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Solving the World Through Equations

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Abstract:

Differential equations are fundamental tools in understanding and modeling the natural and engineered world. From describing the motion of celestial bodies to predicting population growth, and from simulating climate change to optimizing economic systems, these equations serve as the mathematical language of change and continuity. This paper explores the diverse and impactful applications of differential equations across multiple disciplines, including physics, biology, engineering, and economics. It highlights how ordinary and partial differential equations translate real-world phenomena into solvable mathematical problems, offering insights, predictions, and control strategies for complex systems. By bridging theory with practice, differential equations not only deepen our scientific understanding but also empower us to solve critical challenges facing society. Through this exploration, the paper underscores the indispensable role of mathematics in shaping and solving the world around us.

Keywords: Differential Equations, Mathematical Modelling, Real-world Applications, Predictive Analysis, Applied Mathematics

1. INTRODUCTION:

Differential equations form the backbone of modern science and engineering, serving as powerful tools to model and understand dynamic systems. From predicting the spread of diseases to designing efficient transportation networks and analyzing climate change, these equations are indispensable for interpreting and solving real-world problems. They bridge theory with application, allowing researchers and professionals to simulate, predict, and optimize processes across disciplines such as physics, biology, economics, and engineering. By transforming complex natural and man-made phenomena into solvable mathematical models, differential equations enable deeper insight and more accurate decision-making. This paper explores how these mathematical constructs play a pivotal role in shaping our understanding of the world and in devising practical solutions to contemporary global challenges.

One physical system in which many important phenomena occur is that where an initial uneven temperature distribution causes heat to flow. As heat flows the temperature distribution changes ,which modifies the heat flow. That is heat flows from hot places to cold ones, and as this happens the temperature of thecold places rises and the temperature of hot places decreases.

There are two rough mathematical rules governing the relation between heat flow and temperature change. Heat flow is proportional to rates of change in temperature distribution. The time of temperature change at any point is proportional to the rate of heat flow into that point

2. PRELIMINARIES AND BASICS:

2.1 Definition: A differential equation is an equation that relates an unknown function and one or more of its derivatives of with respect to one or more independent variables. For instance, the equation $\frac{dy}{dx} = -5x$ relates the first derivative of y with respect to x, with x. Here x is the independent variable and y is the unknown function (dependent variable).

If $\frac{d^n y}{dx^n}$ is the n-th derivative of y with respect to x. One can also use the notation y (n) to denote $\frac{d^n y}{dx^n}$. It is further convenient to write $y' = y(1)$ and $y'' = y(2)$. In physics, the notation involving dots is also common, such that $\dot{y}$ denotes the first-order derivative.

Different classifications of differential equations are possible and such classifications make the analysis of the equations more systematic.

2.2 Classification of Differential Equations

There are various classifications for differential equations. Such classifications provide remarkably simple ways of finding the solutions (if they exist) for a differential equation. Differential equations can be of the following classes.

2.2.1 Ordinary Differential Equations

If the unknown function depends only on a single independent variable, such a differential equation is ordinary. The following is an ordinary differential equation:

$$\mathrm{L}\frac{\mathrm{d}^2}{\mathrm{dt}^2}\mathrm{Q(t)} + R\frac{\mathrm{d}}{\mathrm{dt}}\mathrm{Q(t)} + \frac{1}{C}Q(t) = E(t)$$

which is an equation which arises in electrical circuits. Here, the independent variable is t.

2.2.2 Partial Differential Equations

If the unknown function depends on more than one independent variables, such a differential equation is said to be partial. Heat equation is an example for partial differential equations:

$$\alpha^2\frac{\partial^2}{\partial \mathrm{x}^2}\mathrm{f(x,t)} = \frac{\partial}{\partial t}\mathrm{f(x,t)}$$

Here x, t are independent variables.

2.2.3 Homogeneous Differential Equations

If a differential equation involves terms all of which contain the unknown function itself, or the derivatives of the unknown function, such an equation is homogeneous. Otherwise, it is non-homogeneous.

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2.2.4 n^{th} order Differential Equations

The order of an ordinary differential equation is the order of the highest derivative that appears in the equation. For example $2y^{(2)} + 3y^{(1)} = 5$, is a second-order equation.

