42	3.43	3.44
Green	0.256	0.512	0.793	1.70	3.03	3.01	2.96
Dark Blue	0.315 (1)	0.629 (1)	0.896 (1)	1.95 (1)	3.54 (2)	3.53 (2=)	3.52

Table 7. ROI over 120 rolls by colour set and level of development (rank among the top three in brackets; = denotes a tie at three significant figures).

Base rent without the full set. Dark Blue, Red and Orange rank highest, and every ROI is below 0.4 because base rents are small compared with purchase prices (Appendix D). Old Kent Road, the first Brown property, has a rent-to-cost ratio of only 0.0333. The ratio rises around the board, reaching 0.100 for Park Lane and 0.125 for Mayfair, the highest of any property, which explains why Dark Blue leads. Red and Orange also perform well because of their high average landing probabilities, 2.72% and 2.77% respectively, even before any development.

Base rent with the full set. The ranking is unchanged. Owning a complete set doubles the base rent of each property, so every ROI also doubles, and Dark Blue, Red and Orange remain the top three.

One house. Dark Blue stays first, followed by Orange and Yellow, with Red a close fourth. Dark Blue's high rents with one house (£175 and £200) outweigh its higher house cost (£200, against £100 for Orange). Orange, with frequent landings and cheaper houses, closes most of the gap.

Two houses. Dark Blue, Yellow and Orange make up the top three. Dark Blue's rent rises steeply at this stage (186% to 200%; Appendix D), which keeps it ahead. Yellow edges past Orange with a 200% rent increase on all three properties, against 175% to 186% for Orange. Brown and Light Blue, despite being cheap, have ROI below 1.2 because their rents at two houses are low (£30 to £60 for Brown and £90 to £100 for Light Blue).

Three houses. Orange moves into first place, ahead of Dark Blue and Red, and ROI rises sharply for every set. The ratio of marginal rent to marginal cost averages about 3.1 at this stage, the highest of any step (Appendix D). Orange combines a high landing probability with houses costing £100, half the price of Dark Blue's, which lets it overtake Dark Blue. With more houses, landing probability matters even more: a single extra landing in 120 rolls now brings in substantial rent.

Four houses. Orange stays first, while Red, Dark Blue and Light Blue are effectively tied for second at 3.53. ROI still rises for six of the eight sets, but more slowly. The rent increase from the fourth house is below 80% for every property and averages about 35%, and the ratio of marginal rent to marginal cost falls to about 1.7 on average (Appendix D). For Green and Dark Blue, whose houses cost £200, ROI levels off or falls slightly. The fourth house therefore still adds to returns for most sets, but much less efficiently than the third.

Hotel. The top three become Orange, Light Blue and Red. Light Blue's second place comes from its very high ratio of marginal rent to marginal cost for the hotel, 3.0, well above the overall average of about 1.6. Brown, Light Blue, Pink, Orange, Red and Yellow all reach their highest ROI with a hotel, while Green and Dark Blue decline slightly. This is consistent with their higher building costs (£200 per house or hotel), which make hotels less efficient: their average ratios of marginal rent to marginal cost are about 0.9 for Green and 1.25 for Dark Blue.

7. Limitations

The model gives a rigorous basis for comparing colour sets, but several simplifications should be noted.

First, it leaves out strategic decisions such as trading, auctions and cash-flow management. In real games these let players change their approach as the game unfolds, which affects who owns and develops which properties in ways the model cannot capture.

Second, houses are assumed to be built as soon as a colour set is complete, without regard to available cash. Real players often delay building until they can afford it, which changes the realised ROI. The game also limits the supply to 32 houses and 12 hotels, so development can be blocked even when a player has the money. This limit is not modelled and may lead to an overestimate of investment opportunities.

Third, the jail rules are simplified: players are assumed always to try for doubles before paying. Early in the game, many players pay immediately in order to get back to buying properties sooner.

Finally, selling houses and mortgaging properties are excluded. These actions are important for managing cash in real play and would alter investment outcomes in ways not reflected here.

8. Conclusion and Future Work

8.1 Conclusion

This study asked how Markov chains can be used to identify the Monopoly colour set with the highest return on investment. A transition matrix was built from the game rules, its steady-state distribution was computed, and the resulting landing probabilities were used to evaluate the long-run profitability of each colour set. The results only partly support the common belief that the most expensive properties give the best returns: this holds while properties are undeveloped, but not once houses are built, when landing probability becomes central to profitability.

The Orange set (Bow Street, Marlborough Street and Vine Street) ranked first at every level of development from the third house onwards. At lower levels of development Dark Blue led, with an ROI of 0.629 with the full set and no houses against 0.523 for Orange, and it stayed ahead up to two houses. Once three houses were built, Orange became the most profitable set and reached an ROI of 4.68 with hotels, the highest value recorded. This is mainly due to its high landing probability: the Markov analysis shows that squares shortly after jail are visited more often. Red benefits from the same effect but ranks above Orange only before any houses are built.

Dark Blue (Park Lane and Mayfair) charges the highest rents, which makes it the strongest set early in the game, but its lower landing probability and £200 building costs hold back its ROI once sets are developed. With hotels it falls behind Orange, Light Blue, Red and Pink, which shows that the most expensive properties are not necessarily the most profitable. ROI instead depends on the balance between rent, building cost and landing probability.

The number of houses also matters. The third house gives by far the largest increase in rent per pound spent, and the gain from each further building is much smaller. For most sets total ROI keeps rising up to a hotel, but for Green and Dark Blue it levels off after the third house, so for these sets three houses is the most efficient stopping point.

Overall, Markov chains provide an effective tool for modelling movement in Monopoly and for calculating landing probabilities and ROI precisely. The results suggest that players who can afford to build should favour frequently visited, moderately priced sets such as Orange, while Dark Blue is most attractive when little money is available for houses. More broadly, the study illustrates how mathematical modelling can support strategic decisions.

8.2 Future work

The model assumes a single player moving indefinitely and leaves out auctions and trades. A more realistic analysis could use Monte Carlo simulation of several players to follow changes of ownership and rent payments. The model also assumes a fixed pattern of building rather than decisions based on available cash; methods such as dynamic programming could capture optimal investment strategies. Further work could also vary player behaviour, for example attitudes to risk when buying property, to give a fuller picture of strategy in Monopoly.

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