2.2.5 Linear Differential Equations

A very important class of differential equations is linear differential equations. A differential equation $F(y, y^{(1)}, y^{(2)}, \ldots, y^{(n)})(x) = g(x)$ is said to be linear if F is a linear function of the variables y, $y^{(1)}, y^{(2)}, \ldots, y^{(n)}$

2.3 Note: If a differential equation is not linear, it is said to be non-linear.

2.4 Solutions of Differential equations

2.4.1 Definition

To say that $y = g(x)$ is an explicit solution of a differential equation $F\left(x, y, \frac{dy}{dx}, \frac{d^n y}{dx^n}\right) = 0$ on an interval I ⊂ R, means that $F(x, g(x), \frac{dg(x)}{dx}, \ldots, \frac{d^n g(x)}{dx^n}) = 0$, for every choice of x in the interval I.

2.4.2 Definition

We say that the relation G(x, y) = 0 is an implicit solution of a differential equation $F\left(x, y, \frac{dy}{dx}, \frac{d^n y}{dx^n}\right) = 0$if for all y = g(x) such that $G(x, g(x)) = 0$, g(x) is an explicit solution to the differential equation on I.

2.4.3 Definition

An n^{th} parameter family of functions defined on some interval I by the relation $h(x, y, c_1, \ldots, c_n) = 0$, is called a general solution of the differential equation if any explicit solution is a member of the family. Each element of the general solution is a particular solution.

2.4.4 Definition

A particular solution is imposed by supplementary conditions that accompany the differential equations. If all supplementary conditions relate to a single point, then the condition is called an initial condition. If the conditions are to be satisfied by two or more points, they are called boundary conditions. Recall that in class we used the falling object example to see that without a characterization of the initial condition (initial velocity of the falling objects), there exist infinitely many solutions. Hence, the initial condition leads to a particular solution, whereas the absence of an initial condition leads to a general solution.

2.4.5 Definition

A differential equation together with an initial condition (boundary conditions) is called an initial value problem (boundary value problem).

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2.5 Basic definitions

- Order: Highest Derivative present in the differential equation
- Degree: Power of the higher derivative after free from radicals and fractions
- Formation of Differential Equation:

1. Consider the given equation $f(x, y, a, b, c \dots d) = 0$
2. Observe the no. of arbitrary constants if it is 'n'
3. Based on Arbitrary constants differentiate the given Equation n times
4. Using the n+1 equations including the given equation

We get a differential equation of nth order.

- The standard form of First order linear differential equation is of the form $\frac{dy}{dx} = f(x, y)$

Homogeneous Function: A function f(x,y) which satisfies the condition that

$$f(kx, ky) = k^n f(x, y)$$

A differential equation is said to be Homogeneous if the f(x, y) is homogeneous of degree zero
For Homogeneous use $y = vx \vee x = vy$substitution based on the equation
For Non-Homogeneous in the case of $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ use $ax + by = t$ as substitution and in the case of $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ take $x = X + h$ & $y = Y + k$ as substitution

- The necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

The solution of the exact differential equation is $\int Mdx + \int Ndy = c$,under the first integral y constant and under the second integral terms do not contain x

The equation is not an exact differential equation if $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ and it can be solved by applying various conditions like homogeneous, inspection, functions in xy, function of x alone, function of y alone and powers of x,y

- The standard form of First order linear differential equation is of the form $\frac{dy}{dx} = f(x, y)$

The linear differential equation of first order and first degree of Leibnitz is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

- The solution of the Leibnitz linear differential equation is $y(I.F) = \int Q(x)(I.F)dx + c$
- The Bernoulli's differential equation is of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$

  To find the solution of Bernoulli's use the substitution as $y^{1-n} = t$

2.Newton's Law of Cooling:

It states that the rate of change of temperature of a body is directly proportional to the difference between temperature of a body and its surrounding medium. i.e $\frac{dT}{dt} \propto (T_t - T_s)$

the Newton's Law of Cooling is given by

$$\frac{dT}{dt} = k\,(T_t - T_s)$$

Where T_t is the temperature at time t and

T_s is the temperature of the surrounding, k is a constant.

The Newton s Law of Cooling Formula is given by

$$T\,(t) = T_s + (T_0 - T_s)\,e^{-kt}$$

Where t is the time taken,

$T(t)$ is the temperature of the given body at time t,

T_s is the surrounding temperature,

T_0 is the initial temperature of the body, k is the constant.

NOTE: The greater the difference in temperature between the system and surrounding, more quickly the body temperature changes.

The Language of Nature: Mathematics

- Galileo once said, "The universe is written in the language of mathematics."
- Every natural phenomenon — from falling raindrops to galaxies rotating — can be translated into an equation.
- Equations are not just tools; they are codes of existence.

2.6. Fundamental Equations That Describe the World

(a) Motion and Gravity

- **Newton's Laws**

$$\mathrm{F} = \mathrm{ma}, \qquad \mathrm{F} = \frac{\mathrm{Gm_1m_2}}{\mathrm{r^2}}$$

Foundation of classical mechanics.

- **Einstein's Relativity**

$$E = mc^2, \qquad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Energy-mass equivalence, spacetime curvature.

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(b) Light and Energy

- **Maxwell's Equations** unify electricity, magnetism, and light.
- Electromagnetic waves explain communication, vision, and cosmic radiation.

One of the greatest achievements in physics was the unification of **electricity, magnetism, and light** through **James Clerk Maxwell's equations** in the 19th century. These four equations form the foundation of classical electromagnetism:

- $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (Gauss's law for electricity)
- $\nabla \cdot \vec{B} = 0$ (Gauss's law for magnetism)
- $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's law of induction)
- $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \in_0 \frac{\partial \vec{E}}{\partial t}$ (Ampe`re–Maxwell law)

- Key Insights: Maxwell showed that **light itself is an electromagnetic wave** — a combination of oscillating electric ($\vec{E}$) and magnetic ($\vec{B}$) fields traveling through space.
- This discovery unified optics (light) with electricity and magnetism, which were once thought to be separate fields.

Applications in the World:

- **Communication:** Radio, television, Wi-Fi, and mobile signals are all electromagnetic waves.
- **Vision:** Human eyes detect a narrow band of EM waves (visible light), allowing us to perceive the world.
- **Cosmic Radiation:** From the cosmic microwave background to gamma rays, electromagnetic radiation carries information about the universe's past and present.
- **Energy Transfer:** Electromagnetic radiation is a primary means of transferring energy across space (e.g., sunlight reaching Earth).

(c) Quantum World

- **Schrödinger Equation**:

$$i\bar{h}\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

Governs atoms, molecules, and modern electronics.

- **Heisenberg's Uncertainty Principle** reminds us that reality has inherent limits.

(d) Entropy and Life

- **Second Law of Thermodynamics**:

$\Delta S \geq 0$ Time flows forward, life exists by maintaining local order against universal disorder.

(e) Cosmic Equations

- **Hubble's Law**:

$$v = H_0 d$$

The universe expands, galaxies recede.

- **Friedman Equations**: govern the dynamics of the cosmos.

2.6.1. Equations as Philosophy

- Equations are **metaphors of existence**:
  - $E = mc^2$ → Matter and energy are two faces of the same coin.
  - $F = ma$ → Change is caused by force.
  - $\Delta S \geq 0$ → The inevitability of change, aging, and transformation.
- Seeing the "world through equation" means understanding that life itself is a **dynamic balance of laws**.

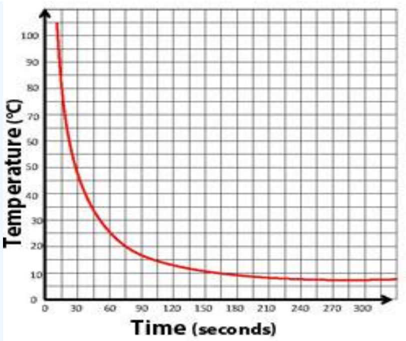
2.6.2. Beyond Science: A Unified World Equation?

- Physicists seek a **Theory of Everything (TOE)** — one master equation unifying gravity, quantum mechanics, and all forces.
- Candidates include **String Theory, Loop Quantum Gravity, and the Wheeler–DeWitt Equation**.
- If found, it would be the **"World Equation"**, a mathematical key to reality.

2.7. Engineering Connection:

Heat transfer is a broad topic used in many branches of engineering. For example, mechanical engineers who design engines from steam locomotives to modern internal combustion engines on a detailed understanding of how heat moves through all types of matter. Industrial engineers use heat transfer concepts to design climate control systems for manufacturing facilities, such as foundries or refrigerated food production facilities, which integrate temperature-sensitive human workers with extreme temperature processes. Moreover, heat transfer is so critical to biological engineering that it has spawned the specialty of "bio heat" transfer, which is the study of normal functioning of the cardiovascular system as well as inherently heated treatments such as cryo-surgery and laser-based therapies.

For example, If you placed a room-temperature can of soda in the refrigerator and waited for it to cool, how would you expect its temperature to change? What kind of trend do you think the temperature would have over time?



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2.8. PROBLEMS

Problem 2.8.1: Cooling of Coffee

A cup of coffee has a temperature of 90^0C in a room where the temperature is 25^0C. After 10 minutes, the coffee cools to 70^0C. Find:

1. The cooling constant k.
2. The temperature of the coffee after 20 minutes.

Solution:

$T\ (t)\ = T_s\ +\ (T_0\ -\ T_s\ )\ e^{-kt}$

At t=10

$70 = 25 + (90 - 25)e^{-10k}$

$\Rightarrow 70 - 25 = 65e^{-10k}$

$\Rightarrow 45 = 65e^{-10k}$

$\Rightarrow e^{-10k} = \frac{45}{65} = 0.6923$

$\Rightarrow -10k = ln(0.6923)$

$\Rightarrow k = 0.0369$ per minute

At t=20t = 20t=20:

$T(20) = 25 + 65e^{-(0.0369).20}$

$\Rightarrow T(20) = 25 + 65e^{-0.738}$

$\Rightarrow T(20) = 25 + 65 \times 0.478$

$\Rightarrow T(20) = 25 + 31.07$

$\Rightarrow T(20) = 56.07^0\mathrm{C}$

Answer:

- Cooling constant k=0.0369k = 0.0369k=0.0369 per minute
- Temperature after 20 minutes ≈ $56.07^0\mathrm{C}$

Problem 2.8.2: Cooling of Metal Rod

A hot metal rod at 150^0C is placed in air at 30^0C. After 5 minutes, the rod cools to 100^0C. Find how long it will take to cool to 60^0C.

Solution:

$T\ (t)\ = T_s\ +\ (T_0\ -\ T_s\ )\ e^{-kt}$

At t=5

$100 = 30 + (150 - 30)e^{-5k}$

$\Rightarrow 70 = 120e^{-10k}$

$\Rightarrow e^{-5k} = \frac{70}{120} = 0.5833$

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$\Rightarrow -10k = ln(0.5833)$
$\Rightarrow k = 0.107$ per minute
At t=5t = 5t=5:
Now for T=60
$60 = 30 + (150 - 30)e^{-0.107t}$
$\Rightarrow 30 = 120e^{-0.107t}$
$\Rightarrow e^{-0.107t} = \dfrac{30}{120} = 0.25$
$\Rightarrow -0.107t = l\,n(0.25) = 0.386$
$\Rightarrow t = \dfrac{0.386}{0.107} =$ 12.95 minutes

Answer:
It will take about **13 minutes** to cool to 60^0C.

Problem 2.8.3: Cooling of Soup (Quick Application)
A bowl of soup at 80^0C is kept in a room at 20^0C. If it cools to 50^0C in 15 minutes, what will its temperature be after 30 minutes?
Solution (Steps):
$T\,(t) \ = T_s \ + \ (T_0 \ - \ T_s\,)\,e^{-kt}$
At t=15
1. $50 = 20 + (80 - 20)e^{-15k} \ \Rightarrow k = 0.0409$ per minute
2. $T(30) = 20 + 60e^{-30k} \ \Rightarrow \mathrm{T}(30) \approx 34.6 \circ \mathrm{C}$

Problem 2.8.4: Newton's Law of Cooling -
A glass of room-temperature water is taken outside where the outdoor temperature is unknown. Initially, the water temperature is 22°C. After 1 minute, it is 26°C. After 2 minutes, it is 28°C. Find the outdoor temperature.

Solution:
Newton's Law of Cooling states:
$$T(t) \ = \ A \ + \ (T_0 \ - \ A)\,e^{-kt}$$
Where:
- A = Ambient temperature = 20°C
- $T(t)$ = Body temperature at time t
- T_0 = Initial temperature difference
- k = Cooling constant

At t = 1 minute:

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T(1) = 26

$$26 = A + (22 - A)e^{-k}$$

At t = 2 minutes:
T(2) = 28
$28 = A + (22 - A)e^{-2k}$ ....(2)
From (1): $e^{-k} = \frac{26 - A}{22 - A}$
From (2): $e^{-2k} = \frac{28 - A}{22 - A}$
So, $(\frac{26 - A}{22 - A})^2 = \frac{28 - A}{22 - A}$
$\Rightarrow (26 - A)^2 = (28 - A)(22 - A)$
$676 - 52A + A^2 = 616 - 50A + A^2$
$676 - 52A = 616 - 50A$
$60 = 2A$
$A = 30$
Therefore, the outdoor temperature is 30°C.

Problem 2.8.5. (Newton's Law of Cooling – Forensic application):
A dead body is discovered at 3:45 pm in a room where the ambient temperature is 20°C. At that time, the body's temperature is 27°C. Two hours later (at 5:45 pm), the body's temperature has dropped to 25.3°C. If the normal body temperature is 37°C, estimate the time of death.
Solution:
Newton's Law of Cooling states:
- $T(t) = A + (T_0 - A)\, e^{-kt}$
  Where:
- A = Ambient temperature = 20°C
- $T(t)$ = Body temperature at time t
- T_0 = Initial temperature difference
- k = Cooling constant

At 3:45 pm, let t = 0. Then T_0 = 27°C.

$$T(t) = 20 + 7e^{-kt}$$

At 5:45 pm (t = 2 hours), T(2) = 25.3°C.
$T(t) = 20 + 7e^{-kt}$
$\Rightarrow 25.3 = 20 + 7e^{-2k}$
$\Rightarrow e^{-2k} = \frac{5.3}{7}$
$\Rightarrow k = -\frac{1}{2}\ln(\frac{5.3}{7})$
$\Rightarrow k = 0.1391\ \text{h}^{-1}$

At time of death, body temperature = 37°C. Let τ be the time before 3:45 pm when death occurred (so we plug $t = -\tau$):

$T(t) = 20 + 7e^{-k(-\tau)}$

$\Rightarrow 37 = 20 + 7e^{k\tau}$

$\Rightarrow 17 = 7e^{k\tau}$

$\Rightarrow e^{k\tau} = \dfrac{17}{7}$

$\Rightarrow \tau = (\frac{1}{k})\, ln(\frac{17}{7}) \approx$ 6.38 hours

Therefore, the estimated time of death is approximately 6 hours 22 minutes before 3:45 pm, which is around 9:22 am on the same day.

Problem 2.8.6.

A thermometer which has been at 70 F inside a house is placed outside where the air is at 10 F.3 min later it is found that thermometer is at 25 F .Find the thermometer reading after 6 minutes?

Problem 2.8.7.

A cup with water at 45 ^{o}C is placed in the cooler held at 5^{o}C .If after 2 min the water temperature is 25 ^{o}C, when will the water temperature be 15^{o} C?

Problem 2.8.8.

A hard-boiled egg at temperature 100^{o} C is placed in 15^{o} C water to cool.5 Minutes later the temp of egg is 55^{o} C. When will the egg be 25 ^{o}C?

2.9. CONCLUSION:

Equations are not merely collections of numbers and symbols; they are the poetry of the universe — concise, elegant, and universal. To view the world through equations is to witness the beauty, order, and unity hidden beneath the apparent complexity of nature. Each equation condenses a profound truth about reality, revealing that the universe itself is a grand mathematical symphony.

In this work, one application of Differential Equations has been explored through Newton's Law of Cooling**.** The derivation of this law demonstrates how mathematics translates a simple physical process — the cooling of a body — into a predictive and usable formula. This connection highlights the essential role of equations in engineering, physics, and daily life, where temperature control and heat transfer are crucial in fields such as mechanical design, chemical processing, and environmental studies.

By solving practical problems using Newton's Law of Cooling, we see how differential equations serve as a bridge between abstract mathematics and real-world applications. Thus, equations are not only intellectual constructs but also practical tools that shape technology, innovation, and scientific progress.
Ultimately, to embrace mathematics through equations is to recognize that every change, every interaction, and every phenomenon can be captured, studied, and understood through the universal

language of mathematics. Newton's Law of Cooling provides a practical method to model how the temperature of an object changes over time when placed in an environment of constant ambient temperature. By observing temperature values at different times, one can determine the cooling constant and predict unknown times or temperatures. This principle has wide applications in physics, engineering, and forensic science, showing how mathematics can be effectively applied to real-life problems.

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